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Detection and Analysis of Rare Events in High-Frequency Financial Data

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Abstract

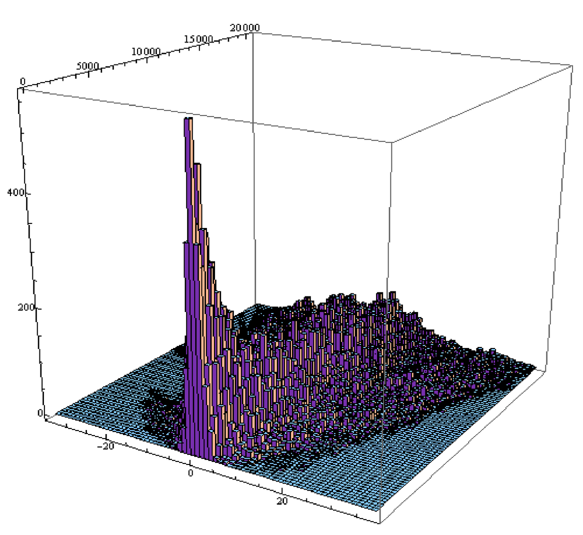
We present a methodology to detect unusual trading activity defined as high price movement with relatively little volume traded. The analysis is applied to high-frequency transactions of thousands of equities and the probability of price recovery in the proximity of these rare events is calculated. Similar results are obtained when analyzing commodities with different expiration dates. The propagation of rare events in the commodity structure and the liquidity problems are addressed.

Specific objectives of the study

- Develop a method to detect large price movements corresponding to small volume of shares traded.
- Analyze the evolution of price after these unusual events and study the probability of price recovery.
- Estimate the expected return if a trade is placed at the detected event.
- Analyze rare events propagation in futures with several expiration dates.
- Liquidity considerations using rare events & aggressor indicator

Methodology

In this analysis we use tick-by-tick data of 5,369 equities (TAQ), US and EUREX futures.



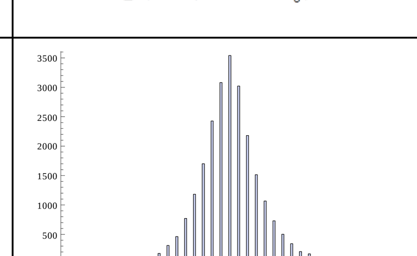
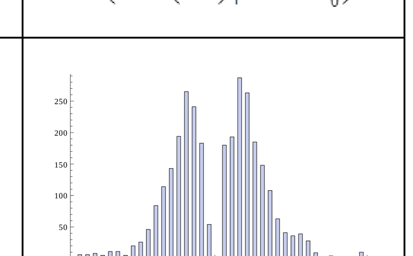
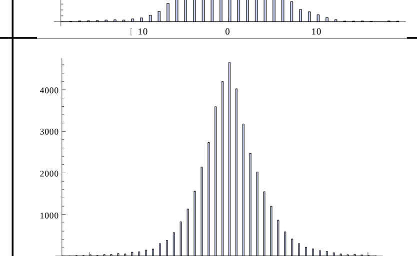
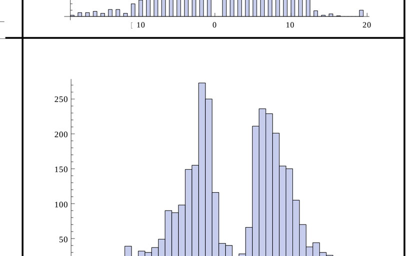
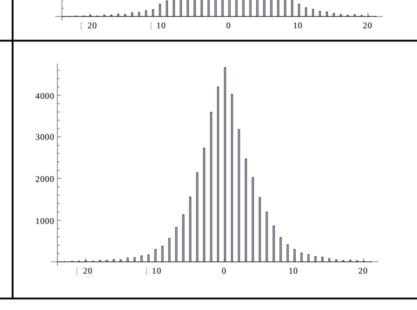
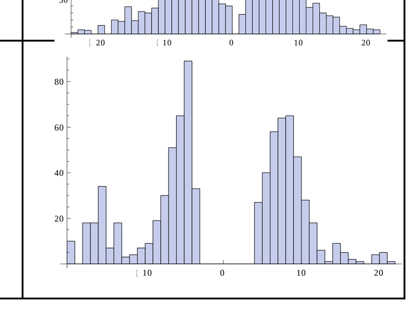
The 3D image on left represents the joint distribution of volume and price movement. The total number of pairs used for this distribution is 159,583. This is an extremely large number of data points to be calculated and analyzed for every stock and for every day.

