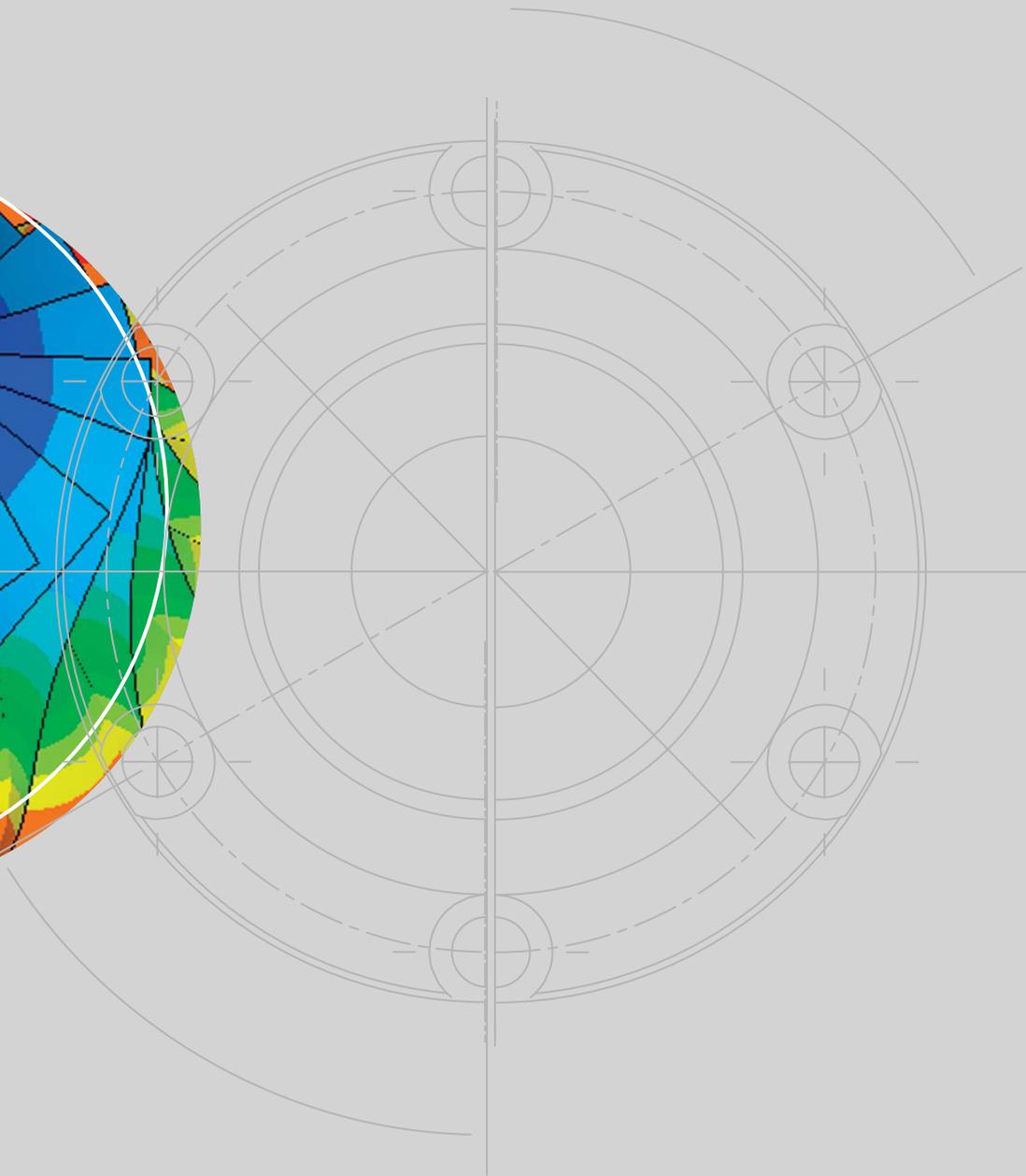
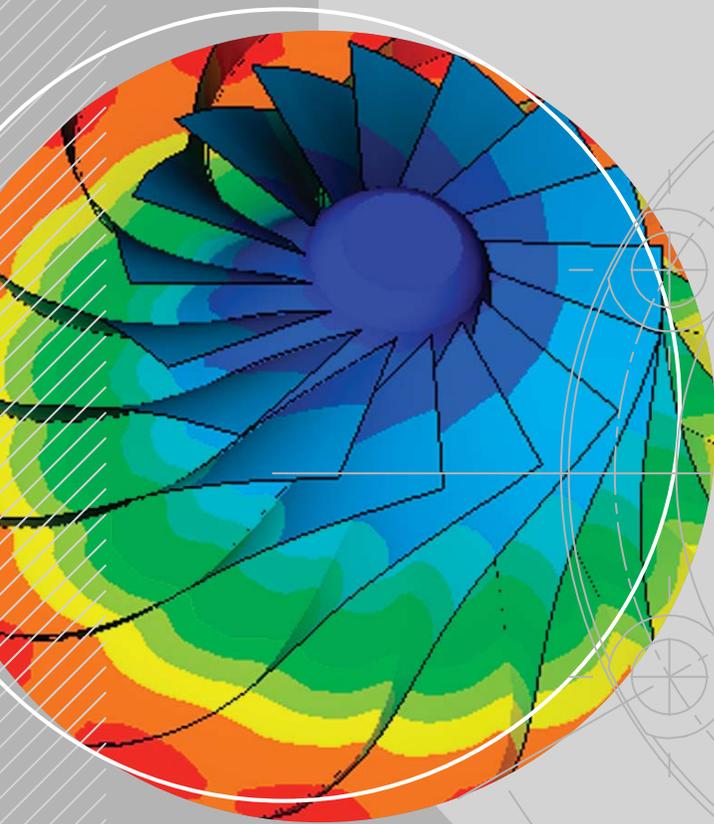


2006

COSMOS[®]

DESIGN ANALYSIS MADE SIMPLE

COSMOSWorks Verification



COSMOSWorks Verification Problems

This document contains verification problems to demonstrate the accuracy of the COSMOSWorks software in comparison to analytical results. Problems are included for linear static, frequency, buckling, thermal, and nonlinear static studies.

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Linear Static Analysis

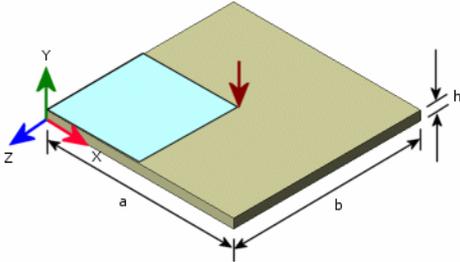
Simply Supported Rectangular Plate

Description

Calculate the deflection at the center of a simply supported isotropic plate subjected to:

- A. Concentrated load $F = 400$ lbs, and
- B. Uniform pressure $P = 1$ psi.

Dimensions of the plate are as follows: $h = 1$ in, $a = b = 40$ in.



File Name: <install_dir>\Examples\Verification\Static_1.SLDPRT

Study Type: Static.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness is 1" - Thin formulation.

Meshing Options: Draft, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use the default Global Size.

Material Properties: Modulus of elasticity = 3×10^7 psi, Poisson's ratio = 0.3.

Modeling Hints: Due to double symmetry in geometry and loads, only a quarter of the plate is modeled.

Results

The deflection at the center of the plate is calculated and compared to analytical solution.

Case	X (in)	Y (in)	Deflection at the center (UY), in	
			Theory	COSMOSWorks
Study A	20	20	0.00270230	0.00270352
Study B	20	20	0.00378327	0.00378535

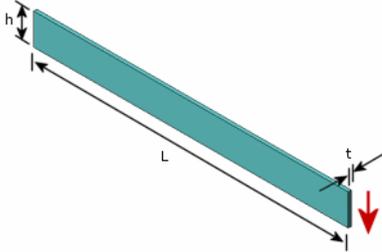
Reference

Timoshenko, S. P. and Woinowsky-Krieger, "Theory of Plates and Shells," McGraw-Hill Book Co., 2nd edition. pp. 143-120, 1962.

Deflection of a Cantilever Beam

Description

A cantilever beam is subjected to a concentrated load ($F = 1 \text{ lb}$) at the free end. Determine the deflections at the free end and the average shear stress. Dimensions of the cantilever are: $L = 10''$, $h = 1''$, $t = 0.1''$.



File Name: <install_dir>\Examples\Verification\Static_2.SLDPRT

Study Type: Static.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 0.1 in - thin shell formulation.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use the default Global Size.

Material Properties: Modulus of elasticity = 3×10^7 psi, Poisson's ratio = 0.

Results

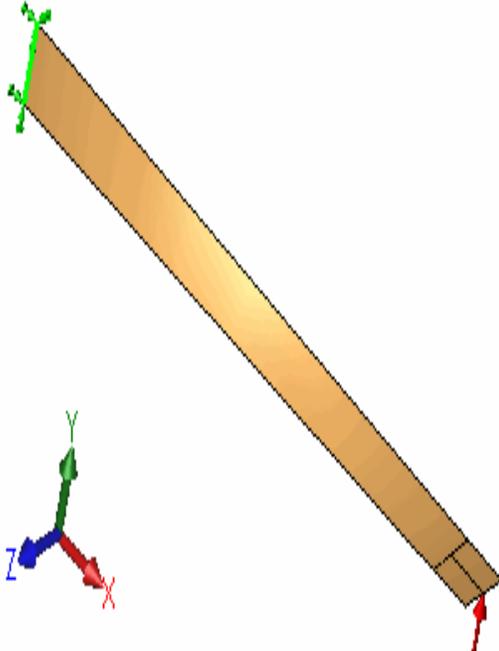
	Theory	COSMOSWorks
Deflection at free edge (UY), inch	0.001333	0.0013413
Average Shear Stress (TauXY), psi	10.0	9.8

NOTE: To generate the above results, plot the displacement in the Y direction (UY) and the shear stress (TauXY) then use the **List Selected** tool  to list the plotted result on the free edge.

Tip Displacements of a Circular Beam

Description

A circular beam fixed at one end and free at the other end is subjected to a 200 lb force as shown in the figure. Determine the deflections in the X, Y direction. Radius of curvature of the beam = 10". The beam width and thickness are 4" and 1" respectively.



File Name: <install_dir>\Examples\Verification\Static_3.SLDPRT

Study Type: Static.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 1 in - Thin formulation.

Meshing Options: Draft, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use the default Global Size.

Material Properties: Modulus of elasticity = 3×10^7 psi, Poisson's ratio = 0.

Results

The UX and UY displacements at the free edge are calculated and compared to analytical values:

	Theory	COSMOSWorks
X Deflection at free edge (UX), inch	0.712E-2	0.713E-2
Y Deflection at free edge (UY), inch	0.714E-2	0.100E-1

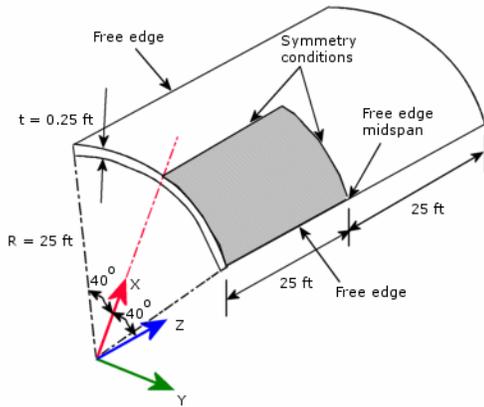
Reference

Warren C. Young, "Roark's Formulas for Stress and Strain," Sixth Edition, McGraw Hill Book Company, New York, 1989.

Cylindrical Shell Roof

Description

Determine the vertical deflections across the midspan of a shell roof under its own weight. Dimensions and boundary conditions are shown in the figure below. The radius of curvature of the shell is $R = 25$ ft.



File Name: <install_dir> \Examples\Verification\Static_4.SLDPRT

Study Type: Static.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 0.25 ft - Thin formulation.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use the default Global Size.

Material Properties: Modulus of elasticity = 3×10^6 psi, Poisson's ratio = 0, Density = 0.2083 lbs/in³.

Modeling Hints: Due to symmetry, a quarter of the shell is considered for modeling. The shell weight is applied as a gravity loading.

Results

The vertical displacement (UX) at midspan of the free edge is compared to analytical value:

	Theory	COSMOSWorks
X Deflection at midspan of the free edge (UX), ft	-0.3024	-0.3018

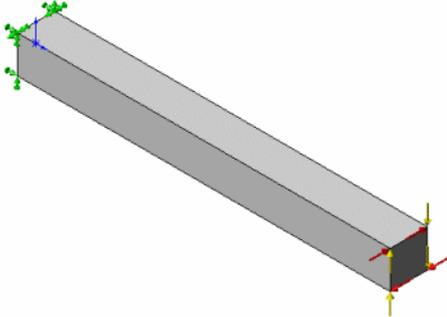
Reference

Pawsley, S. F., "The Analysis of Moderately Thick to Thin Shells by the Finite Element Method," Report No. USCEM 70-12, Dept. of Civil Engineering, University of California, 1970.

Torsion of a Square Box Beam

Description

Find the shear stress and the angle of twist for the square box beam shown in the figure. The free end is subjected to a 300 lb-in torque. The beam has a length of 1500". The beam cross section is a square with a side length of 150" and a thickness of 3".



