In problems 1, 3, 5 and 7, the auxiliary equation for the given differential equation has complex roots. Find a general solution.

1.) \( y'' + 9y = 0 \)

Auxiliary equation
\[ r^2 + 9 = 0 \]
\[ r = \pm 3i \]
\[ \alpha = 0, \beta = 3 \]
\[ y(t) = c_1 e^{0t} \cos 3t + c_2 e^{0t} \sin 3t \]
\[ y(t) = c_1 \cos 3t + c_2 \sin 3t \]

2.) \( z'' - 6z' + 10z = 0 \)

Auxiliary equation
\[ r^2 - 6r + 10 = 0 \]
\[ r = 3 \pm i \]
\[ \alpha = 3, \beta = 1 \]
\[ z = c_1 e^{3t} \cos t + c_2 e^{3t} \sin t \]

5.) \( w'' + 4w' + 6w = 0 \)

Auxiliary equation
\[ r^2 + 4r + 6 = 0 \]
Using the quadratic equation \( r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)
\[ r = \frac{-4 \pm \sqrt{16 - 24}}{2} = -2 \pm \sqrt{2} i \]
\[ \alpha = -2, \beta = \sqrt{2} \]
\[ w(t) = c_1 e^{-2t} \cos \sqrt{2} t + c_2 e^{-2t} \sin \sqrt{2} t \]

7.) \( 4y'' - 4y' + 26y = 0 \)

Auxiliary equation
\[ 2r^2 - 2r + 13 = 0 \]
\[ r = \frac{1}{2} \pm \frac{5}{2} i \]
\[ \alpha = \frac{1}{2}, \beta = \frac{5}{2} \]
\[ y(t) = c_1 e^{t/2} \cos 5t/2 + c_2 e^{t/2} \sin 5t/2 \]

In problem 17, find a general solution.

17.) \( y'' - y' + 7y = 0 \)

Auxiliary equation
\[ r^2 - r + 7 = 0 \]
\[ r = \frac{1}{2} \pm \frac{3\sqrt{3}}{2} i \]
\[ \alpha = 1/2, \beta = 3\sqrt{3}/2 \]
\[ y(t) = c_1 e^{t/2} \cos(\frac{3\sqrt{3}}{2} t) + c_2 e^{t/2} \sin(\frac{3\sqrt{3}}{2} t) \]

In problem 27, solve the given initial value problem.
\[ y''' - 4y'' + 7y' - 6y = 0, \quad y(0) = 1, \quad y'(0) = 0, \quad y''(0) = 0 \]
Auxiliary equation:
\[ r^3 - 4r^2 + 7r - 6 = 0 \]
Examining the divisors of \(-6\), that is, \(\pm 1, \pm 2, \pm 3, \pm 6\), we find that \(r = 2\) satisfies the equation. Next, we divide \[ r^3 - 4r^2 + 7r - 6 = (r-2)(r^2 - 2r + 3) \Rightarrow r = 2; r = 2 \pm \frac{\sqrt{13} - 1} {2} = 1 \pm \sqrt{2} i \]
A general solution to the differential equation is given by
\[ y(t) = c_1 e^{2t} + c_2 e^{t} \cos(\sqrt{2} t) + c_3 e^{t} \sin(\sqrt{2} t) \]
\[ y' = 2c_1 e^{2t} + c_2 e^{t} \cos(\sqrt{2} t - \sqrt{2} c_3 e^{t} \sin(\sqrt{2} t) + c_3 e^{t} \cos(\sqrt{2} t + \sqrt{2} c_3 e^{t} \cos(\sqrt{2} t) = 2c_2 e^{t} + c_1 e^{t} \cos(\sqrt{2} t - \sqrt{2} c_3 e^{t} \sin(\sqrt{2} t) + c_3 e^{t} \sin(\sqrt{2} t + \sqrt{2} c_3 e^{t} \cos(\sqrt{2} t) \]
\[ y'' = 4c_1 e^{2t} + c_2 e^{t}(-\cos(\sqrt{2} t - 2\sqrt{2} \sin(\sqrt{2} t) + c_3 e^{t}(-\sin(\sqrt{2} t + 2\sqrt{2} \cos(\sqrt{2} t) \]
\[ \Rightarrow c_1 + c_2 + c_3 = 0; \]
\[ 4c_1 - c_2 + 2\sqrt{2} c_3 = 0 \]
\[ \Rightarrow y(t) = e^{2t} - \sqrt{2} e^{t} \sin(\sqrt{2} t) \]

In problem 29b, find a general solution to the higher-order equation.
\[ y''' + 2y'' + 5y' - 26y = 0 \]
Auxiliary equation:
\[ r^3 + 2r^2 + 5r - 26 = 0 \]
Examining the divisors of \(-26\), that is, \(\pm 1, \pm 2, \pm 3, \pm 13\), we find that \(r = 2\) satisfies the equation. Next, we divide \[ r^3 + 2r^2 + 5r - 26 = (r - 2)(r^2 + 4r + 13) \Rightarrow r = 2; \quad r = -2 \pm 3i \]
A general solution to the differential equation is given by
\[ y(t) = c_1 e^{2t} + c_2 e^{-2t} \cos(3t) + c_3 e^{-2t} \sin(3t) \]