To further simplify we retain only the maximum price movement with respect to change in volume for each window with $\Delta V < V_0$. Consequently, all the analysis that follows is based on the following distribution:

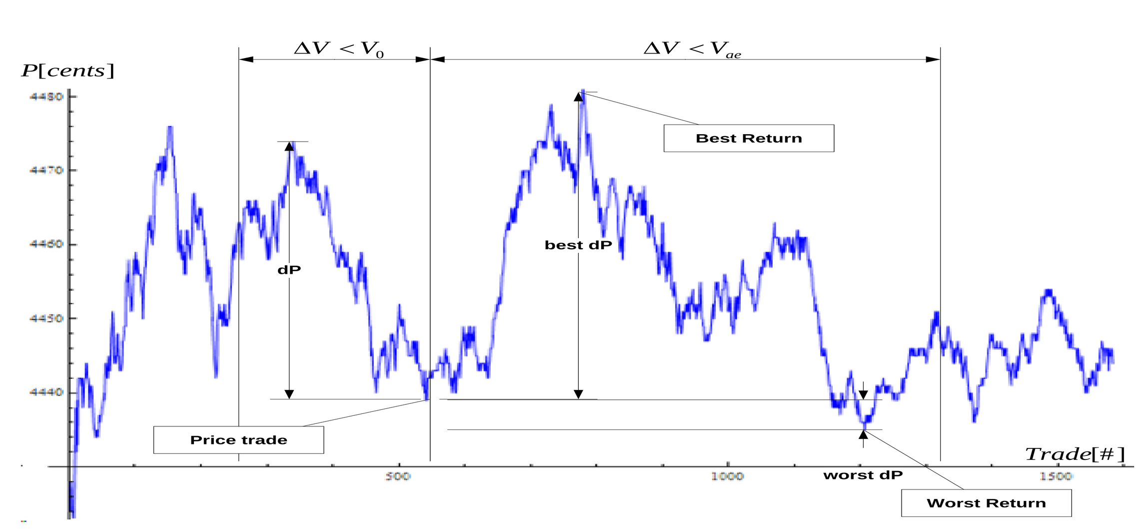
$$h(Max(\Delta P) \mid \Delta V < V_0)$$

For example for IFF stock, we obtain 1,570 observations for $V_0 = 3{,}000$ shares, 1,562 observations for $V_0 = 5{,}000$ shares, 1,544 observations for $V_0 = 10{,}000$ shares.

The proposed sampling technique generates new distributions that share similar behavior of the tails and provides a computationally feasible approach to this analysis.

$< V_0$	$g(\Delta P \mid \Delta V < V_0)$	$h(Max(\Delta P) \mid \Delta V < V_0)$
$< 3{,}000$		
$< 5{,}000$		
$< 10{,}000$		

