Spectrum Efficiency of Cognitive Relay Transmissions with Cooperative Diversity in Cognitive Radio Networks

Yulong Zou and Yu-Dong Yao
ECE Department, Stevens Institute of Technology, Hoboken 07030, USA
Email: {Yulong.Zou, Yu-Dong.Yao}@stevens.edu

Abstract—Cognitive radio is a promising technology that enables unlicensed users to communicate with each other over a licensed band (through spectrum holes). Typically, each cognitive transmission process consists of two required phases, i.e., the spectrum sensing and the data transmission phases. To improve the overall system performance of cognitive transmissions, we propose a cognitive relay scheme where a cognitive relay is used for both the spectrum sensing and the data transmission phases. By jointly considering the two phases, we derive an exact closed-form expression of the spectrum efficiency for the proposed scheme, which is to quantify the percentage of spectrum holes utilized by cognitive users for their successful data transmissions. Numerical results show that a significant performance improvement is obtained in terms of the spectrum efficiency by using the proposed cognitive relay transmission scheme. In addition, it is shown that a maximum spectrum efficiency can be achieved through a tradeoff in determining the time durations for the spectrum sensing and data transmission phases.

I. INTRODUCTION

Cognitive radio allows unlicensed users to utilize licensed bands in a way of opportunistic access [1]. As stated in [2] and [3], a cognitive source node requires two essential phases to complete a cognitive transmission process: 1) a spectrum sensing phase (also referred to as a spectrum hole detection phase), in which the cognitive source attempts to detect an available spectrum hole within a certain time duration; and 2) a data transmission phase, in which data is transmitted to the destination through the detected spectrum hole. The analyses of the two individual phases have been studied extensively in terms of different sensing [4] - [7] or different transmission [8] - [11] techniques. However, very little research has been conducted to investigate the two individual phases as a whole, which is significant and essential to cognitive radio since the two phases affect each other and can not be optimized in isolation [2]. In [2], we have explored the detection-and-transmission tradeoff in cognitive radio networks, where two cognitive transmission scenarios (i.e., non-relay and cognitive relay schemes) are presented along with their performance analyses.

However, the cognitive relay transmission scheme proposed in [2] considers the use of cognitive relay for data transmission only without its application to spectrum hole detection. Such a cognitive relay scheme is thus referred to as a direct-detection and relay-transmission (DDRT) scheme. As is known [6], [7], the performance of spectrum hole detection degrades greatly in fading environments and the cooperative relaying has been shown as an effective approach to combat the fading effect. To that end, in this paper, we propose a relay-detection and relay-transmission (RDRT) scheme, in which a cognitive relay is utilized for both the spectrum hole detection and the data transmission. Besides, we derive an exact closed-form expression of the spectrum efficiency for the RDRT scheme over Rayleigh fading channels to evaluate its performance.

II. SYSTEM DESCRIPTION AND MODELING

A. System Description

As shown in Fig. 1, we consider a cognitive radio network, where a cognitive source (CS) transmits data to a cognitive destination (CD) through opportunistic access, i.e., CS first attempts to detect a spectrum hole to seek an available transmission opportunity and then transmits data to CD over a detected hole. In this paper, we propose a relay-detection and relay-transmission scheme through the use of cognitive relay (CR) for both the spectrum hole detection and the data
transmission. As seen from Fig. 2, each cognitive transmission process of the proposed RDRT scheme includes two phases (i.e., the hole detection and the data transmission phases), where the parameter $\alpha$ is referred to as spectrum sensing overhead, which can be adjusted to optimize the performance of cognitive transmissions.

In the detection phase, there are two sub-phases, called signal detection sub-phase and data fusion sub-phase, respectively. Specifically, CS and CR independently detect whether or not there is a spectrum hole in the signal detection sub-phase. Then, CR forwards its detection result to CS in the subsequent sub-phase so that CS fuses the initial detection results based on a given fusion rule (such as, “AND” and “OR” rules [6]) and determine the availability of the channel. Following [6], [7], we assume that there is a common control channel (also called dedicated channel) when CR forwards its initial detection result to CS. Notice that a parameter $\beta$ is used to determine the time allocation between the two sub-phases, which is to be optimized. We do not use equal time allocation between the two sub-phases since only few bits are transmitted in the fusion sub-phase.