File Name: <install_dir> \Examples\Verification\Static_5.SLDPRT

Study Type: Static.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 3 in - Thin formulation.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use a Global Size of 75 in.

Material Properties: Modulus of elasticity = 7.5 psi, Poisson's ratio = 0.3.

Results

	Theory	COSMOSWorks
Shear stress (τ), psi	0.0021365	0.00213645
Rotation (θ), radians	0.01541	0.01538

NOTE: The angle of rotation θ is calculated as: $\theta = \text{Sin}^{-1}(\text{resultant displacement of a vertex on the free cross section}/\text{distance from that vertex to the center of the cross section})$

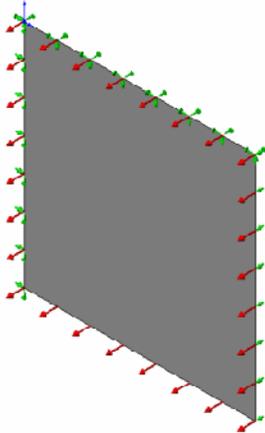
Reference

Timoshenko, S. P., and Goodier, J. N., "Theory of Elasticity," McGraw-Hill, New York, 1951, p. 299.

Effect of Transverse Shear on Maximum Deflection

Description

Find the effect of transverse shear on maximum deflection of an isotropic simply supported square plate of side (a) = 24" subjected to a constant pressure (q) = 30 psi. The plate thickness (H) varies according to the table shown below (in terms of the ratio H/a).



File Name: <install_dir> \Examples\Verification\Static_6.SLDPRT

Study Type: Static.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: The input shell thickness is 0.001" - Thin/Thick formulations.

Meshing Options: Draft, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use a Global Size of 1 in.

Material Properties: Modulus of elasticity = 3×10^7 psi, Poisson's ratio = 0.3.

Modeling Hints: Due to symmetry, only one quarter of the plate is considered. Thick shell formulation is used. A design scenario with 3 thickness values is defined to automate the processing. To run the design scenarios, right-click the study icon and select **Run Design Scenario**. To list the maximum resultant displacements, right-click the **Design Scenario Results** folder and select **Show Summary**.

Results

Thickness (inches)	Thickness ratio (H/a)	Maximum Deflection*	β^{**}	
			Reissner Theory	COSMOSWorks
1.2	0.05	0.008989	0.044936	0.046817
2.4	0.1	0.0012096	0.046659	0.050399
3.6	0.15	0.00039104	0.049533	0.054989

* The resultant displacement (not UZ) is used for the maximum deflection.

** The Reissner coefficient (β) is given by: $\beta = E H^3 W_{\max} / q a^4$, where W_{\max} is the maximum deflection.

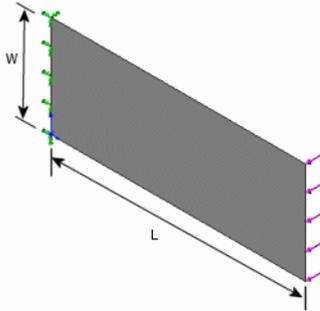
Reference

Pryor, Charles W., Jr., and Barker, R. M., "Finite Element Bending Analysis of Reissner Plates," Engineering Mechanics Division, ASCE, EM6, December, 1970, pp. 967-983.

Reactions and Deflections of a Cantilever Beam

Description

Calculate reactions and deflections of a cantilever beam subjected to an 8 lb force acting on the free end of the cantilever as shown in the figure below. The cantilever dimensions are: $L = 10''$ and $W = 4''$.



File Name: Click <install_dir> \Examples\Verification\Static_7.SLDPRT

Study Type: Static.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 1 in - Thin formulation.

Meshing Options: Draft, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use a Global Size of 1 in.

Material Properties: Modulus of elasticity = 3×10^7 psi, Poisson's ratio = 0.3.

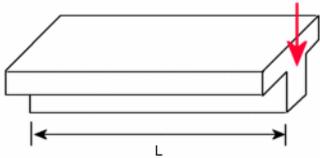
Results

	Theory	COSMOSWorks
Maximum deflection at tip, in	2.667e-4	2.570e-4
Total reaction force, lb	8	8

Bending of a T Section Beam

Description

Calculate the deflections of a cantilever T beam of length $L = 2000''$ subjected to a force of 100 lbs acting on its free end.



File Name: <install_dir>\Examples\Verification\Static_8.SLDPRT

Study Type: Static.

Mesh Type: Shell mesh using mid-surfaces.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use a Global Size of 25 in.

Material Properties: Modulus of elasticity = 1×10^{11} psi, Poisson's ratio = 0.3.

Results

The following analytical solutions are used to calculate displacements and rotations: δ (displacement) = $PL^3/3EI$, ϕ (rotation) = $PL^2/2EI$.

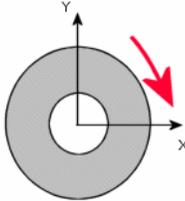
Where P is the value of the applied force, E is the cantilever's modulus of elasticity, and I is the moment of inertia of the beam cross section.

		Theory	COSMOSWorks
Free end	Y-displacement (in)	-5.546e-6	-5.43449e-6
	X-Rotation (rad)	4.159e-9	4.07129e-9

Bending of a Circular Plate with a Center Hole

Description

A circular plate with a center hole is fixed along the inner edge. The outer edge of the plate is subjected to bending by a moment $M = 10$ in-lb/in. Determine the maximum deflection and the maximum slope of the plate. The plate thickness is 0.25" and the outer and inner radii of the plate are 30" and 10" respectively. Due to symmetry of the problem, a 10° wedge is modeled. The applied moment is equivalent to applying a moment of 52.359 lb-in per 10° segment.



File Name: <install_dir>\Examples\Verification\Static_9.SLDPRT

Study Type: Static.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 0.25 in - Thin formulation.

Meshing Options: Draft, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use a Global Size of 1 in.

Material Properties: Modulus of elasticity = 3×10^7 psi, Poisson's ratio = 0.3.

Results

	Theory	COSMOSWorks
Maximum deflection (UZ), inch	0.04906	0.04787
Maximum rotation, rad	0.0045089	0.0045239

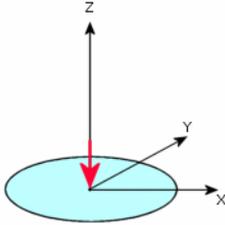
Reference

Timoshenko, S., "Strength of Materials, Part II, Advanced Theory and Problems," 3rd Edition, D. Van Nostrand Co., Inc., New York, 1956.

Circular Plate Under a Concentrated Load

Description

A circular thick plate of radius 5" is subjected to a load of 4 lb at its center. The plate is clamped at its boundary. Determine the transverse displacement along the radius r .



File Name: <install_dir> \Examples\Verification\Static_10.SLDPRT

Study Type: Static.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 2 in - Thin formulation.

Meshing Options: Draft, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use a Global Size of 1 in.

Material Properties: Modulus of elasticity = 1.09×10^6 psi, Poisson's ratio = 0.3.

Modeling Hints: Due to axial symmetry of the model, only a quarter of the plate is modeled.

Results

Distance from the plate center, r (in)	Analytical	COSMOSWorks
0.0	---	5.48E-06
1.0	3.53748E-06	3.60E-06
2.0	2.19719E-06	2.22E-06
3.0	1.14364E-06	1.16E-06
4.0	3.88628E-07	3.90E-07
5.0	0	0.00E+00

The following formula was used to generate the analytical results:

$$U(r) = \frac{PR^2}{16\pi D} [1 - (r/R)^2 - 2(r/R)^2 \ln(R/r) - 8D \ln(r/R) / (KGtR^2)]$$

where:

$U(r)$ = the displacement at distance r from the plate center

R = the radius of the plate

P = the force value

$D = \frac{EX t^3}{12(1-\nu_{XY}^2)}$, where EX and ν_{XY} are the modulus of elasticity and the Poisson's ratio of the plate and t is the thickness of the plate

$G = \frac{EX}{2(1+\nu_{XY})}$

$K = 0.8333$ (shear correction factor)

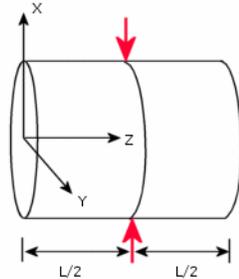
Reference

Hughes, T. J. R., Taylor, R. L., and Kanoknukulchai, W. A., "Simple and Efficient Finite Element for Plate Bending," I.J.N.M.E., 11, 1529-1543, 1977.

Test of a Pinched Cylinder with Diaphragm

Description

A cylindrical shell of thickness 3" is covered at both ends with rigid diaphragms to allow displacement only along its axial direction. At the cylinder mid-span, a load of 1 lb is applied as shown in the figure below. Determine the radial deflection at the point where the load is applied. The radius and length of the cylinder are 300" and 600" respectively.



File Name: <install_dir> \Examples\Verification\Static_11.SLDPRT

Study Type: Static.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 3" - Thin formulation.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use a Global Size of 25 in.

Material Properties: Modulus of elasticity = 3×10^6 psi, Poisson's ratio = 0.3.

Modeling Hints: Due to symmetry, only one-eighth of the cylinder is modeled. To simulate the rigid diaphragm, translations in the radial and circumferential directions and rotations about the axial direction are all set to zero.

Results

	Theory	COSMOSWorks
Radial deflection (UX), in	1.8248e-5	1.7796e-5

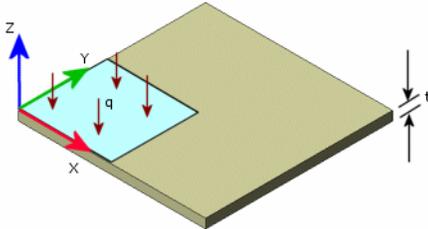
Reference

Dvorkin, E. N., and Bathe, K. J., "A Continuum Mechanics Based Four Node Shell Element for General Nonlinear Analysis," Engineering Computations, 1, 77-78, 1984.

Clamped Square Plate Under Uniform Loading

Description

Determine the deflection at the center of a square plate of side 2" and thickness (t). The plate is clamped at its boundaries and subjected to uniform pressure (q). Various span-to-thickness ratios are investigated.



File Name: <install_dir> \Examples\Verification\Static_12.SLDPRT

Study Type: Static.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Thin shell formulation.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use a Global Size of 0.25 in.

Material Properties: Modulus of elasticity = 1×10^7 psi, Poisson's ratio = 0.3.

Modeling Hints: Due to symmetry, only one quarter of the plate is modeled. A design scenario was defined to model the cases under investigation. To run the analysis, right-click the study icon and select

Run Design Scenario.

Results

Span/Thickness Ratio	Deflection (in)	
	Theory	COSMOSWorks
10 (q = 1.0 psi)	-2.7518E-6	3.2861E-6
100 (q = 1.0 psi)	-2.7518E-3	2.7688E-3
1000 (q = 0.01 psi)	-2.7518E-2	2.7613E-2

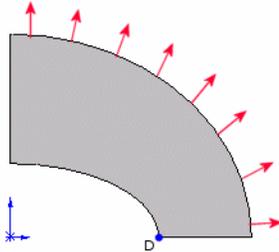
Reference

Timoshenko, S. P. and Woinowsky-Krieger, S., "Theory of Plates and Shells," 2nd Ed., McGraw Hill, New York, 1959.

Analysis of an Elliptic Membrane Under Pressure

Description

Calculate the stresses at point (D) of an elliptic membrane under a uniform outward pressure of magnitude 10 MPa.



File Name: <install_dir> \Examples\Verification\Static_13.SLPRT

Study Type: Static.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 0.1 m - Thin shell formulation.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use a Global Size of 0.14 m.

Material Properties: Modulus of elasticity = 210×10^3 MPa, Poisson's ratio = 0.3.

Modeling Hints: Due to symmetry, only a quarter of the plate is modeled.

Results

	Theory	COSMOSWorks
σ_y at point D (MPa)	92.70	92.75

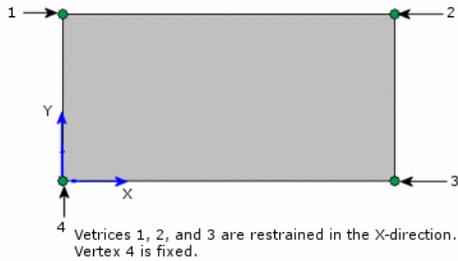
Reference

Barlow, J., and Davis, G. A. O., "Selected FE Benchmarks in Structural and Thermal Analysis," NAFEMS Rept. FEBSTA, Rev. 1, October, 1986, Test No. LG1.

Thermal Stress Analysis of a 2D Structure

Description

A rectangular plate of dimensions 1"X2"X0.1" is subjected to a uniform temperature rise of 100° F. The plate is restrained as shown in the figure. Determine the maximum Y displacement and the normal stress in the X direction.



File Name: <install_dir> \Examples\Verification\Static_14.SLDPRT

Study Type: Static.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 0.1 in - Thin shell formulation.

Meshing Options: Draft, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use a Global Size of 1 in.

Material Properties: Modulus of elasticity = 3×10^7 psi, Poisson's ratio = 0.25, Thermal expansion coefficient = $6.5 \times 10^{-6}/F$.

