

Assignment 11

This homework is due *Thursday* Nov 21.

There are total 24 points in this assignment. 21 points is considered 100%. If you go over 21 points, you will get over 100% for this homework and it will count towards your course grade (up to 115%).

Collaboration is welcome. If you do collaborate, make sure to write/type your own paper *and give credit to your collaborators in your pledge*. Your solutions should contain full proofs. Bare answers will not earn you much.

This assignment covers sections 5.4, 6.1 in Bartle–Sherbert.

- (1) [2pt] (5.4.2) Show that function $f(x) = 1/x^2$ is uniformly continuous on $A = [1, \infty)$, but that it is not uniformly continuous on $B = (0, \infty)$.
- (2) (5.4.3) Use the Nonuniform Continuity Criterion to show that the following functions are not uniformly continuous on the given sets.
 - (a) [2pt] $f(x) = x^2$, $A = [0, \infty)$.
 - (b) [2pt] $g(x) = \sin(1/x)$, $B = (0, \infty)$.
- (3) [3pt] (5.4.6) Show that if f and g are uniformly continuous on $A \subseteq \mathbb{R}$, and if they are *both* bounded on A , then their product fg is uniformly continuous on A .
- (4) (Part of 6.1.1) Use the definition to find derivative of each of the following functions:
 - (a) [2pt] $f(x) = x^3$, $x \in \mathbb{R}$,
 - (b) [2pt] $f(x) = 1/\sqrt{x}$, $x > 0$.
 (*Hint:* You can use any of the definitions of the derivative that were discussed in class, but the shortest here probably is the limit of ratio definition.)
- (5) [2pt] (~6.1.2) Show that $f(x) = x^{1/10}$, $x \in \mathbb{R}$, is not differentiable at $x = 0$.
- (6) [3pt] (~6.1.4) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^3$ for x rational, $f(x) = 0$ for x irrational. Show that f is differentiable at $x = 0$, and find $f'(0)$. (*Hint:* Use the limit definition of derivative.)
- (7) [3pt] (6.1.7) Suppose that $f : \mathbb{R} \rightarrow \mathbb{R}$ is differentiable at $c = 0$ and that $f(c) = 0$. Show that $g(x) = |f(x)|$ is differentiable at c if and only if $f'(c) = 0$.
- (8) [3pt] (6.1.10) Let $g : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $g(x) = x^2 \sin(1/x^2)$ for $x \neq 0$, and $g(0) = 0$. Show that g is differentiable for all $x \in \mathbb{R}$. Also show that the derivative g' is not bounded on the interval $[-1, 1]$.