There are also two sub-phases in the data transmission phase. If a spectrum hole was detected earlier (in the hole detection phase), CS will start transmitting its data to both CD and CR in the first sub-phase. Then, CR forwards its detection result to CS in the second sub-phase. Specifically, if CR decodes the received signal successfully, it will notify CS and forward the decoded signal to CD; otherwise, CS, instead of CR, will repeat its signal transmission. It is noted that we use equal channel allocation between the two sub-phases of data transmission [9] - [11], which is reasonable since the two sub-phases carry the same amount of data information.

For notational convenience, let $H_p(k)$ denote, for time slot $k$, whether or not the licensed band is occupied by a primary user (PU), i.e., $H_p(k) = H_0$ represents the band being unoccupied by PU and, otherwise, $H_p(k) = H_1$. The time-bandwidth products of the licensed channel and dedicated channel are denoted by $BT$ and $B_zT_d$, respectively. In addition, we model $H_p(k)$ as a Bernoulli random variable (RV) with parameter $P_a$ (the probability of the licensed band being available for CS), i.e., $Pr(H_p(k) = H_0) = P_a$ and $Pr(H_p(k) = H_1) = 1 - P_a$. Furthermore, we assume that the primary traffic status is constant during one time slot.

### B. Signal Modeling

Now, we formulate the signal model for the proposed RDRT scheme. Note that the transmit powers of the primary user and secondary user are $P_p$ and $P_s$, respectively. In the signal detection sub-phase of time slot $k$, the signals received at CS and CR can be expressed as

$$y_s(k,1) = \sqrt{T_p}h_{ps}(k)\theta(k,1) + n_s(k,1) \quad (1)$$

and

$$y_r(k,1) = \sqrt{T_p}h_{ps}(k)\theta(k,1) + n_r(k,1) \quad (2)$$

where

$$\theta(k,1) = \begin{cases} 0, & H_p(k) = H_0 \\ x_p(k,1), & H_p(k) = H_1 \end{cases}$$

where $x_p(k,1)$ is the transmit signal of PU in the first sub-phase (i.e., signal detection sub-phase) of time slot $k$. Notice that $H_p(k) = H_0$ denotes that the channel band is unoccupied by PU and nothing is transmitted from PU, and $H_p(k) = H_1$ represents that a PU signal is transmitted. Based on the received signals as given by Eq. (1) and Eq. (2), CS and CR obtain their initial detection results, denoted by $\hat{H}_s(k,1)$ and $\hat{H}_r(k,1)$, respectively. Then, at the subsequent sub-phase, CR forwards its initial detection result to CS over its dedicated channel, i.e.,

$$y_s(k,2) = \sqrt{T_r}h_{rs}(k)\hat{H}_r(k,1) + n_s(k,2) \quad (3)$$

Hence, CS attempts to decode $\hat{H}_r(k,1)$ from Eq. (3) and the decoded result is denoted by $\hat{H}_s(k,2)$. According to the coding theorem, if no channel outage occurs, i.e., channel capacity is larger than data rate, the receiver can decode successfully; otherwise, the receiver is deemed to fail to decode no matter what encoder/decoder is adopted. Thus, we obtain

$$\hat{H}_s(k,2) = \begin{cases} \hat{H}_r(k,1), & \Theta_{rs}(k,2) = 0 \\ \text{rand}(H_0, H_1), & \Theta_{rs}(k,2) = 1 \end{cases} \quad (4)$$

where $\text{rand}(H_0, H_1)$ indicates the random selection of an element from $\{H_0, H_1\}$, $\Theta_{rs}(k,2) = 0$ denotes that no outage occurs over the channel from CR to CS, and $\Theta_{rs}(k,2) = 1$ denotes an outage event occurring over the channel from CR to CS. Finally, CS combines $\hat{H}_s(k,1)$ and $\hat{H}_s(k,2)$ by using a given fusion rule, leading to the final decision $\hat{H}_s(k)$. Throughout this paper, we consider the “AND” rule [6] and thus $\hat{H}_s(k)$ is expressed as