Results

	Theory	COSMOSWorks
Y - displacement (in)	0.001083	0.001081
SX - stress (psi)	26000	25935

Clamped Beam Subject to Prescribed Displacement

Description

Determine the end forces of a clamped beam due to a 1 inch settlement at the right end. The length of the beam is 80" and the beam has a square cross section of dimensions 2"X2".



File Name: <install_dir> \Examples\Verification\Static_15.SLDPRT

Study Type: Static.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 2 in - Thick shell formulation.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use the default Global Size.

Material Properties: Modulus of elasticity = 3×10^7 psi.

Results

	Theory	COSMOSWorks
End stress (psi)	28125	28357
End reaction (lb)	937.50	959.48

Reference

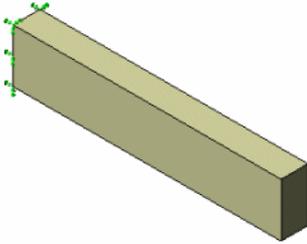
Gere, J. M. and Weaver, W. Jr., "Analysis of Framed Structures," D. Van Nostrand Co., 1965.

Bending of a Solid Beam

Description

A 10" long cantilever beam has a rectangular cross section of 1" width and 2" height. Find the deflection of the free end under the effect of the following loads:

- an end moment of 2000 in-lb, and
- a shear force of 300 lbs.



File Name: <install_dir>\Examples\Verification\Static_16.SLDPRT

Study Type: Static.

Mesh Type: Solid mesh.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use a Global Size of 2 in.

Material Properties: Modulus of elasticity = 3×10^7 psi, Poisson's ratio = 0.

Results

Y-displacement at the free end (UY), in	Theory	COSMOSWorks
End moment (Moment Study)	-0.005	-0.005007
Shear force (Force Study)	0.005	0.005089

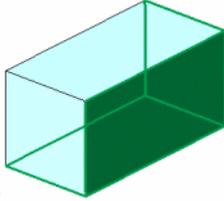
Reference

Roark, R. J., "Formulas for Stress and Strain," 4th Edition, McGraw-Hill Book Co., New York, 1965, pp. 104-106.

Thermal Stress Analysis of a 3D Structure

Description

Determine the maximum displacements of a solid rectangular block of dimensions 1"X1"X2" due to a uniform temperature rise of 100 °F. Three orthogonal faces of the block are restrained in their normal directions.



File Name: <install_dir> \Examples\Verification\Static_17.SLDPRT

Study Type: Static.

Mesh Type: Solid mesh.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use a Global Size of 1 in.

Material Properties: Modulus of elasticity = 3×10^7 psi, Poisson's ratio = 0.25, Thermal expansion coefficient = 6.5×10^{-6} /°F.

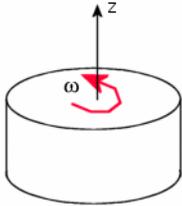
Results

Z-translation (UZ), in	Theory	COSMOSWorks
At Mid plane	$6.5e-4$ ($1 \times 100 \times 6.5e-6$)	$6.5e-4$
At Free End	$1.3e-3$ ($2 \times 100 \times 6.5e-6$)	$1.3e-3$

Rotating Solid Disk

Description

A solid disk of radius 9" and thickness 1" rotates about its center with angular velocity $\omega = 25$ rad/sec. Determine the stress distribution in the disk.



File Name: <install_dir> \Examples\Verification\Static_18.SLDPRT

Study Type: Static.

Mesh Type: Solid mesh.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use a Global Size of 0.25 in.

Material Properties: Modulus of elasticity = 3×10^7 psi, Poisson's ratio = 0.3, Density = 7.7244 lb/in³.

Modeling Hints: Due to symmetry, only a 45° wedge is analyzed.

Results

		Theory	COSMOSWorks
Location (r = 0.5")	Radial stress (SX), psi	416.37	404.3
	Tangential stress (SY), psi	416.91	415.2
Location (r = 8.5")	Radial stress (SX), psi	45.12	44.76
	Tangential stress (SY), psi	203.16	202.7

Tip: To obtain the above results, define the SX and SY plots using Axis1 as a reference then use the

Probe tool  to probe these plots at the desired locations (X = 0.5" and X = 8.5").

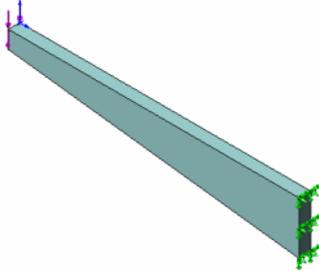
Reference

S. P. Timoshenko and J. N. Goodier, "Theory of Elasticity," McGraw-Hill, New York, 1970, p. 80.

Laterally Loaded Tapered Beam

Description

A cantilever beam of width 2" and length 50" has a depth which tapers uniformly from 3" at the tip to 9" at the wall. The cantilever beam is loaded by a 4000 lb force at the tip. Find the maximum bending stress at the mid-span of the cantilever.



File Name: <install_dir> \Examples\Verification\Static_19.SLDPRT).

Study Type: Static.

Mesh Type: Solid mesh/shell mesh using mid-surface.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use the default Global Size.

Material Properties: Modulus of elasticity = 3×10^7 psi, Poisson's ratio = 0.

Results

Bending stress at midspan (SX), psi	Solid mesh	Shell mesh using mid-surfaces
Theory	8333	8333
COSMOSWorks	8373	8380

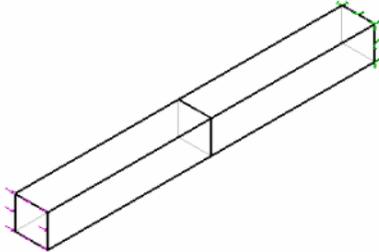
Reference

Crandall, S. H., and Dahl, N. C., "An Introduction to the Mechanics of Solids," McGraw-Hill Book Co., Inc., New York, 1959, pp. 342.

Bending of a Cantilever Beam

Description

Calculate the maximum deflection and maximum rotation (θ) of a cantilever beam loaded by a shear force of magnitude 1 lb acting on the free end of the cantilever. The length of the cantilever is 10" and the dimensions of its cross section are 1"X1". The cantilever beam is modeled as two identical cantilevers connected at their common interface with a **Bonded** contact condition.



File Name: <install_dir> \Examples\Verification\Static_20.SLDASM

Study Type: Static.

Mesh Type: Solid mesh.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use the default Global Size.

Material Properties: Modulus of elasticity = 1×10^6 psi, Poisson's ratio = 0.

Results

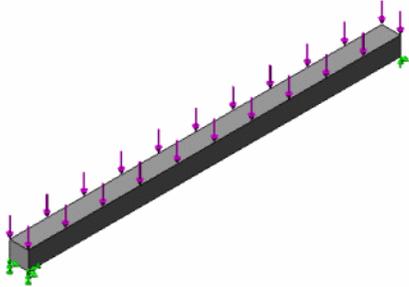
	Theory	COSMOSWorks
Displacement at the free end (UX), in	-0.004	-0.004
Rotation at the free end, rad	-0.0006	-0.0006

The following relation was used to calculate the end rotations: $\theta = 3 \delta / (2 * L)$, where δ is the deflection at the free end.

Deformation of a Uniformly Loaded Beam

Description

Determine the maximum displacement in Y-direction of a uniformly loaded beam with a fixed support at one end and a simple support at the other end. The length of the beam is 20" and the beam section is a square of side 1".



File Name: <install_dir> \Examples\Verification\Static_22.SLDPRT

Study Type: Static.

Mesh Type: Solid mesh.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use the default Global Size.

Material Properties: Modulus of elasticity = 3×10^7 psi, Poisson's ratio = 0.28.

Results

	Theory	COSMOSWorks
Maximum Displacement in the Y-direction (UY), inch	1.729e-3	1.755e-3

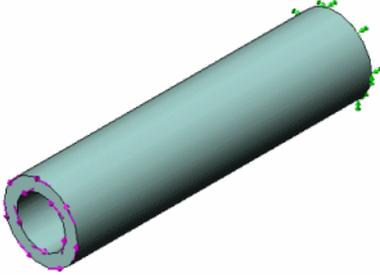
Reference

Shigley, J. E. and Mischke, C. R., "Mechanical Engineering Design", 5th Edition, Table A-9, Page 741, Item 13.

Shear Stress in Hollow Cylinder

Description

Determine the shear stress in a hollow concentric cylinder fixed at one end and subjected to a torque of 10 lb-in at the other end. The inner and outer radii of the cylinder are 1" and 2" respectively and the length of the cylinder is 12".



File Name: <install_dir>\Examples\Verification\Static_23.SLDPRT

Study Type: Static.

Mesh Type: Solid mesh.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use the default Global Size.

Material Properties: Alloy Steel.

Results

	Theory	COSMOSWorks
Maximum Shear Stress (TYZ), psi	2.353	2.377

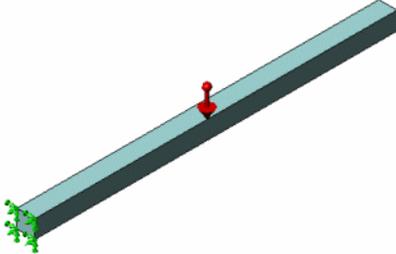
Reference

Young, Y. C. and Budynas, R. G., "Roark's Formulas for Stress and Strain", 7th Edition, Table 10.1, Page 404, Item 10.

Deflection of a Cantilever Under Gravity

Description

Determine the maximum displacement in Y-direction of a cantilever, fixed at one end, under its own weight. The length of the cantilever is 20" and its section is a square of side 1".



File Name: <install_dir> \Examples\Verification\Static_24.SLDPRT

Study Type: Static.

Mesh Type: Solid mesh.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use the default Global Size.

Material Properties: Modulus of elasticity = 3×10^7 psi, Poisson's ratio = 0.28, Density = 0.2782 lb/in³.

Results

	Theory	COSMOSWorks
Maximum Displacement in the Y-direction (UY), inch	2.2197e-3	2.221e-3

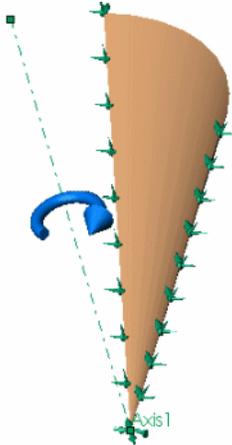
Reference

Shigley, J. E. and Mischke, C. R., "Mechanical Engineering Design", 5th Edition, Table A-9, Page 736, Item 3.

Conical Shaped Vessel Under Centrifugal Load

Description

Determine the hoop stress in a thin walled conical shaped vessel subjected to centrifugal load due to angular velocity of 5 rad/sec.



File Name: <install_dir> \Examples\Verification\Static_25.SLDPRT

Study Type: Static.

Mesh Type: Shell mesh.

Shell Parameters: Shell thickness = 0.001" - Thin formulation.

Meshing Options: Draft, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use the default Global Size.

Material Properties: Alloy Steel.

Modeling Hints: Due to symmetry, a quarter of the vessel is modeled.

Results

	Theory	COSMOSWorks
Hoop Stress (SY), psi	6.979e-4	7.121e-4

Reference

Young, Y. C. and Budynas, R. G., "Roark's Formulas for Stress and Strain", 7th Edition, Table 10.1, Page 596, Item 2f.

Tensile Stress in a Steel Bar

Description

Determine the Maximum tensile stress in a steel bar (shaped like a truncated cone with radii 1.5" and 0.5" and a height of 24") rigidly held at both ends and subjected to a temperature drop of $\Delta T = 50$ F.



File Name: <install_dir> \Examples\Verification\Static_26.SLDPRT

Study Type: Static.

Mesh Type: Solid mesh.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use the default Global Size.

Material Properties: Modulus of elasticity = 3×10^7 psi, Poisson's ratio = 0.3, Thermal expansion coefficient = $6.5 \times 10^{-6}/F$.

Modeling Hints: Use the FFEPlus or the Direct sparse solver with the **Use soft springs to stabilize model** flag on.

Results

	Theory	COSMOSWorks
Maximum Tensile Stress (SY), psi	2.9265e4	3.0719e4

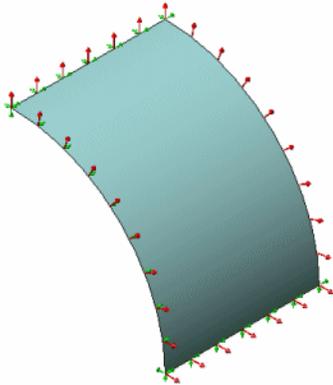
Reference

Young, Y. C. and Budynas, R. G., "Roark's Formulas for Stress and Strain", 7th Edition, Page 113, Item 2.

Hoop Stress in Thin-Walled Pressure Vessel

Description

Determine the hoop stress in a thin-walled pressure vessel under uniform radial pressure of magnitude 100 psi. The radius and thickness of the pressure vessel are 1" and 0.01" respectively.



File Name: <install_dir> \Examples\Verification\Static_28.SLDPRT

Study Type: Static.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 0.01 in - Thin formulation.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use the default Global Size.

Material Properties: Alloy Steel.

Modeling Hints: Due to symmetry, only a quarter of the model is considered.

