EXAMPLE 1.2

A typical exterior frame wall (made up of 2 × 4 studs) of a house contains the materials shown in the table below. Let us assume an inside room temperature of 70°F and an outside air temperature of 20°F, with an exposed area of 150 ft². We are interested in determining the temperature distribution through the wall.

<table>
<thead>
<tr>
<th>Items</th>
<th>Resistance hr·ft²·°F/Btu</th>
<th>U-factor Btu/hr·ft²·°F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Outside film resistance (winter, 15-mph wind)</td>
<td>0.17</td>
<td>5.86</td>
</tr>
<tr>
<td>2. Studding, wood (1/2 × 8 lapped)</td>
<td>0.81</td>
<td>1.23</td>
</tr>
<tr>
<td>3. Sheathing (1/2 in regular)</td>
<td>1.32</td>
<td>0.76</td>
</tr>
<tr>
<td>4. Insulation batt (3 – 31/2 in)</td>
<td>11.0</td>
<td>0.091</td>
</tr>
<tr>
<td>5. Gypsum wall board (1/2 in)</td>
<td>0.45</td>
<td>2.22</td>
</tr>
<tr>
<td>6. Inside film resistance (winter)</td>
<td>0.68</td>
<td>1.47</td>
</tr>
</tbody>
</table>

Preprocessing Phase

1. **Discretize the solution domain into finite elements.**
   We will represent this problem by a model that has seven nodes and six elements, as shown in Figure 1.7.

2. **Assume a solution that approximates the behavior of an element.**
   For Example 1.2, there are two modes of heat transfer (conduction and convection) that we must first understand before we can proceed with formulating the conductance matrix and the thermal load matrix. The steady-state thermal behavior of the elements (2), (3), (4), and (5) may be modeled using Fourier’s law. When...
ME345 - Fall 2006
Frank's MathCad code to solve equations for heat transfer problem

NOTE changed the 'default' index variable to ONE - so do NOT have subscripts of zero

\[
\begin{bmatrix}
5.88 \\
1.23 \\
0.76 \\
0.091 \\
2.22 \\
1.47
\end{bmatrix} +
\]

\[
\begin{bmatrix}
-\ell_1 + \ell_2 - \ell_2 \\
\ell_2 - \ell_2 + \ell_3 - \ell_3 \\
0 - \ell_3 + \ell_4 - \ell_4 - \ell_4 \\
0 - \ell_4 + \ell_5 - \ell_5 \\
0 - \ell_5 + \ell_6 \\
0 + 150 \cdot \ell_6
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 + 150 \cdot \ell_1 \\
0 \\
0
\end{bmatrix}
\]

\[
\begin{bmatrix}
-20.59 \\
23.409 \\
27.972 \\
66.079 \\
67.641
\end{bmatrix}
\]
ME345 - Fall 2000
Rank's MathCAD code to solve equations for heat transfer problem

NOTE changed the 'default' index variable to ONE - so do NOT have subscripts of zero

\[
\begin{bmatrix}
3.88 \\
1.23 \\
0.76 \\
0.091 \\
2.22 \\
1.47
\end{bmatrix}
\]

\( u := \)

To solve in Matlab, I need to supply initial 'guesses' for the solution

\[
T_2 := 20 \quad T_3 := 20 \quad T_4 := 20 \quad T_5 := 20 \quad T_6 := 20
\]

\[
q_1 := 10 \quad q_7 := 10
\]

Given

\[
\begin{align*}
\frac{q_1}{150} &= u_1 - u_5 - T_2 \\
0 &= -u_1 + (u_1 + u_2) T_2 - u_5 T_3 \\
0 &= -u_2 T_2 + (u_2 + u_3) T_3 - u_5 T_4 \\
0 &= -u_3 T_3 + (u_3 + u_4) T_4 - u_5 T_5 \\
0 &= -u_4 T_4 + (u_4 + u_5) T_5 - u_5 T_6 \\
0 &= -u_5 T_5 + (u_5 + u_6) T_6 - u_6 T_7 - 0.70 \\
\frac{q_7}{150} &= -u_5 T_5 + u_5 T_6 + u_6 T_7
\end{align*}
\]

\[
\text{find} \begin{bmatrix} T_2, T_3, T_4, T_5, T_6, q_1, q_7 \end{bmatrix} = \begin{bmatrix} 20.59 \\
28.409 \\
27.972 \\
66.079 \\
67.641 \\
-520.161 \\
-520.161 \end{bmatrix}
\]