$$\hat{H}_s(k) = \hat{H}_s(k,1) \otimes \hat{H}_s(k,2) \quad (5)$$

where $\otimes$ represents the logic “AND” operation. Next, we address signal modeling for the data transmission phase. In the front part of the data transmission phase, i.e., the third sub-phase of time slot $k$, the signals received at CR and CD can be expressed as

$$y_r(k,3) = h_{sr}(k)\sqrt{T_s}\beta(k,3) + h_{ps}(k)\sqrt{T_p}\theta(k,3) + n_r(k,3) \quad (6)$$

and

$$y_d(k,3) = h_{sd}(k)\sqrt{T_s}\beta(k,3) + h_{pd}(k)\sqrt{T_p}\theta(k,3) + n_d(k,3) \quad (7)$$

where the definition of $\theta(k,3)$ is similar to $\theta(k,1)$ and

$$\beta(k,3) = \begin{cases} x_s(k), & \hat{H}_s(k) = H_0 \\ 0, & \hat{H}_s(k) = H_1 \end{cases}$$

where $x_s(k)$ is the transmit signals of CS. Notice that $\hat{H}_s(k) = H_0$ indicates that CS detects a spectrum hole and transmits $x_s(k)$ over the detected hole. If CR succeeds in decoding $x_s(k)$ from its received signal as given by Eq. (6),
we have case $\Theta = 1$ as described below (in an information-theoretic sense [9] - [11])

\[
\Theta = 1 : \quad \frac{1 - \alpha}{2} \log \left( 1 + \frac{|h_{sr}(k)|^2 \gamma_s |\beta(k,3)|^2}{|h_{pr}(k)|^2 \gamma_p |\theta(k,3)|^2 + 1} \right) > R
\]

(8)

Notice that case $\Theta = 1$ occurs possibly only when $\beta(k,3) = x_s(k)$; otherwise, there is no possibility of successfully decoding $x_s(k)$ at CR. In the given case $\Theta = 1$, CR will forward its correctly decoded result $x_s(k)$ to CD in the fourth sub-phase of time slot $k$. By combining the two received signal copies at CD using the maximum ratio combining (MRC) method, the signal-to-interference-and-noise ratio (SINR) of such an enhanced signal is given by

\[
\text{SINR}_d(\Theta = 1) = \frac{|h_{sd}(k)|^2 \gamma_s + |h_{rd}(k)|^2 \gamma_s}{|h_{pd}(k)|^2 \gamma_p |\theta(k,3)|^2 + |h_{pd}(k)|^2 \gamma_p |\theta(k,4)|^2 + 2}
\]

(9)

In obtaining Eq. (9), we have used $\beta(k,3) = x_s(k)$, which is true in the given case $\Theta = 1$. If CR fails to decode the transmit signal of CS, case $\Theta = 2$ is described as the following event

\[
\Theta = 2 : \quad \frac{1 - \alpha}{2} \log \left( 1 + \frac{|h_{sr}(k)|^2 \gamma_s |\beta(k,3)|^2}{|h_{pr}(k)|^2 \gamma_p |\theta(k,3)|^2 + 1} \right) < R
\]

(10)

CS, instead of CR, will repeat the transmission of $x_s(k)$ to CD in the fourth sub-phase of time slot $k$. Therefore, in the given case $\Theta = 2$, the SINR received at CD can be found as

\[
\text{SINR}_d(\Theta = 2) = \frac{|h_{sd}(k)|^2 \gamma_s |\beta(k,3)|^2 + |h_{sd}(k)|^2 \gamma_s |\beta(k,4)|^2}{|h_{pd}(k)|^2 \gamma_p |\theta(k,3)|^2 + |h_{pd}(k)|^2 \gamma_p |\theta(k,4)|^2 + 2}
\]

(11)

where the definition of $\beta(k,4)$ is similar to $\beta(k,3)$.