Results

	Theory	COSMOSWorks
Hoop Stress (SY), psi	10005	9959

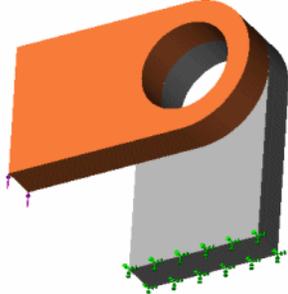
Reference

Young, Y. C. and Budynas, R. G., "Roark's Formulas for Stress and Strain", 7th Edition, Page 592, Table 13.1, Item 1b.

Pin with Rotational Spring

Description

The joint shown in the figure provides a 1000 lb.in/radian resistance to relative rotation. The rotation of the moving part is verified against the theoretical value.



File Name: <install_dir>\ Examples\ Verification\ Static_28.sldasm

Study Type: Static.

Mesh Type: Solid mesh.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Default with global contact set to **Touching Faces: Free**.

Modeling Hints: Define a pin between the two cylindrical holes with a specified rotational stiffness of $K = 1,000$ lb.in/radian.

Results

- View the UY displacement with reference to the axis of the cylindrical faces. The UY value shown in the table corresponds to the average value of UY on the cylindrical face of the moving arm.
- UY (theoretical) = (Moment/K)* Radius = $(5 \times 2 / 1000) \times 0.5 = 0.005$ in

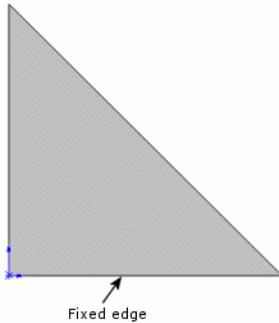
	COSMOSWorks	Theory
UY Tangential Displacement (in)	5.0027e-3	5.0000e-3

Frequency Analysis

Frequencies of a Triangular Wing

Description

Calculate the natural frequencies of a right-angle isosceles triangle wing. The equal sides of the triangle are 6" in long and the thickness is 0.034". One of the equal sides of the triangle is fixed.



File Name: <install_dir> \Examples\Verification\Frequency_1.SLDPRT

Study Type: Frequency.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 0.034 in - Thin formulation.

Meshing Options: High/Draft, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use the default Global Size.

Material Properties: Modulus of elasticity = 6.5×10^6 psi, Poisson's ratio = 0.3541, Density = 0.06411 lb/in³.

Results

Frequency No.	Theory (Hz)	COSMOSWorks (Hz)	
		Draft mesh	High quality mesh
1	55.9	55.910	55.903
2	210.9	211.20	211.24
3	293.5	292.81	293.00

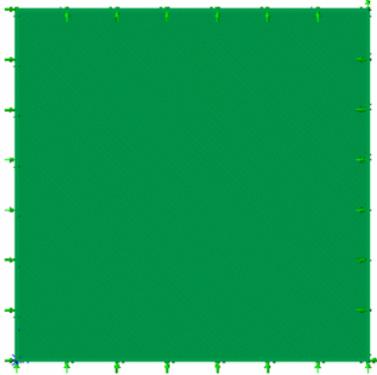
Reference

"ASME Pressure Vessel and Piping 1972 Computer Programs Verification, by I. S. Tuba and W. B. Wright, ASME Publication I-24, Problem 2.

Frequency Analysis of a Simply Supported Plate

Description

Calculate the first natural frequency of a simply supported rectangular plate of dimensions 40"X40"X1".



File Name: <install_dir> \Examples\Verification\Frequency_2.SLDPRT

Study Type: Frequency.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 1 in - Thin formulation.

Meshing Options: High/Draft, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use the default Global Size.

Material Properties: Modulus of elasticity = 3×10^7 psi, Poisson's ratio = 0.3, Density = 115.866 lb/in³.

Modeling Hints: The first mode is symmetrical and a quarter of the model could have been used.

Results

Frequency No.	Theory (Hz)	COSMOSWorks (Hz)	
		Draft mesh	High quality mesh
1	5.940	5.912	5.903

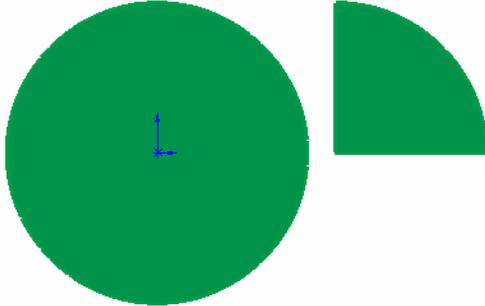
Reference

Leissa, A.W. "Vibration of Plates," NASA, sp-160, p. 44.

Frequency Analysis of a Clamped Circular Plate

Description

Obtain the first three natural frequencies of a 40"-radius, 1" thick circular plate clamped at its edge.



Three shell studies are created as follows:

Study name	Description
Full	Full model
Quarter	Quarter model
Full_Quarter	Detached full and quarter models

This verification problem illustrates the limitation of using symmetry in frequency analysis as unsymmetrical modes are not detected. The Full_Quarter study demonstrates the ability of the solvers to solve detached models simultaneously.

File Name: <install_dir> \Examples\Verification\Frequency_3.SLDPRT

Study Type: Frequency.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 1 in - Thick formulation.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use the default Global Size.

Material Properties: Modulus of elasticity = 3×10^7 psi, Poisson's ratio = 0.3, Density = 0.282072 lb/in³.

Modeling Hints: Due to the double-symmetry, only a quarter of the plate is modeled.

Results

Mode No.	COSMOSWorks (Hz)		
	Full Model	Quarter	Full and Quarter Models
1	62.199	62.226	62.224
2	129.210	211.850	62.226
3	129.23	241.52	129.33
4	211.52	421.73	129.33
5	211.54	511.47	211.85
6	241.08	538.70	211.85
7	308.71	688.57	211.87
8	308.77	843.16	241.50
9	367.61	924.58	241.52
10	367.70	950.81	309.46

Animate the modes to see that the quarter study does not detect unsymmetric modes like the 129.33 Hz mode. Also animate all the modes in the Full_Quarter study to see the sequence of mode extraction. Results agree closely with the reference.

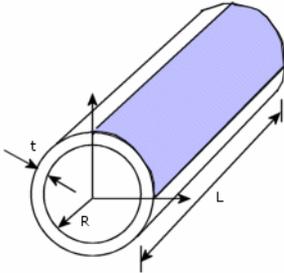
Reference

Leissa A.W., "Vibration of Plates," NASA sp-160, p. 8.

Frequencies of a Cylindrical Thin Shell

Description

Determine the first three natural frequencies of the cylindrical shell shown in the figure. The model dimensions are: $L = 6$ in, $R = 3$ in, and $t = 0.01$ in. Consider symmetric modes only.



File Name: <install_dir>\Examples\Verification\Frequency_4.SLDPRT

Study Type: Frequency.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 0.01 in - Thin formulation.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use the default Global Size.

Material Properties: Modulus of elasticity = 30,000,000 psi, Poisson's ratio = 0.3, Density = 0.282072 lb/in³.

Modeling Hints: Since symmetric modes are considered, 1/8 of the cylinder is modeled.

Results

Frequency No.	Theory (Hz)	COSMOSWorks (Hz)
1	552	538
2	736	727
3	783	768

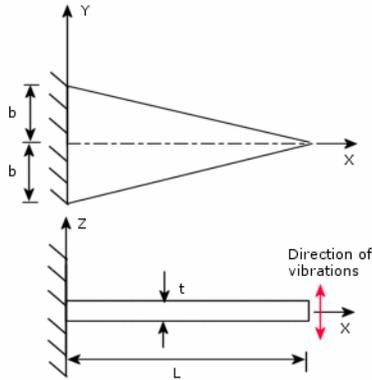
Reference

Kraus, "Thin Elastic Shells," John Wiley & Sons, Inc., p. 307.

Vibration of a Clamped Wedge

Description

Determine the fundamental frequency of the lateral vibrations of a cantilever wedge-shaped plate. The plate has the dimensions shown in the figure. The dimensions of the wedge are as follows: $t = 1$ in, $b = 2$ in, $L = 16$ in.



File Name: <install_dir>\Examples\Verification\Frequency_5.SLDPRT

Study Type: Frequency.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 1 in - Thin formulation.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use a Global Size of 0.75".

Material Properties: Modulus of elasticity = 3×10^7 psi, Density = 0.2812 lb/in³.

Results

The first in-plane natural frequency is calculated by: $f_1 = (5.315 b/2\pi L^2)*(E/3\rho)^{1/2}$.

The first and second natural frequencies are calculated using Ritz approximate method by: $f_1 = (5.319 b/2\pi L^2)*(E/3\rho)^{1/2}$ and $f_2 = (17.301 b/2\pi L^2)*(E/3\rho)^{1/2}$. Where E is the modulus of elasticity and ρ is the density.

Frequency No.	Theory (Hz)	COSMOSWorks (Hz)
1	774.55	778.36
2	2521.3	2264.6

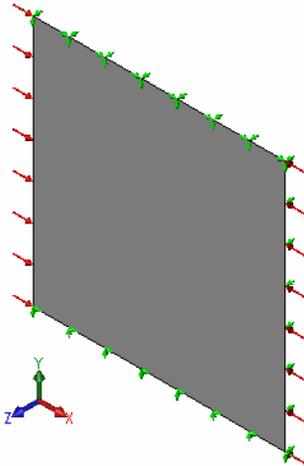
Reference

Timoshenko, S., and Young, D. H., "Vibration Problems in Engineering," 3rd Edition, D. Van Nostrand Co., Inc., New York, 1955, p. 392.

Simply Supported Rectangular Plate With Inplane Pressure

Description

Obtain the fundamental frequency of a 40"X40"X1" simply supported square plate subjected to an in-plane pressure of 33.89 psi as shown in the figure.



File Name: <install_dir>\Examples\Verification\Frequency_6.SLDPRT

Study Type: Frequency.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 1 in - Thin formulation.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use a Global Size of 7.5".

Material Properties: Modulus of elasticity = 3×10^4 psi, Poisson's ratio = 0.3, Density = 0.1158 lb/in³.

Modeling Hints: The Direct sparse solver is used with the **Use in-plane effect** flag turned on.

Results

Frequency No.	Theory (Hz)	COSMOSWorks (Hz)
1	4.20	4.19

The study **WithoutLoad** shows the fundamental frequency to be 6.095 Hz when the compressive load is not considered. Compressive loads lower the natural frequencies.

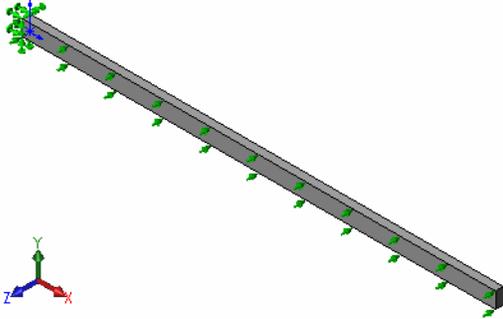
Reference

Leissa, A.W., "Vibration of Plates," NASA SP-160, p. 277.

Frequency Analysis of a Cantilever Beam

Description

Find the first two natural frequencies of a cantilever beam in the X-Y plane. The cantilever is 6" long and has a rectangular cross section of 0.1" by 0.2".



File Name: <install_dir> \Examples\Verification\Frequency_7.SLDPRT

Study Type: Frequency.

Mesh Type: Solid (Study 1) - Shell mesh using mid-surface (Study 2).

Meshing Options: High/Draft (Study 1 & Study 2), Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use the default Global Size.

Material Properties: Modulus of elasticity = 1×10^7 psi, Poisson's ratio = 0.3, Density = 0.0946239 lb/in³.

Results

		1 st Mode (Hz)	2 nd Mode (Hz)
Theory		181.17	1136.29
COSMOSWorks (Solid mesh)	High	184.10	1148.00
	Draft	202.77	1264.90
COSMOSWorks (Shell mesh)	High	181.23	1129.90
	Draft	184.77	1151.80

The following formulas were used to calculate the natural frequencies of the beam:

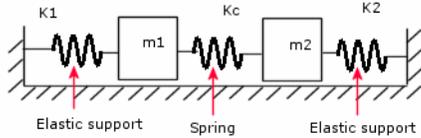
$$f_1 = (1.875)^2 (EI/ML^3)^{1/2}/2\pi \text{ and } f_2 = (4.694)^2 (EI/ML^3)^{1/2}/2\pi. \text{ Where:}$$

E: Modulus of elasticity, I: Moment of inertia of the section of the beam, M: Mass of the beam, and L: the length of the beam.

Natural Frequencies of a Two-Mass Spring System

Description

Determine the normal modes and natural frequencies of the spring-mass system shown below.



File Name: <install_dir> \Examples\Verification\Frequency_8.SLDASM

Study Type: Frequency.

Mesh Type: Solid mesh.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use a Global Size of 1 in.

Spring and Mass Properties: $m_2 = 2m_1 = 1 \text{ lb-sec}^2/\text{in}$, $k_2 = k_1 = 200 \text{ lb/in}$, $k_c = 4k_1 = 800 \text{ lb/in}$.

Modeling Hints: A spring connectors is used to simulate the spring between the two masses. Elastic supports are used to simulate the springs between the masses and the walls. Small cubes are used to model the masses.