Visualization of the quantities used in the study

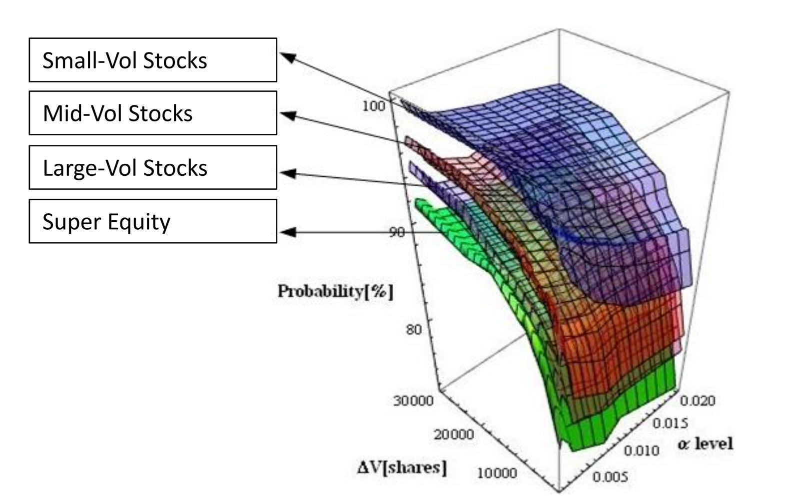


Comparative Study of Equity Groups

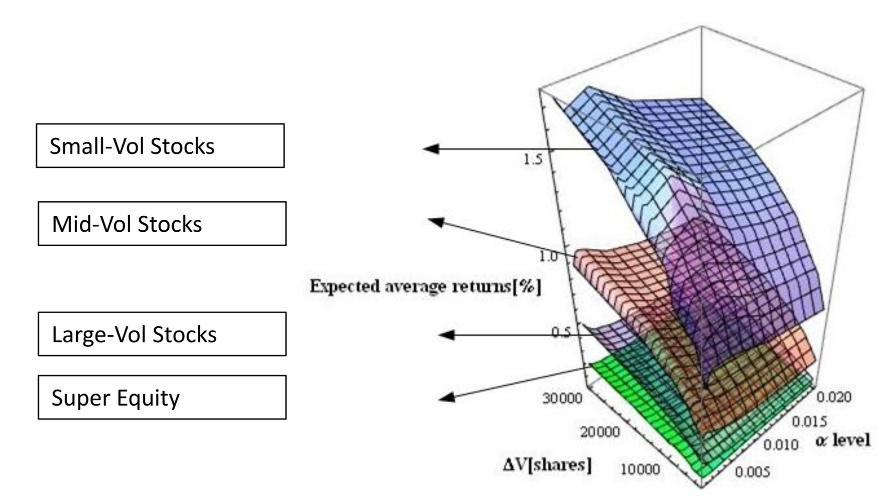
We analyze the change in price from the volume perspective. We classify stocks into classes based on the average daily traded volume. We refer to this classification as the ***multi-scale volume classification*** of the 5,369 stocks considered in this study.

Table 1: Asset Classification

	Class	Average daily volume (shares)	Number equities
1		$ADV \le 30{,}000$	1,305
2	Small-Vol Stocks	$30{,}000 < ADV \le 100{,}000$	1,088
3	Mid-Vol Stocks	$100{,}000 < ADV \le 1{,}000{,}000$	2,117
4	Large-Vol Stocks	$1{,}000{,}000 < ADV \le 10{,}000{,}000$	799
5	Super Equity	$10{,}000{,}000 < ADV$	60



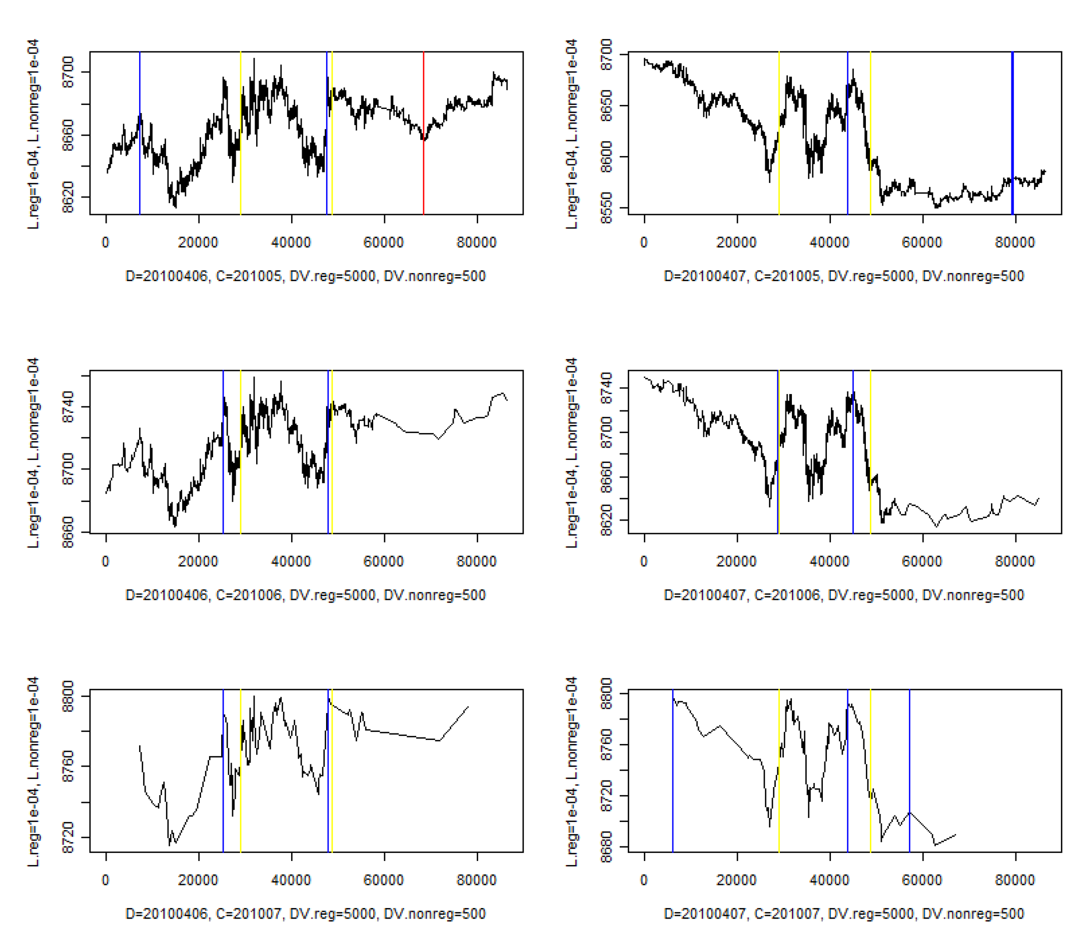
Probability of elastic behavior for each asset class