III. PERFORMANCE ANALYSIS OF THE RDRT SCHEME

According to the coding theorem, an outage event is considered to occur when channel capacity falls below data rate $R$. Thus, we can calculate the outage probability for the RDRT scheme as

\[
P_{\text{out}} = \text{Pr} \left\{ \frac{1 - \alpha}{2} \log (1 + \text{SINR}_d) < R \right\}
\]

(12)

where SINR$_d$ is the received SINR at CD and the factor $1/2$ in front of log$(\cdot)$ results from a half-duplex relay constraint [9], [10]. Equivalently, we can rewrite Eq. (12) as

\[
P_{\text{out}} = \text{Pr} \{ \text{SINR}_d(\Theta = 1) < \gamma_s \Delta, \Theta = 1 \}
\]

+ \text{Pr} \{ \text{SINR}_d(\Theta = 2) < \gamma_s \Delta, \Theta = 2 \}
\]

(13)

where $\Delta = [2^R(1/\alpha) - 1] \gamma_s$, SINR$_d(\Theta = 1)$ and SINR$_d(\Theta = 2)$ are given by Eq. (9) and Eq. (11), respectively. According to Eq. (8) and Eq. (9), the term Pr{SINR$_d(\Theta = 1) < \gamma_s \Delta, \Theta = 1$} at the right hand side of Eq. (13) can be expanded as

\[
P_{\text{out}} \text{Pr} \{ \text{SINR}_d(\Theta = 1) < \gamma_s \Delta, \Theta = 1 \}
\]

\[
= P_a P_{d_a} \text{Pr} \left\{ |h_{ad}(k)|^2 + |h_{rd}(k)|^2 < 2 \Delta \right\}
\]

\[
\times \text{Pr} \left\{ |h_{sr}(k)|^2 > \Delta \right\}
\]

\[
+ (1 - P_a) P_f \text{Pr} \left\{ |h_{sr}(k)|^2 - |h_{pr}(k)|^2 \gamma_p \Delta > \Delta \right\}
\]

\[
\times \text{Pr} \left\{ |h_{sd}(k)|^2 + |h_{rd}(k)|^2 - 2 |h_{pd}(k)|^2 \gamma_p \Delta < 2 \Delta \right\}
\]

(14)

where $P_a = \text{Pr} \{ H_p(k) = H_0 \}$ is the probability that there is a spectrum hole, $P_{d_a} = \text{Pr} \{ H_s(k) = H_0 | H_p(k) = H_0 \}$ and $P_{f_a} = \text{Pr} \{ H_s(k) = H_0 | H_p(k) = H_1 \}$ are the probability of correct detection and the probability of false detection of spectrum holes at CS, respectively. Similarly, based on Eq. (10) and Eq. (11), the term Pr{SINR$_d(\Theta = 2) < \gamma_s \Delta, \Theta = 2$} can be rewritten as

\[
P_{\text{out}} \text{Pr} \{ \text{SINR}_d(\Theta = 2) < \gamma_s \Delta, \Theta = 2 \}
\]

\[
= P_a P_{d_a} \text{Pr} \left\{ |h_{ad}(k)|^2 < \Delta \right\} \text{Pr} \left\{ |h_{sr}(k)|^2 < \Delta \right\}
\]

\[
+ (1 - P_a) P_f \text{Pr} \left\{ |h_{ad}(k)|^2 - |h_{pd}(k)|^2 \gamma_p \Delta < \Delta \right\}
\]

\[
\times \text{Pr} \left\{ |h_{sr}(k)|^2 < \gamma_p \Delta \right\}
\]

+ $P_a (1 - P_d) + (1 - P_a) (1 - P_f)$

(15)

Substituting $\hat{H}_s(k)$ from Eq. (5) into $P_{d_a} = \text{Pr} \{ H_s(k) = H_0 | H_p(k) = H_0 \}$ yields

\[
P_{d_a} = P_{d_a,1} \times P_{d_a,2}
\]

(16)

where $P_{d_a,1} = \text{Pr} \{ \hat{H}_s(k,1) = H_0 | H_p(k) = H_0 \}$ and $P_{d_a,2} = \text{Pr} \{ H_s(k,2) = H_0 | H_p(k) = H_0 \}$. Similarly, from Eq. (5), $P_f$ is calculated as

\[
P_f = P_{f_a,1} \times P_{f_a,2}
\]