Results

Frequency No.	Theory (Hz)	COSMOSWorks (Hz)
1	2.581	2.606
2	8.326	8.335

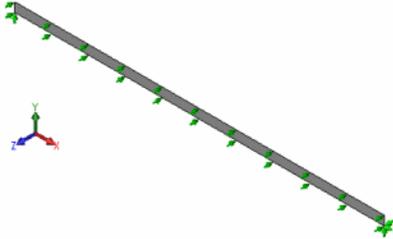
Reference

Thomson, W. T., "Vibration Theory and Application," Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 2nd printing, 1965, p. 163.

Frequency Analysis of a Simply Supported Beam

Description

Determine the natural frequencies of a simply supported 80" long beam with 2"X2" square cross-section. Consider modes in the XY plane only.



File Name: <install_dir> \Examples\Verification\Frequency_9.SLDPRT

Study Type: Frequency.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 2 in - Thick formulation.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use the default Global Size.

Material Properties: Modulus of elasticity = 30,000,000 psi, Poisson's ratio = 0.3, Density = 0.282072 lb/in³.

Results

Frequency No.	Theory (Hz)	COSMOSWorks (Hz)
1	28.78	28.68
2	115.12	113.80
3	259.00	251.79

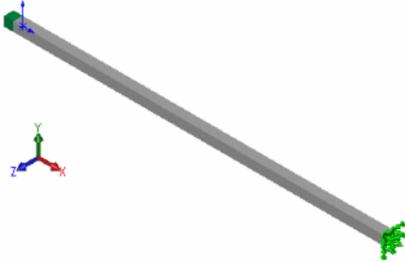
Reference

Thomson, W. T., "Vibration Theory and Applications," Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 2nd printing, 1965, p. 18.

Frequency of a Cantilever Beam with Lumped Mass

Description

A 10" long cantilever beam has a square cross-section of 0.25" X 0.25". A mass of 10 lbs is attached to the free end of the cantilever as shown in the figure. Determine the natural frequency of the system if the mass is displaced slightly and released.



File Name: <install_dir> \Examples\Verification\Frequency_10.SLDASM

Study Type: Frequency.

Mesh Type: Solid mesh.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use the default Global Size.

Material Properties: Modulus of elasticity = 30,000,000 psi, Poisson's ratio = 0, Density = 0.282072 lb/in³.

Modeling Hints: A small block of artificially high density is used to model the mass.

Results

Two studies are used: study **Nodensity** does not consider the mass of the beam (density = 0). Study **Density** considers the mass of the beam.

	Frequency, Hz
Theory	5.355
COSMOSWorks (Nodensity)	5.351
COSMOSWorks (Density)	5.341

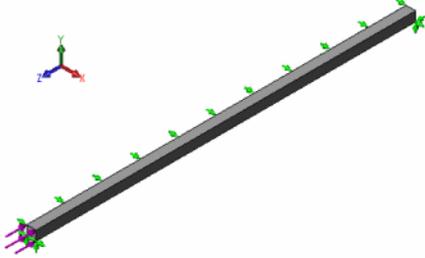
Reference

William, W. Seto, "Theory and Problems of Mechanical Vibrations," Schaum's Outline Series, McGraw-Hill Book Co., Inc., New York, 1964, p. 7.

Lateral Vibration of an Axially Loaded Bar

Description

Determine the first three natural frequencies of lateral vibration of a simply supported beam. The plate has a rectangular cross-section 2"X2" and its length is 80". An axial force of 40,000 lbs is applied at the roller support.



File Name: <install_dir>\Examples\Verification\Frequency_12.SLDPRT

Study Type: Frequency.

Mesh Type: Solid mesh.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use a Global Size of 2 in.

Material properties: Modulus of elasticity = 3×10^7 psi, Poisson's ratio = 0.3, Density = 0.2812683 lb/in³.

Modeling Hints: Use the **Sparse solver** with the **Use In-plane Effect** flag on.

Results

	f ₁ (Hz)	f ₂ (Hz)	f ₃ (Hz)
Theory	17.055	105.32	249.39
COSMOSWorks	17.068	104.68	245.19

The study **NoLoadEffect** shows the fundamental frequency to be 28.729 Hz when the load is not considered. Compressive loads lower the natural frequencies.

Reference

Timoshenko, S., and Young, D. H., "Vibration Problems in Engineering," 3rd Edition, D. Van Nostrand Co., Inc., New York, 1955, p. 374.

Natural Frequencies of a Long Bar

Description

Calculate the first three natural frequencies of a long bar fixed at one end. The bar is 50" long and has a square cross section of side 0.9".



File Name: <install_dir>\Examples\Verification\Frequency_13.SLDPRT

Study Type: Frequency.

Mesh Type: Solid mesh.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use the default Global Size.

Material properties: Modulus of elasticity = 3×10^7 psi, Poisson's ratio = 0.3, Density = 0.2835 lb/in^3 .

Results

	f_1 (Hz)	f_2 (Hz)	f_3 (Hz)
Theory	11.79	74.47	208.54
COSMOSWorks	11.77	73.62	205.62

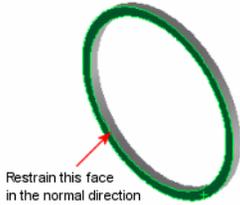
Reference

Flugge, W., "Handbook of Engineering Mechanics", McGraw Hill Book Co. Ltd., 1962, pp61-6, 61-9.

Natural Frequencies of a Ring

Description

Calculate the natural frequencies of an Alloy Steel ring with an outer diameter of 40", inner diameter of 36", and a thickness of 2 inches.



File Name: <install_dir>\ Examples\ Verification\ Frequency_14.sldprt

Study Type: Frequency.

Mesh Type: Solid mesh.

Meshing Options: High, Standard, 4 Points, Automatic transition, and Smooth surface.

Mesh Parameters: Default Global Size and Tolerance.

Number of modes: The number of modes is set to 6 in the properties of the study.

Restraints: A planar face is constrained in its normal direction.

Results:

NOTE: The first 3 modes are rigid body modes (0 frequency).

Mode shape no.	Theory (Hz)	COSMOSWorks (Hz)
4	139.87	141.11
5	139.87	141.13
6	392.15	395.76
7	392.15	395.85
8	742.79	750.01
9	742.79	750.16

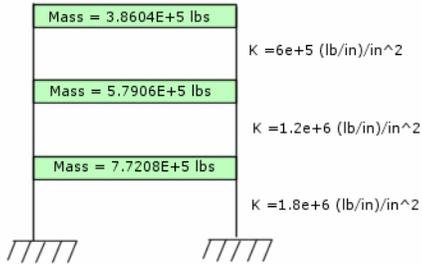
Reference

Robert D. Blevins, "Formulas for Frequency and Mode Shape," 1984.

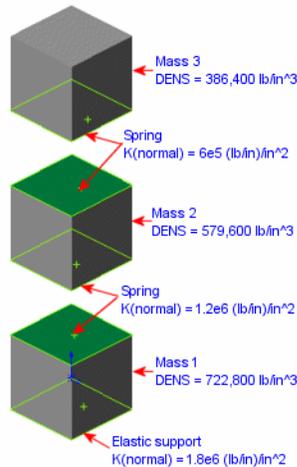
Frequency Analysis of a Building Frame

Description

A 3-story building frame is shown in the figure. The mass of the frame is lumped in the girders, with values as shown, and the columns are assumed to be weightless. Also, the girders are assumed to be rigid, so that the columns in each story act as simple lateral springs with stiffness values as indicated. It is desired to calculate the resonance frequencies of the frame.



The problem is idealized as the mass-spring system shown below:



File Name: <install_dir>\ Examples\ Verification\ Frequency_15.sldasm

Study Type: Frequency.

Mesh Type: Solid mesh.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use a global element size that corresponds to the leftmost position on the mesh slider. A larger element size can be used since only rigid body modes are of interest.

Modeling Hints: Artificial material densities are used to account for proper mass values (mass = volume*density). The **Spring** and **Elastic Support** connectors are used to define column stiffness values.

Results

	Theory	COSMOSWorks
First Mode (rad/sec)	14.5	14.4
Second Mode (rad/sec)	31.1	31.2
Third Mode (rad/sec)	46.1	46.6

Reference

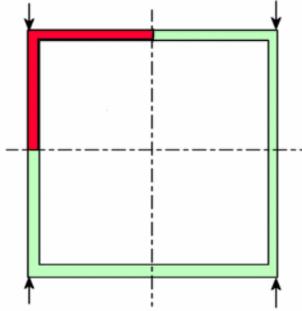
Ray W. Clough, J. Penzien, "Dynamics of Structures," McGraw-Hill Book Company, 1975, Chapter 12, pp 177.

Buckling Analysis

Buckling of a Frame

Description

Two parallel sides of a 100" square frame are loaded axially as shown. The frame has a uniform square cross section of 1"X1". Find the symmetrical buckling load in the plane of the frame.



File Name: <install_dir> \Examples\Verification\Buckling_1.SLDPRT

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 1" - Thin formulation.

Meshing Options: High/Draft, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use the default Global Size.

Material Properties: Modulus of elasticity = 1×10^7 psi.

Modeling Hint: A quarter of the frame is modeled due to double symmetry and since only the symmetrical modes are considered.

Results:

		Buckling Load (lbs)
Theory		1372
COSMOSWorks	High	1381
	Draft	1421

The theoretical value is calculated from: $P_{cr} = 16.47EI / L^2 = 1372.45$, where:

P_{cr} = critical (buckling) force, E = Modulus of elasticity, I = Moment of inertia of the cross section, L = Length of the frame side.

The critical load from COSMOSWorks is calculated as:

$$P_{cr} (\text{COSMOSWorks}) = (\text{Applied force}) * (\text{Buckling load factor})$$

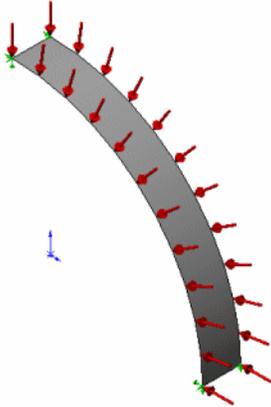
Reference:

Timoshenko, S. P. and Gere J. M., "Theory of Elastic Stability," McGraw-Hill Book Co., New York, 1961.

Buckling of a Quarter Ring Under a Radial Force

Description

A quarter ring has a radius of 5" , a width of 1", and a thickness of 0.1". The quarter ring is supported symmetrically. Find the buckling load applied radially as shown.



File Name: <install_dir> \Examples\Verification\Buckling_2.SLDPRT

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 0.1" - Thin formulation.

Meshing Options: High/Draft, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use the default Global Size.

Material Properties: Modulus of elasticity = 1×10^7 psi.

Results:

		Buckling Load (lbs)
Theory		26.66
COSMOSWorks (Shell mesh)	High	26.66
	Draft	26.67

The theoretical value is calculated from Donnell approximation as: $P_{cr} = 4EI / R^3 = 26.66$ lb/in, where:

P_{cr} = critical (buckling) force, E = Modulus of elasticity, I = Moment of inertia of the cross section of the shell, R = Radius.

The critical load from COSMOSWorks is calculated as:

$$P_{cr} \text{ (COSMOSWorks)} = (\text{Applied force}) \times (\text{Buckling load factor})$$

Reference:

Brush, D. O., and Almroth, B. O., "Buckling of Bars, Plates, and Shells," McGraw-Hill Inc., New York, 1975, p. 139.

Buckling Analysis a Cantilever Beam

Description

A cantilever beam of length 50" and a square cross section of side 1" is fixed from one end and subjected to a compressive normal force of magnitude 1 lb. Calculate the buckling load factor.



File Name: <install_dir> \Examples\Verification\Buckling_3.SLDPRT

Mesh Type: Solid mesh.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use the default Global Size.

Material Properties: Modulus of elasticity = 3×10^7 psi, Poisson's ratio = 0.

Results

	Buckling Load Factor
Theory	2467.4
COSMOSWorks	2466.7

Reference

Brush, D. O., and Almroth, B. O., "Buckling of Bars, Plates, and Shells", McGraw-Hill Inc., New York, 1975, p.22.

Buckling of a Bar Under End and Distributed Axial Loads

Description

A 20"-uniform bar of 1"X1" square cross section, made of Alloy Steel, is fixed at one end and subjected to an axial load of magnitude $P=50$ lbs at the other free end. Another nonuniform axial load that varies linearly from a value of $p=10$ lbs at the fixed end to a zero value at a distance $a=10$ " from the fixed end is also applied. Assuming that P does not cause buckling on its own, calculate the value of the product $(p*a)'$ that will cause buckling.



File Name: <install_dir>\Examples\Verification\Buckling_4.sldprt

Study Type: Buckling.

Mesh Type: Solid mesh.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Mesh Parameters: Use the default Global Size.

Modeling Hints: A coordinate system (Coordinate System1) was created at the middle point of the left edge of the bar to define the nonuniform axial force. Both axial loads are acting along plane Dir 2.

Results

The following equation calculates the product $(p*a)'$ that will cause buckling: $(p*a)' = K \pi^2 E I/L$, where $(p*a)'$ is the theoretical critical value of the product $(p*a)$, K is an empirical factor that depends on the ratio a/L and $P/(p*a)$. The value of K is looked up from table 34 of the reference cited below as 0.49. E , I , and L are the Elasticity modulus of Alloy Steel, the moment of inertia of the cross section of the bar, and the length of the bar respectively.

