Assignment 2

This homework is due *Friday*, September 23.

There are total 21 points in this assignment. 18 points is considered 100%. If you go over 18 points, you will get over 100% for this homework and it will count towards your course grade.

Collaboration is welcome. If you do collaborate, make sure to write/type your own paper. Your solutions should contain full proofs/calculations. Bare answers will not earn you much.

This assignment covers section 1.6 in O'Neill.

(1) (1.6.1) Let $\varphi = yzdz + dz$, $\psi = \sin z\ dx + \cos z\ dy$, $\xi = dy + zdz$. Find the standard expression (in terms of $dxdy, \dots$) for
 (a) [2pt] $\varphi \wedge \psi$, $\psi \wedge \xi$, $\xi \wedge \varphi$,
 (b) [2pt] $d\varphi$, $d\psi$, $d\xi$.

(2) (a) [2pt] (1.6.3) For any function f show that $d(df) = 0$.
 (b) [2pt] (1.6.3) For any 1-form φ show that $d(d\varphi) = 0$.
 (In this exercise you may for simplicity assume that the mentioned forms and functions are in $\mathbb{R}^3$.)

(3) [2pt] (1.6.3) Deduce from problem 2a that $d(fdg) = df \wedge dg$.

(4) (1.6.4bcd) Simplify:
 (a) [2pt] $d((f-g)(df+dg))$,
 (b) [2pt] $d(fdg \wedge gdf)$,
 (c) [2pt] $d(gfdf) + d(fdg)$.

(5) [3pt] (1.6.5) For any three 1-forms $\varphi_i = \sum_j f_{ij} dx_j$, $i = 1, 2, 3$, prove

$$\varphi_1 \wedge \varphi_2 \wedge \varphi_3 = \begin{vmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{vmatrix} dx_1 dx_2 dx_3.$$

(6) [2pt] (1.6.6) If r, θ, z are the cylindrical coordinate functions on $\mathbb{R}^3$, then $x = r\cos\theta$, $y = r\sin\theta$, $z = z$. Compute the volume element $dxdydz$ of $\mathbb{R}^3$ in cylindrical coordinates. (That is, express $dxdydz$ in terms of the functions r, θ, z, and their differentials.)