PROBLEM 3. (22 points TOTAL)

Consider the one dimensional problem shown in the Figure to the right, consisting of two springs, a damper, and a mass. Assume gravity acts in the +y direction as shown.

Note that the damper and the spring k2 are in series.

Derive the first order state equations describing the system behavior. Clear and legible work will be eligible for partial credit. (22 points)

\[ f_{S1} = k_1 x_{S1} \]
\[ f_{b1} = b_1 v_{cl1} \]
\[ f_{S2} = k_2 x_{S2} \]

\[ v_{S1} = v_m \]
\[ v_{cl} + v_{S2} = v_m \]

Figure. Schematic for Problem 3.

\[ f_{S1} \uparrow \quad f_{S2} \uparrow \]
\[ \begin{align*}
\sum F_y &= m a_y \\
mg - f_{S1} - f_{S2} &= ma_y
\end{align*} \]

1. SU's: \( x_{S1}, x_{S2}, v_m \)
2. \( x_{S1}' = v_{S1} = \frac{v_m}{b} \)
3. \( x_{S2}' = v_{S2} = v_m - \frac{v_d}{b} = \frac{v_m - f_{S2}}{b} = \frac{v_m - \frac{k_2 x_{S2}}{b}}{b} \)
4. \( v_m' = a_m = \frac{1}{m} \left( mg - f_{S1} - f_{S2} \right) = \frac{1}{m} \left( mg - k_1 x_{S1} - k_2 x_{S2} \right) \)