	COSMOSWorks	Analytical Solution
Buckling value $(p*a)$, lbs	31202	30680

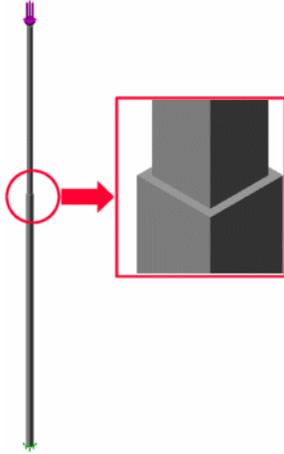
Reference

Roark and Young, "Formulas for Stress and Strains," Fifth Edition, McGraw-Hill Book Company, Chapter 14, pp 540.

Buckling of Columns with Variable Cross Sections

Description

A long column consists of two portions each with a constant cross section. Both cross sections are squares with sides 1" and 0.795" with the thicker portion in the bottom. The thicker portion is fixed at its base and a force of 100 lbs is applied to the free end of the thinner portion. The lengths of the thicker and thinner portions are 60" and 40" respectively. Calculate the buckling load of the column.



File Name: <install_dir> \Examples\Verification\Buckling_5.SLDPRT

Mesh Type: Solid mesh.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use the default Global Size.

Material Properties: Modulus of elasticity = 3×10^7 psi, Poisson's ratio = 0.3.

Results

	Critical (Buckling) Load
Theory	530.00
COSMOSWorks	531.58

The buckling load is calculated as: $P_{cr} = 2.12 EI_2/L^2$, where:

P_{cr} = critical (buckling) force, E = Modulus of elasticity, I_2 = Moment of inertia of the cross section of the thicker portion of the column, L = total length of the column.

The critical load from COSMOSWorks is calculated as:

P_{cr} (COSMOSWorks) = (Applied force)*(Buckling load factor)

Reference

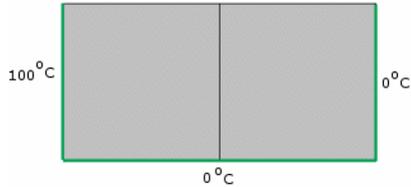
Wang, C. T., "Applied Elasticity," McGraw-Hill Book Company, New York, 1953, pp 231.

Thermal Analysis

Steady State Heat Conduction in a Plate

Description

The temperature at one of the edges of a square plate 4mX4mX0.1m is maintained at 100 °C. The temperature of the other 3 edges is maintained at 0 °C. Find the temperature at the center of the plate.



File Name: <install_dir> \ Examples \ Verification \ Thermal \ Thermal_1.SLDPRT

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 0.1 m - Thin shell formulation.

Meshing Options: High/draft (2 studies), Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use the default Global Size.

Material Properties: Thermal conductivity = 43 w/(m.°C).

Modeling Hints: One half of the plate is modeled. The plate is split into two surfaces so that the center can be easily identified.

Results:

		Temperature at Center (°C)
Theory		25
COSMOSWorks	High	25
	Draft	25

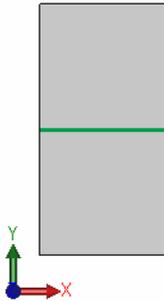
Reference:

Carslaw, H. S., and Jaeger, J. C., "Conduction of Heat in Solids," 2nd edition, Oxford University Press, 1959.

Steady State Heat Flow in an Orthotropic Plate

Description

A 1mX2mX0.1m rectangular plate is generating heat at a rate of $Q = 100 \text{ w/m}^3$. Two adjacent edges are insulated and the two other edges are dissipating heat to the atmosphere at 0°C . The plate has orthotropic properties. Determine the steady state temperature distribution in the plate.



File Name: <install_dir>\ Examples\ Verification\ Thermal\ Thermal_2.SLDPRT

Study Type: Steady state thermal analysis.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 0.1 m - Thin shell formulation.

Meshing Options: Draft, Standard, 4 Points, and Smooth surface.

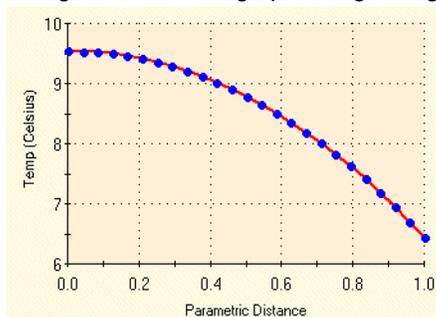
Meshing Parameters: Use the default Global Size.

Material Properties and Other Inputs: Thermal conductivity in X = $KX = 10 \text{ w/(m}\cdot^\circ\text{C)}$, Thermal conductivity in Y = $KY = 20 \text{ w/(m}\cdot^\circ\text{C)}$. Convection coefficient along the long edge = $h_1 = 10 \text{ w/m}^2\cdot^\circ\text{C}$. Convection coefficient along the short edge = $h_2 = 1 = 20 \text{ w/m}^2\cdot^\circ\text{C}$.

Modeling Hint: One half of the plate is modeled. Insulated conditions are automatic when no other condition is applied.

Results

The graph of the temperature variation in the X direction along the bottom edge of the model is shown in the figure below. The graph is in good agreement with reference results.



TIP: Plot the temperatures, then right-click on the plot, choose **List Selected**, select the bottom edge, click **Update**, and then click **Plot** to generate the graph.

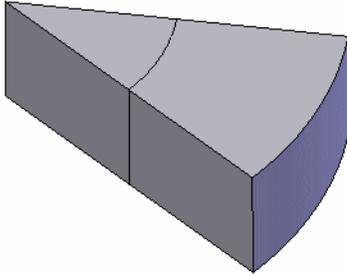
Reference

M. N. Ozisik, "Heat Conduction," Wiley, New York, 1980.

Transient Heat Conduction in a Long Cylinder

Description

A long aluminum cylinder, 50 mm in diameter is initially at 200 °C, is suddenly exposed to a convection environment at 70 °C and a convection coefficient of 525 W/m²°C. Calculate the temperature at a radius of 12.5 mm, one minute after the cylinder is exposed to the environment.



File Name: <install_dir>\Examples\Verification\Thermal\Thermal_3.SLDPRT

Study Type: Transient thermal analysis.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 10 mm - Thin shell formulation.

Meshing Options: Draft, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use the default Global Size.

Material properties: Thermal conductivity = 215 W/m°C, Density = 2700 Kg/m³, Specific heat = 936.8 J / Kg°C.

Modeling Hint: A small wedge away from the ends of the cylinder is used. The effect of convection of the circular end faces of the cylinder is ignored. The model is insensitive to thickness as the heat dissipation per unit length of the cylinder is constant. The problem is solved using solid and shell elements. Split lines are used so that the results along the 12.5 mm radius are easily evaluated.

Results

		Temperature at radius 0.0125 m	Error %
Theory		118.40	-
COSMOSWorks	Shell	119.49 (average)	1%
	Solid	119.49 (average)	1%

The listed temperature is the average temperature at a radius of 12.5 mm at 60 seconds.

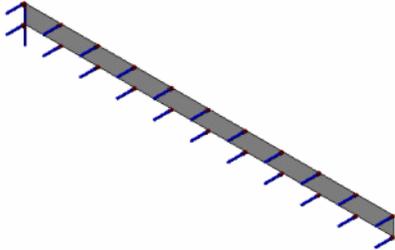
Reference

Carslaw, H. S., and Jaeger, J. C., "Conduction of Heat in Solids," 2nd edition, Oxford University Press, 1959.

Thermal Analysis with Phase Change

Description

A uniform infinite slab of liquid is considered to be initially at zero degree temperature. Suddenly, the temperature of the surface $x = 0$ is reduced to -45 F and maintained constant. Calculate the temperature distribution in the slab and the time variation (response) at $x = 1$ ".



File Name: <install_dir> \Examples\Verification\Thermal_4.SLDPRT

Study Type: Transient thermal analysis.

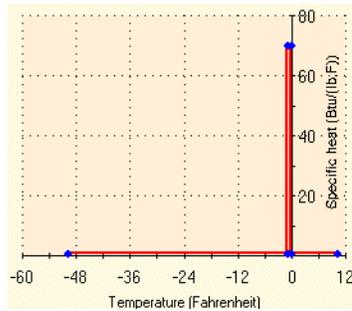
Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 1" - Thin formulation.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

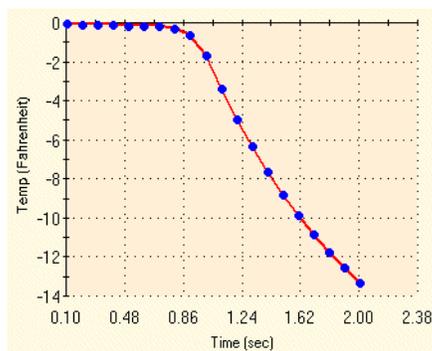
Meshing Parameters: Use a Global Size of 0.1".

Material Properties: Density = 1 lb/in³, Thermal conductivity = 1.08 BTU/(in-sec-F). The specific heat of the liquid (C) has the following temperature variation:



Modeling Hints: A zero temperature is initially applied to the model. The study properties are set to **Transient** with a **Total time** of 2 sec and 0.1 sec **Time increment**.

Results: The following response at $x = 1$ " is found to be in good agreement with the results reported in the reference mentioned below.



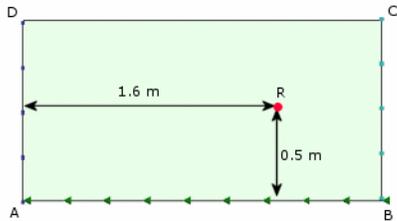
Reference

K. Morgan, R. W. Lewis, and O. C. ZienKiewicz, "An Improved Algorithm for Heat Conduction Problems with Phase Change," International Journal of Numerical Methods in Engineering, Volume 12, pp. 1191-1195, 1978.

Temperature Distribution in a Slab Exchanging Radiation with Ambient Atmosphere

Description

A slab of dimensions 2mX1mX1m has the following boundary conditions: heat flux of 10.4 W/m^2 on the face AD, convection on the face AB with heat transfer coefficient of $1.2 \text{ W/(m}^2\text{-K)}$, and a radiation boundary condition on the face BC with emissivity of 0.5. The ambient temperature for convection and radiation is $100 \text{ }^\circ\text{C}$. Calculate the steady state temperature at point R (1.6m,0.5m).



File Name: <install_dir> \Examples\Verification\Thermal_5.SLDPRT

Study Type: Steady state thermal analysis.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 1 m - Thin formulation.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use a Global Size of 0.1 m.

Material Properties: Thermal conductivity = 1 W/(m.K) .

Results

	Theory	COSMOSWorks
Temperature at point R, $^\circ\text{C}$	101.69	101.69

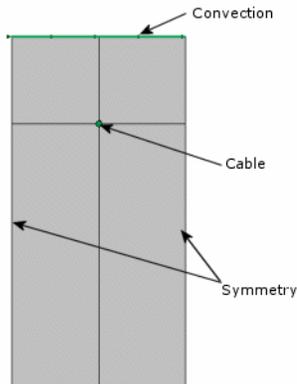
Reference

R. Siegel and J. R. Howell, "Thermal Radiation Heat Transfer,".

Heat Conduction Due to Heating Cables

Description

A series of heating cables have been placed in a conducting medium. The medium has conductivities of $KX=10 \text{ W/(m}\cdot\text{K)}$ and $KY=15 \text{ W/(m}\cdot\text{K)}$. The lower surface is bounded by an insulating medium. Assuming that each cable is a point source of 250 W, determine the temperature distribution in the medium.



File Name: <install_dir> \Examples\Verification\Thermal_6.SLDPRT

Study Type: Steady state thermal analysis.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 1 m - Thin shell formulation.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use a Global Size of 0.5 m.

Material Properties: Thermal conductivity in the x direction $KX = 10 \text{ W/(m}\cdot\text{K)}$, Thermal conductivity in the y direction $KY = 15 \text{ W/(m}\cdot\text{K)}$.

Modeling Hints: Since the cables are uniformly distributed throughout the medium, the problem can be simplified by analyzing only the section shown in the figure. Because of symmetry, the two vertical sides are insulated.

Results

	Theory	COSMOSWorks
Temperature at cable location, °C	299.1	298.8

Reference

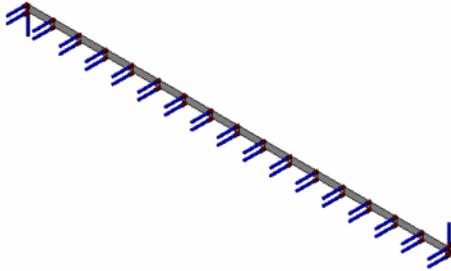
J. N. Reddy, "An Introduction to the Finite Element Method," McGraw-Hill Book Company, 1984, pp. 260.

Transient Heat Conduction in a Slab of Constant Thickness

Description

A large plate of thickness 62.8 cm is initially at a temperature of 50°C. Suddenly, both of its faces are raised to and held at 550°C. Determine:

1. The temperature at a plane 15.7 cm from the left surface, 5 sec after the sudden change in surface temperature.
2. Instantaneous heat flux at the left surface at the end of 5 seconds.



