EXAMPLE 5: Example problem from Previous Quiz

Consider the one dimensional problem shown in the Figure to the right, consisting of two springs, a damper, and a mass. Assume gravity in the +y direction as shown.

Note that the damper and spring $k_2$ are in series.

Derive the first order state equations describing the system behavior. Clear and work will eligible for partial credit.

1) CL  $f_{s1} = k_1x_{s1}$  $f_{b1} = b v_{d1}$  $f_{s2} = k_2x_{s2}$

2) GC  $v_{s1} = v_m$

&emsp;&emsp; $v_d + v_{s2} = v_m$

3) FBD

\[ \sum F_y = ma_y = 0 \]

\[ f_d = f_{s2} \]

\[ f_{s1} \text{ up} \]

\[ f_{s2} \text{ up} \]

\[ mg \text{ down} \]

\[ \downarrow \sum F_y^+ = ma_y \]

\[ mg - f_{s1} - f_{s2} = ma_m \]

Note that the equation on the right could also be obtained due to the fact that the elements are in series.
4) SVs: \( x_{s1}, x_{s2}, v_m \)

5) \( x_{s1}' = v_{s1} = v_m \)

\[
x_{s2}' = v_{s2} = v_m - v_d = v_m - \frac{f_d}{b} = v_m - \frac{f_{s2}}{b} = v_m - \frac{k_2 x_{s2}}{b}
\]

\[
v_m' = a_m = \frac{1}{m} \left( mg - f_{s1} - f_{s2} \right) = \frac{1}{m} \left( mg - k_1 x_{s1} - k_2 x_{s2} \right)
\]