Example 2

1. CL \[ f_d = bV_d \]
2. GC \[ V_d = V_m \]
3. \[ \begin{align*} \sum F_x &= ma_m \\ -f_d &= ma_m \end{align*} \]
4. SV: \[ V_m \]
5. \[ V_m' = a_m = -\frac{f_d}{m} = -bV_d = -bV_m \]

\[ V_m' = -\frac{b}{m} V_m \]

Analytical solution: Guess \( V_m(t) = A e^{rt} \).
Follow procedure from before

\[ V_m(t) = A e^{-\frac{b}{m} t} \]

\(-\) general solution.
EXAMPLE 3
(parachute problem)

1. \( CL \quad f_d = b \cdot v_d \)

2. \( GC \quad v_d = v_m \)

3. \( \dot{v} = \frac{m g - f_d}{m} = \frac{1}{m} (mg - b \cdot v_d) \)

4. \( SV: \quad v_m \)

5. State equation(s):

\[
V_m' = a_m = \frac{1}{m} (mg - f_d) = \frac{1}{m} (mg - b \cdot v_d)
\]

\[
(\quad V_m' = \frac{1}{m} (mg - b v_m) \quad \text{appropriate state equation})
\]
Example 4

1. CL: \( f_s = k x_s \quad \text{and} \quad f_d = b u_d \)
2. GC: \( v_s = v_d \quad \text{and} \quad v_s = v_m \)
3. FBD
   \[ \begin{align*}
   v_{rope} & \quad \downarrow \quad \text{force} \\
   v_{mg} & \quad \downarrow \quad \text{weight}
   \end{align*} \]
   \[ \begin{align*}
   \sum F &= ma \\
   -f_{rope} + mg &= ma_m
   \end{align*} \]
   \[ f_{rope} - f_s - f_d = 0 \]
4. SV: \( x_s, v_m \) (here we have two state variables... need two state equations)
5. Solve for each state equation separately:
   \[ x_s' = v_s = v_m \quad \checkmark \]
   \[ v_m' = a_m = \frac{1}{m} (f_{rope} + mg) = \frac{1}{m} [ (f_s + f_d) + mg ] \]
   \[ = \frac{1}{m} [ -k x_s - bu_d + mg ] \]
   \[ \checkmark \]
Thus the state equations are:

\[
\begin{align*}
    x_s' &= v_m \\
    v_m' &= \frac{1}{m} (mg - kx_s - bv_m)
\end{align*}
\]

Can write in matrix form as

\[\begin{bmatrix}
x_s' \\
v_m'
\end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m}\end{bmatrix} \begin{bmatrix} x_s \\
v_m\end{bmatrix} + \begin{bmatrix} 0 \\
1 \end{bmatrix} q
\]

Can also write in second order form:

\[\begin{align*}
x_s'' &= ? \\
x_s'' &= \frac{d}{dt}(x_s') = \frac{d}{dt}(v_m') \\
x_s'' &= v_m' = \frac{1}{m} (mg - kx_s - bv_m)
\end{align*}\]

Now \(v_m\) is not a SV!

\[\begin{align*}
x_s''' &= \frac{1}{m} (mg - kx_s - bx_s')
\end{align*}\]

This is all lower order derivative.

Can re-write in standard form:

\[x_s'' + \frac{b}{m} x_s' + \frac{k}{m} x_s = g\]

In math terms, this is "non-homogeneous."
EXAMPLE 5

PROBLEM 3. (22 points TOTAL)

Consider the one dimensional problem shown in the Figure to the right, consisting of two springs, a damper, and a mass. Assume gravity acts in the +y direction as shown.

Note that the damper and the spring $k_2$ are in series.

Derive the first order state equations describing the system behavior. Clear and legible work will be eligible for partial credit. (22 points)

$$\begin{align*}
1) \quad & f_{s1} = k_1 x_{s1} \\
& f_{b1} = b v_{d1} \\
& f_{s2} = k_2 x_{s2} \\
2) \quad & v_{s1} = v_m \\
& v_d + v_{s2} = v_m \\
3) \quad & f_{s1} = m a_y \\
& f_{s2} = m a_y \\
& \text{(also, because in series)} \\
4) \quad & s.t.s: x_{s1}, x_{s2}, v_m \\
5) \quad & x_{s1}' = v_{s1} = v_m \\
& x_{s2}' = v_{s2} = v_m - v_d = v_m - \frac{f_d}{b} = v_m - \frac{f_{s2}}{b} = v_m - \frac{k_2 x_{s2}}{b} \\
& v_m' = a_m = \frac{1}{m} (mg - f_{s1} - f_{s2}) = \frac{1}{m} (mg - k_1 x_{s1} - k_2 x_{s2})
\end{align*}