File Name: <install_dir> \Examples\Verification\Thermal_7.SLDPRT

Study Type: Transient thermal analysis.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 1 cm - Thin formulation.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use a Global Size of 3.9 cm.

Material Properties: Density = 23.2 Kg/m³, Thermal conductivity = 46.4 W/(m.K), Specific heat = 1000 J/(Kg.K).

Modeling Hints: Since the other dimensions of the plate are infinitely large, conduction occurs through the plate thickness, i.e., along the x axis. For the transient study properties, the total solution time is set to 5 sec and the time increment is set to 0.05 sec.

Results

	Theory	COSMOSWorks
Temperature at x = 15.7 cm, °C	183.9	183.4
Heat flux, W/m ²	130880	132180

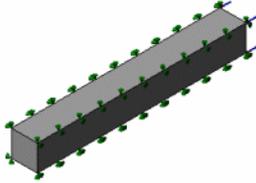
Reference

Gupta, C. P., and Prakash, R., "Engineering Heat Transfer," Nem Chand and Bros., India, 1979, pp. 155-157.

Heat Transfer from Cooling Fin

Description

A cooling fin of square cross section of area 0.0069 ft^2 , and length 0.667 ft extends from a wall maintained at temperature 100°F . The surface convection coefficient between the fin and the surrounding air is $1.929\text{e-}6 \text{ BTU}/(\text{s.in}^2.\text{F})$, the air temperature is 0°F and the tip of the fin is insulated. Determine the temperature of the fin tip.



File Name: <install_dir> \Examples\Verification\Thermal_8.SLDPRT

Study Type: Steady state thermal analysis.

Mesh Type: Solid mesh.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use a Global Size of 0.4 ".

Material Properties: Thermal conductivity = $0.000579 \text{ BTU}/(\text{s.in.F})$.

Results

	Theory	COSMOSWorks
Temperature of the fin tip, °F	68.594	68.581

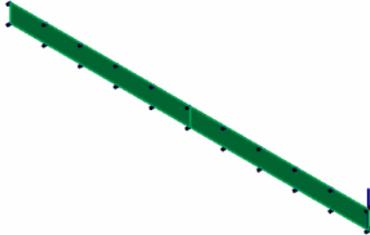
Reference

Kreith, F., "Principles of Heat Transfer," International Textbook Co., Scranton, Pennsylvania, 2nd printing, 1959.

Heat Conduction with Temperature-Dependent Conductivity

Description

Determine the temperature distribution in a slab of dimensions (2m x 0.1m x 1m) which is insulated on one face, and subjected to a constant temperature of 100°C on the other face. Assume a constant internal heat generation in the slab of $1e5 \text{ W/m}^3$ and a linear variation of thermal conductivity.



File Name: <install_dir> \ Examples \ Verification \ Thermal \ Thermal_9.SLDPRT

Study Type: Steady state thermal analysis.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 1 m - Thin shell formulation.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use a Global Size of 0.026 m.

Material Properties: Thermal conductivity has the following temperature dependency: $K(T) = 50(1+2T)$ W/(m.°C).

Results

	Theory	COSMOSWorks
Temperature at the right end, °C	118.24	118.25
Temperature at the middle of the slab, °C	113.96	113.96

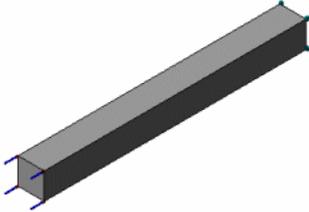
Reference

Ozisik, M., "Heat Conduction," John Wiley & Sons Inc., 1980, pp. 440-443.

Radiation from a Rod

Description

Determine the temperature distribution in a rod of dimensions 10m x 1m x 1m in which one end has a fixed temperature of 1000°C and the other end is radiating heat into an ambient atmosphere of temperature 10°C. Use a value of 1 for both radiation view factor and emissivity of the radiating surface.



File Name: <install_dir> \Examples\Verification\Thermal_10.SLDPRT

Study Type: Steady state thermal analysis.

Mesh Type: Solid mesh.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use a Global Size of 0.4 m.

Material Properties: Thermal conductivity = 45 W/(m.°C).

Results

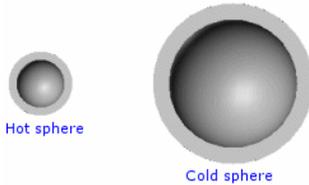
	Theory	COSMOSWorks
Average temperature of the radiating end, °C	235.77	234.34
Total heat entering the rod, W	3439	3445.5
Total heat leaving the rod, W	3439	3445.5

The analytical solution for this problem consists of solving the 1D heat transfer differential equation with the boundary conditions: at $x = 0$, $T = 1000^\circ\text{C}$ and at $x = 10$ m, $T = T_{10}$. Where T_{10} is the temperature at the radiating face. Then a heat balance equation at the radiating face is written such that the rate of heat conduction at that face equals the rate of heat lost to the environment by radiation. The resulting fourth order algebraic equation in T_{10} is solved using the Newton-Raphson method. The heat entering and leaving the rod are calculated from the gradients calculated at both ends of the rod.

Surface to Surface Radiation

Description

The thermal energy exchange through radiation between two spherical faces is compared with theoretical results. The two spheres are hollow with fixed temperatures at the inner faces. The two faces are assumed to radiate as black bodies (emissivity = 1.0). Radiation to the atmosphere is not considered (closed system).



File Name: <install_dir> \Examples\Verification\Thermal_11.sldasm

Study Type: Steady state thermal analysis.

Mesh Type: Solid mesh.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Default element size and tolerance. Identical local mesh controls are defined on the outer faces of the two spheres.

Results

- The theory calculates the Radiation View Factor (RVF) of each sphere to the other. The RVF from the smaller sphere to the larger one is calculated as **0.0459**. The theoretical thermal energy (E) received by the colder sphere, which is equal to the thermal energy radiated by the hotter sphere, is calculated from:

$$E = F_{\text{small} \rightarrow \text{large}} * \text{Area of small sphere} * \text{Emissivity} * (T_{\text{smallsphere}}^4 - T_{\text{largesphere}}^4) = 5203.6 \text{ Watts}$$

- Use the **List Selected** tool to list the resultant heat flux on each of the outer faces of the two spheres. The two values should be equal in magnitude but opposite in sign.

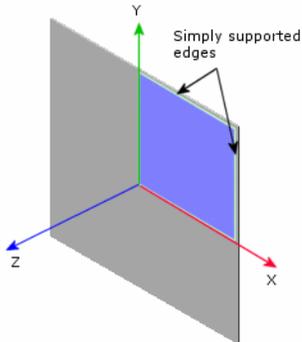
	COSMOSWorks	Analytical Solution
Thermal heat exchange at steady state (Watts)	For the large sphere = +5138.2 Watts For the small sphere = -5051.4 Watts	5203.6 Watts

Nonlinear Static Analysis

Simply Supported Rectangular Plate Under Normal Pressure

Description

Calculate the large deformation response of a simply supported square plate of side 2" and thickness 0.12" subjected to uniform pressure of magnitude 0.0625 psi.



File Name: <install_dir>\Examples\Verification\Nonlinear_Static_1.sldprt

Study Type: Nonlinear.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 0.12" - thin shell formulation.

Meshing Options: Draft, Standard, 4 Points, and Smooth surface.

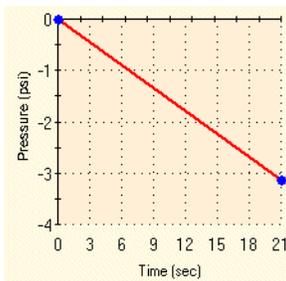
Meshing Parameters: Use the default Global Size.

Material Model: Linear elastic isotropic.

Material Properties: Modulus of elasticity = 1×10^7 psi, Poisson's ratio = 0.3162.

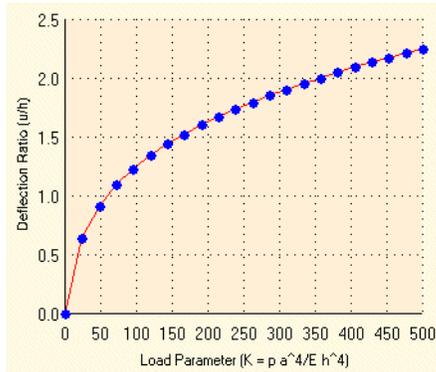
Modeling Hints: Due to symmetry, only a quarter of the plate is modeled. Draft quality shell mesh is used.

Restraints and Loads: All edges are simply supported. The pressure load is applied as follows:



Results

The load parameter versus the deflection ratio at the center of the plate is in good agreement with analytical results.



NOTE: The results of the nonlinear analysis have been processed by a graphing software to generate the above graph.

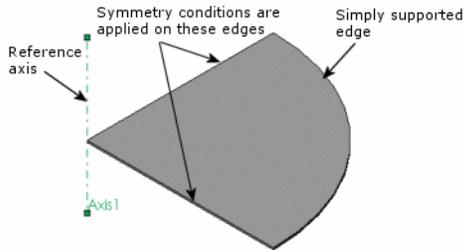
Reference

Levy, S., "Bending of Rectangular Plates With Large Deflections," Technical Notes, National Advisory Committee for Aeronautics. No. 846, 1942.

Large Displacement of a Circular Plate

Description

Investigate the large displacement behavior of a simply supported circular plate of radius 100 cm and thickness 1 cm with radially movable edges. The plate is subjected to a normal pressure of magnitude 400 N/cm².



File Name: <install_dir>\ Examples\ Verification\ Nonlinear_Static_2.sldprt

Study Type: Nonlinear.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 1 cm - thin shell formulation.

Meshing Options: Draft, Standard, 4 Points, and Smooth surface.

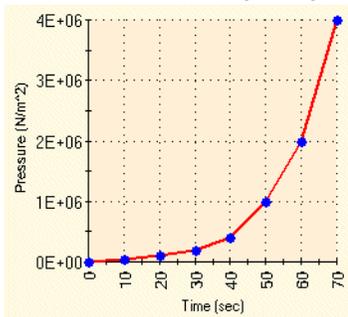
Meshing Parameters: Use the default Global Size.

Material Model: Linear elastic isotropic.

Material Properties: Modulus of elasticity = 2×10^8 N/cm², Poisson's ratio = 0.25.

Modeling Hints: Due to axial symmetry, only a wedge of the plate is modeled.

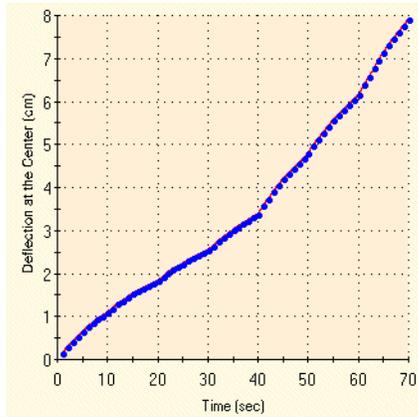
Restraints and Loads: The circular edge of the plate is simply supported. Symmetry boundary conditions are applied to the straight edges. Pressure is gradually applied normal to the plate as shown:



Results

	Theory	COSMOSWorks
Deflection at the plate center (cm)	7.88	7.905

The deflection at the center of the plate as a function of pseudo time is shown below:



The time history of the displacement at the center of the plate is in good agreement with analytical results.

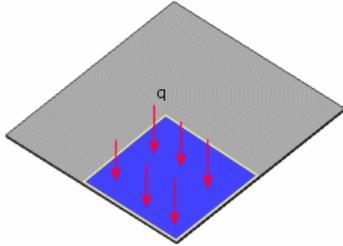
Reference

Timoshenko, S., and Woinowsky-Kreiger, S., "Theory of Plates and Shells," pp.411.

Large Deflection of a Clamped Square Plate

Description

Calculate the displacement of a clamped square plate of side 2 m and thickness 0.01 m under a normal pressure $q = 2 \times 10^4 \text{ N/m}^2$.



File Name: <install_dir>\Examples\Verification\Nonlinear_Static_3.sldprt

Study Type: Nonlinear.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 0.01 m - thin shell formulation.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

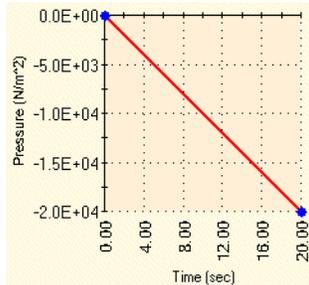
Meshing Parameters: Use the default Global Size.

Material Model: Linear elastic isotropic.

Material Properties: Modulus of elasticity = $10.92 \times 10^{10} \text{ N/m}^2$, Poisson's ratio = 0.3.

Modeling Hints: Due to double symmetry, only a quarter of the plate is modeled.

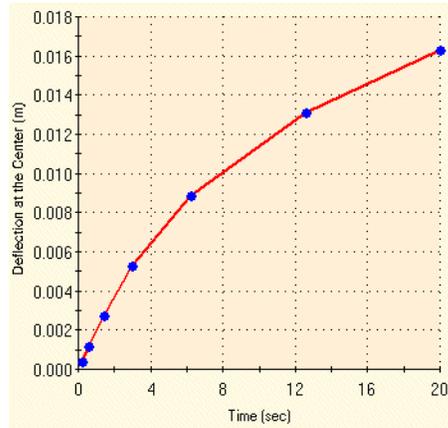
Restraints and Loads: The two outer edges are fixed. Symmetry restraints are applied to the other two edges. Pressure is linearly incremented to its full value of $q = 2 \times 10^4 \text{ N/m}^2$.