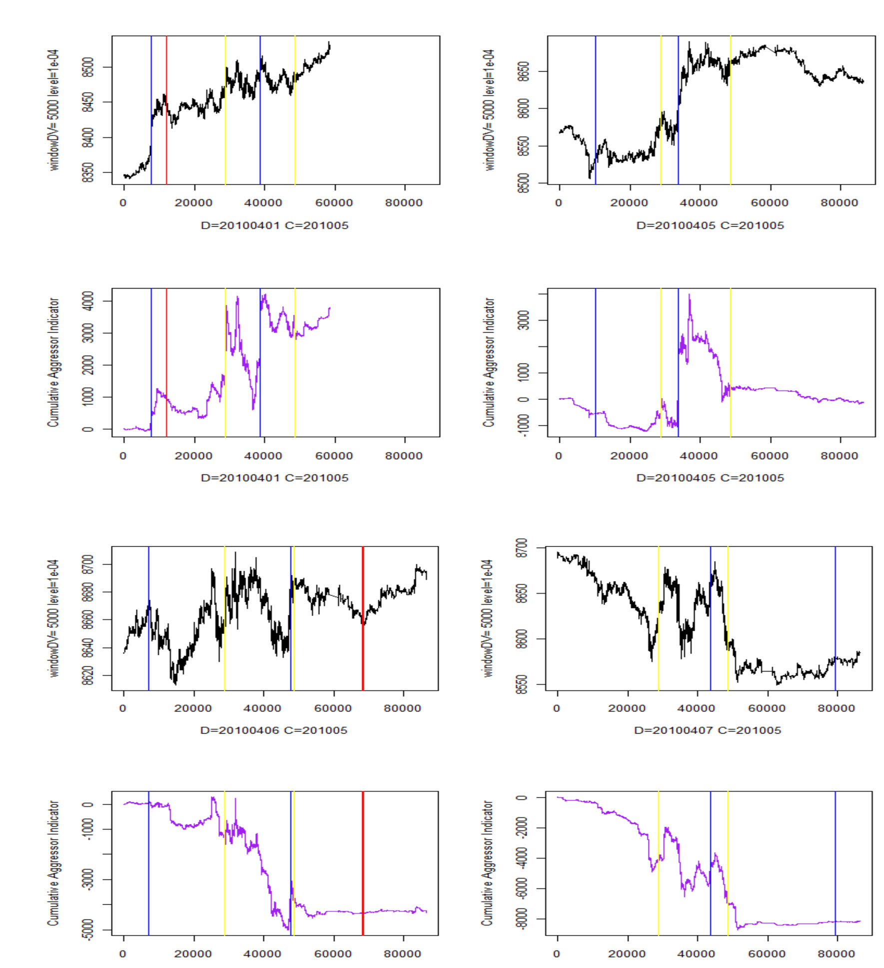
Expected returns for a simple trading rule

Application to Futures

The Rare Events Detection is implemented for several expirations dates of the same underlying futures contract. The propagation of the rare events in the future structure is influenced by the trading activity and results in asynchronous behavior.