(17)

where $P_{f_a,1} = \text{Pr} \{ \hat{H}_s(k,1) = H_0 | H_p(k) = H_1 \}$ and $P_{f_a,2} = \text{Pr} \{ H_s(k,2) = H_0 | H_p(k) = H_1 \}$. Notice that throughout this paper, the energy detector [4] is considered in evaluating the performance of the spectrum hole detection phase. From Eq. (1), we obtain

\[
P_{f_a,1} = \frac{1 - [1 - \ln(1 - P_{d_a,1})] (1 - P_{d_a,1})}{2 (1 - P_{d_a,1})}
\]

\[
= \frac{1 - 2 - \alpha \sigma_{\gamma_p}^2 \gamma_p}{\alpha \sigma_{\gamma_p}^2 \gamma_p - 2 (1 - P_{d_a,1})}
\]

(18)

Also, substituting $\hat{H}_s(k,2)$ from Eq. (4) into $P_{d_a,2} = \text{Pr} \{ H_s(k,2) = H_0 | H_p(k) = H_0 \}$ gives

\[
P_{d_a,2} = P_{d_a,1} \text{Pr} \{ \Theta_{rs}(k,2) = 0 \} + \frac{1}{2} \text{Pr} \{ \Theta_{rs}(k,2) = 1 \}
\]

(19)

Similarly, we obtain

\[
P_{f_a,2} = P_{f_a,1} \text{Pr} \{ \Theta_{rs}(k,2) = 0 \} + \frac{1}{2} \text{Pr} \{ \Theta_{rs}(k,2) = 1 \}
\]

(20)
Based on the derived outage probability, the spectrum efficiency is defined as

$$\eta = (1 - P_{\text{Out}})/P_a$$  \hspace{1cm} (21)$$

where $1 - P_{\text{Out}}$ indicates the quantity of spectrum holes that are utilized by the cognitive source (CS) for its successful data transmissions (without channel outage) and $P_a$ implies the total quantity of spectrum holes available for CS. Accordingly, the spectrum efficiency can be seen as a measure to quantify the percentage of spectrum holes utilized by the cognitive source for its successful data transmission.

IV. NUMERICAL RESULTS AND ANALYSIS

In Fig. 3, we compare the spectrum efficiency performance of the DDDT, DDRT and RDRT schemes [2]. From Fig. 3, one can observe that DDRT and RDRT scheme perform worse than DDDT in the low SNR regions. This is because that we adopt the half-duplex relay, rather than the full-duplex relay, for DDRT and RDRT, which would degrade the system performance at some extent. However, in higher SNR regions, the relay benefits achieved overtake the performance costs due to half-duplexing and thus DDRT and RDRT outperform DDDT in terms of spectrum efficiency. Also, it is seen from Fig. 3 that the RDRT’s performance is better than that of DDRT.

In Fig. 4, we depict the spectrum efficiency versus $(\alpha, \beta)$ with a three-dimensional surface plot of Eq. (21). As shown in Fig. 4, the plotted surface is convex and an optimal vector $(\alpha_{\text{opt}}, \beta_{\text{opt}})$ exists to maximize the spectrum efficiency. Based on the numerical results with the specified parameters, the optimal vector $(\alpha_{\text{opt}}, \beta_{\text{opt}})$ is given by $(\alpha_{\text{opt}} = 0.62, \beta_{\text{opt}} = 0.93)$ and the corresponding maximum spectrum efficiency is $\eta = 0.94$.

V. CONCLUSION

In this paper, we have proposed a relay-detection and relay-transmission scheme in cognitive radio networks, where a cognitive relay is used to assist a cognitive source for detecting spectrum holes and transmitting data. We have derived a closed-form expression of the spectrum efficiency for the proposed RDRT scheme over Rayleigh fading channels. Numerical results have demonstrated the performance advantage of the proposed RDRT scheme in terms of spectrum efficiency. Furthermore, it has been shown that a maximum spectrum efficiency can be achieved through a tradeoff between the hole detection phase and the data transmission phase.

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