MA 221 Homework Solutions
Due February 11, 2014

Section 4.2

37) For problem 37, find three linearly independent solutions.

\[ y''' + y'' - 6y' + 4y = 0 \]

The auxiliary equation is

\[ r^3 + r^2 - 6r + 4 = 0 \]

\( r = 1 \) is clearly a root. So \( r^3 + r^2 - 6r + 4 = (r-1)(r^2 + 2r - 4) = 0 \)

Thus

\[ r = 1 \quad \text{and} \quad r = \frac{-2 \pm \sqrt{4 - 4(1)(-4)}}{2} = -1 \pm \sqrt{5} \]

Hence

\[ y(t) = c_1 e^t + c_2 e^{(-1+\sqrt{5})t} + c_3 e^{(-1-\sqrt{5})t} \]

For problem 43, solve the initial value problem.

43.) \[ y''' - y' = 0, y(0) = 2, y'(0) = 3, y''(0) = -1 \]

\[ r^3 - r = 0 \]

\( r = 1, -1, 0 \)

\[ y(t) = c_1 e^t + c_2 e^{-t} + c_3 e^t = c_1 + c_2 e^{-t} + c_3 e^t \]

\[ y'(t) = -c_2 e^{-t} + c_3 e^t \]

\[ y''(t) = c_2 e^t + c_3 e^t \]

\[ y(0) = c_1 + c_2 + c_3 = 2 \]

\[ y'(0) = -c_2 + c_3 = 3 \]

\[ y''(0) = c_2 + c_3 = -1 \]

\( c_1 = 3 \)

\( c_2 = -2 \)

\( c_3 = 1 \)

\( \Rightarrow y(t) = 3 - 2e^{-t} + e^t \)

Section 4.4

Find a particular solution to the differential equation.

10)
\[ y'' + 3y = -9 \]

By inspection we see that \( y_p = -3 \).

11) 
\[ y''(x) + y(x) = 2^x \]

Let \( y_p = A_0 2^x \Rightarrow y'_p = A_0 (\ln 2) 2^x \) and \( y''_p = A_0 (\ln 2)^2 2^x \). The DE implies

\[ y''_p + y_p = A_0 (\ln 2)^2 2^x + A_0 2^x = A_0 \left[ (\ln 2)^2 + 1 \right] 2^x = 2^x \]

Hence

\[ A_0 = \frac{1}{(\ln 2)^2 + 1} \]

and

\[ y_p = \frac{2^x}{(\ln 2)^2 + 1} \]

14) 
\[ 2z'' + z = 9e^{2t} \]

The auxiliary equation for the homogeneous equation is \( p(r) = 2r^2 + 1 = 0 \). Thus \( e^{2t} \) is not a homogeneous solution. Hence the formula

\[ z_p = \frac{Ae^{\alpha t}}{p(\alpha)} \]

and \( A = 9, \alpha = 2 \) so

\[ z_p = \frac{9e^{2t}}{9} = e^{2t} \]