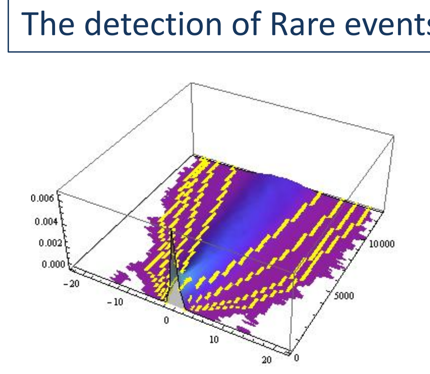
Three futures (different expirations) on the same commodity.
Blue line indicates unusual increase in price relative to the volume, Red unusual decrease. Open Pit trading hours are between the yellow lines.



The most liquid future (black line), Aggressor Indicator (purple line), for 4 different days. These two are strongly correlated, but the later is not visible to traders. Rare events are capable to detect sharp liquidity drops (bid or ask) by only analyzing the price/volume movement.

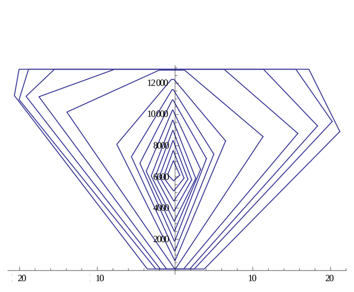
Comparison of Rare Events Detection with other, computationally expensive Algorithms

The detection of Rare events is compared with p-level points generation and zonoid trimmed regions method.



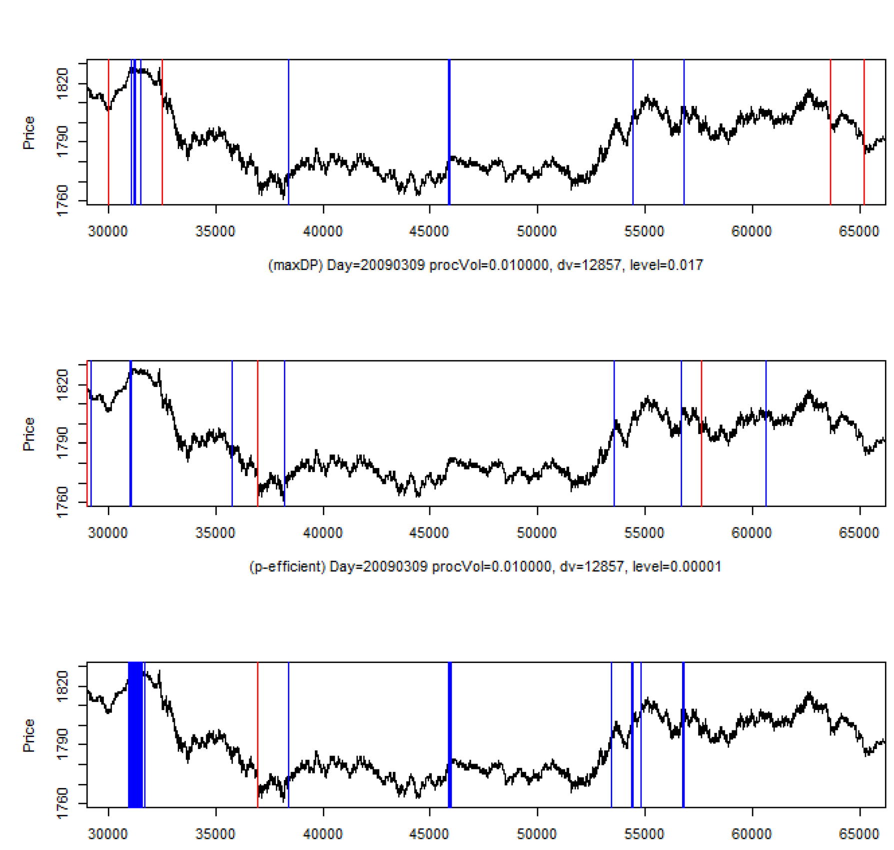
p-level points
(all points more extreme than p-efficient points)

$$D_p(x_1,\dots,x_n) = \{x_i : P(x_i \le \xi_i, i = 1,\dots,n) \le p\} \bigcup \{x_i : P(x_i \ge \upsilon_i, i = 1,\dots,n) \le 1-p\}$$



zonoid trimmed regions

$$D_\alpha(x_1,\dots,x_n) = \left\{ \frac{1}{n\alpha}\sum_{i=1}^{n}\lambda_i x_i : \sum_{i=1}^{n}\lambda_i = n\alpha, 0 \le \lambda_i \le 1 \text{ for all } i \right\}$$



Publications

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