Results

	Theory	COSMOSWorks
Deflection at the plate center (m)	0.01594	0.0163

The following graph shows the deflection at the center of the plate versus time:



The results from the above curve are in agreement with analytical results.

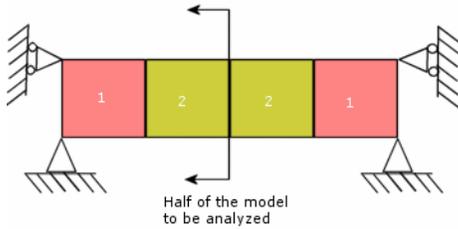
Reference

Timoshenko, S., and Woinowsky-Kreiger, S., "Theory of Plates and Shells," pp. 422-423.

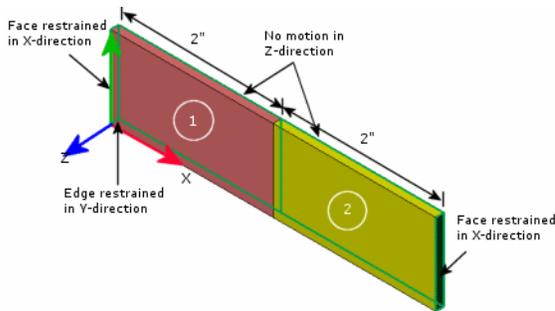
Thermal Stress - Temperature-Dependent Properties

Description

A plate is made of two different materials as shown in the figure. The plate thickness is 0.1". Determine the thermal stresses due to temperature changes of 100 °F and 200 °F. The full model is shown below:



The following figure depicts the loads and restraints applied to the half model.



File Name: <install_dir> Examples\ Verification\ Nonlinear_Static_4.sldasm

Study Type: Nonlinear.

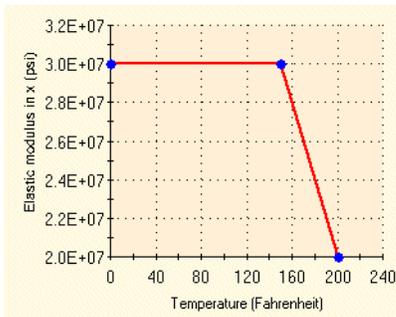
Mesh Type: Solid mesh.

Meshing Options: High, standard, 4 Points, and Smooth surface.

Meshing Parameters: Use a Global Size of 0.5".

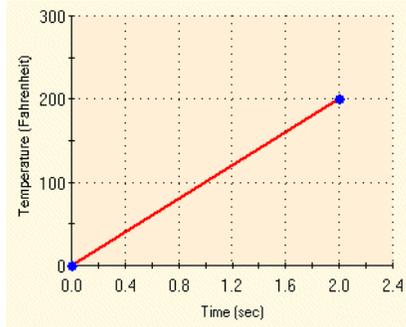
Material Model: Linear elastic isotropic material with temperature dependency.

Material Properties: Poisson's ratio = 0, Coefficient of thermal expansion = $1 \times 10^{-5} / ^\circ\text{F}$. Modulus of elasticity for part 1 of the plate is = 3×10^7 psi. The modulus of elasticity of part 2 is temperature-dependent as follows:



Restraints and Loads: Due to symmetry, only one half of the plate is modeled. The right face of the plate is simply supported and the back faces are restrained in the Z-direction. The lower right edge is restrained in the Y-direction.

Two uniform changes in temperature ($\Delta T_1 = 100\text{ }^\circ\text{F}$ and $\Delta T_2 = 200\text{ }^\circ\text{F}$) have been applied, one at a time, to both parts of the model. Each temperature rise followed the time curve shown below:



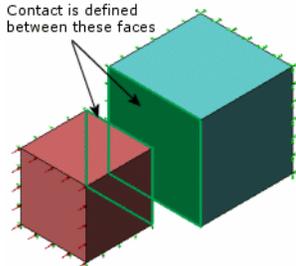
Results

Maximum Stresses in X-direction (SX), psi	$\Delta T_1 = 100\text{ }^\circ\text{F}$	$\Delta T_2 = 200\text{ }^\circ\text{F}$
Theory	-30,000	-48,000
COSMOSWorks	-30,000	-48,006

Contact Between Two Cubes

Description

A cube is pushed against another cube as shown by 4×10^6 psi. Calculate the maximum displacement.



File Name: <install_dir>\ Examples\ Verification\ Nonlinear_Static_5.sldasm

Study Type: Nonlinear.

Mesh Type: Solid mesh.

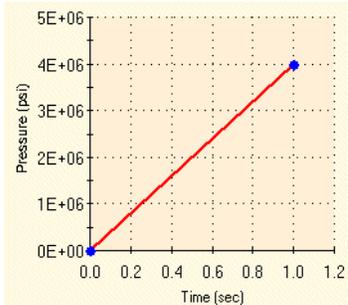
Meshing Options: Draft, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use default Global Size.

Material Model: Linear elastic isotropic.

Material Properties: Modulus of elasticity = 3×10^7 psi, Poisson's ratio = 0.

Restraints and Loads: The back face of the larger cube is fixed. Right and bottom faces of the two cubes are prevented from moving in their normal directions. A pressure is applied incrementally on the free face of the smaller cube according to the following time curve:



Results

NOTE: Use a deformation scale factor of 1.0 to plot results on the deformed shape. Animate the results to see how the two cubes move together after contact.

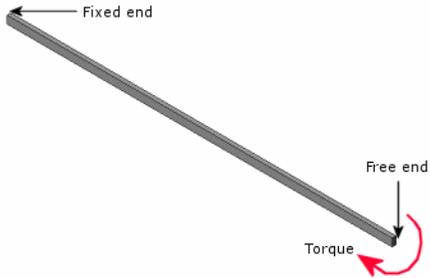
The linear theoretical elastic maximum displacement is calculated as $U = PL_1/(A_1E_1) + PL_2/(A_2E_2)$, where the subscripts refer to cubes 1 and 2. L is the length, A is the area ($A=L^2$) and E is the Modulus of elasticity.

	Maximum Displacement (UZ, inch)
Linear Theory	0.22857
COSMOSWorks (Node to node)	0.2168
COSMOSWorks (Node to surface)	0.2083

Deflection of a Nonlinear Elastic Cantilever Beam

Description

Determine the deflection of a cantilever beam under a torque applied to its free end. The beam has a length of 100" and a rectangular cross section of dimensions 2"X1".



The material is nonlinear elastic (defined by a stress-strain curve).

File Name: <install_dir>\Examples\ Verification\ Nonlinear_Static_6.sldprt

Study Type: Nonlinear.

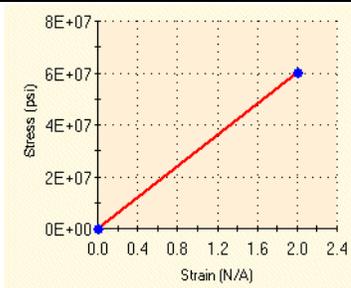
Mesh Type: Solid mesh.

Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use the default Global Size.

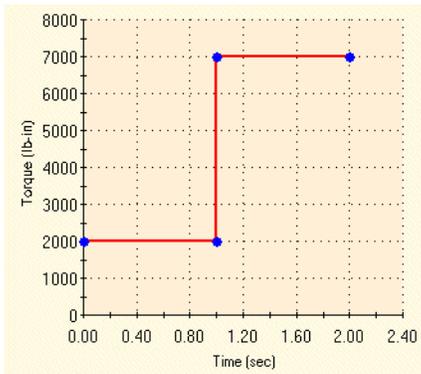
Material Model: Nonlinear elastic. The material has the following stress-strain curve:

Stress (psi)	Strain
0	0
3E3	0.001
60.003E6	2.001



Material Properties: Poisson's ratio = 0.

Restraints and Loads: The bar is fixed at one end and a torque is applied at the other end as follows:



Results

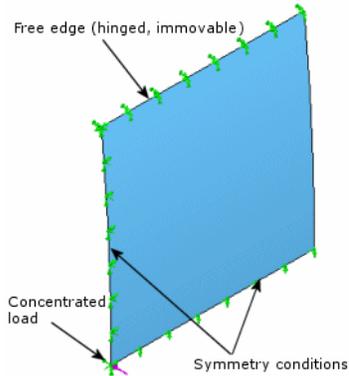
Deflection at free end (inch)	Linear elasticity range (Torque = 2000 lb-in)	Nonlinear elasticity range (Torque = 7000 lb-in)
Theory	-4.9998	-7.5156
COSMOSWorks	-4.9748	-7.4939

The theoretical solution is based on thin beam theory.

Large Displacement of a Cylindrical Shell - Force

Description

A shallow cylindrical shell is subjected to concentrated load at its center. The curved edges are free and the straight edges are hinged. Due to symmetry, only a quarter of the plate (shown below) is modeled. Determine the deflection.



File Name: <install_dir>\Examples\Verification\Nonlinear_Static_7.sldprt

Study Type: Nonlinear.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 12.7 mm - thick shell formulation.

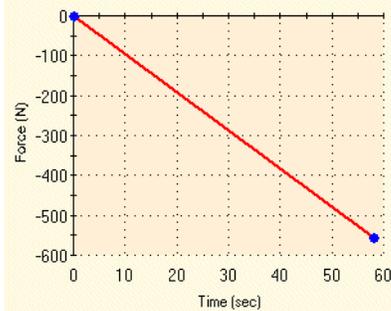
Meshing Options: Draft, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use a Global Size of 0.8".

Material Model: Linear elastic isotropic.

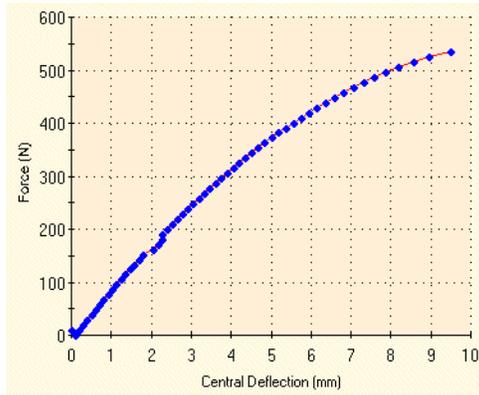
Material Properties: Modulus of elasticity = 3102.75 N/mm², Poisson's ratio = 0.3.

Restraints and Loads: The shell is fixed along its straight edges and symmetry conditions are applied to the edges of symmetry. A force is applied at the center as shown:



Results

The force versus central deflection curve shown below is in agreement with analytical results.



NOTE: The results of the nonlinear analysis have been processed by a graphing software to generate the above graph.

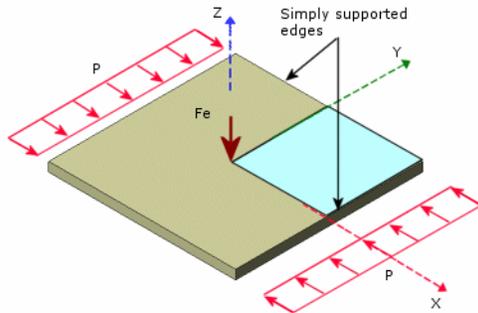
Reference

Horrigmo, G., "Finite Element Instability Analysis of Free-Form Shells," Report No. 77-1, the Norwegian Institute of Technology, the University of Trondheim, Norway (1977).

Buckling and Post Buckling of a Simply Supported Plate

Description

A simply supported isotropic square plate of side 40" and thickness 1" is subjected to in-plane uniform pressure p on two opposite sides. Find the buckling load and investigate the post buckling behavior.



File Name: <install_dir>\Examples\ Verification\ Nonlinear_Static_8.sldprt

Study Type: Nonlinear.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 1 in - thick shell formulation.

Meshing Options: Draft, Standard, 4 Points, and Smooth surface.

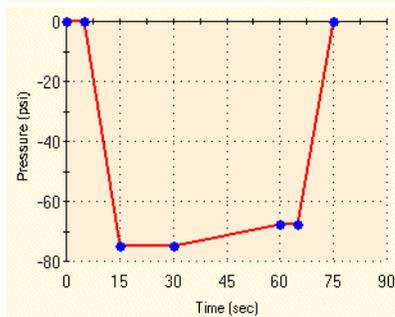
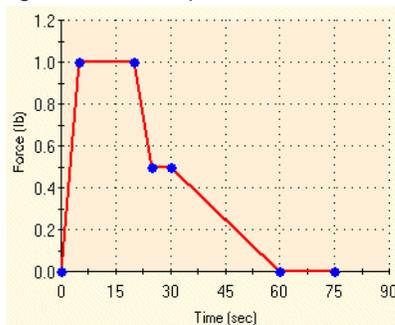
Meshing Parameters: Use a Global Size of 2.5 in.

Material Model: Linear elastic isotropic.

Material Properties: Modulus of elasticity = 3×10^4 psi, Poisson's ratio = 0.3.

Modeling Hints: Due to double symmetry, only a quarter of the plate is modeled. The post buckling behavior is studied by applying a transverse force (F_e) at the center of the plate.

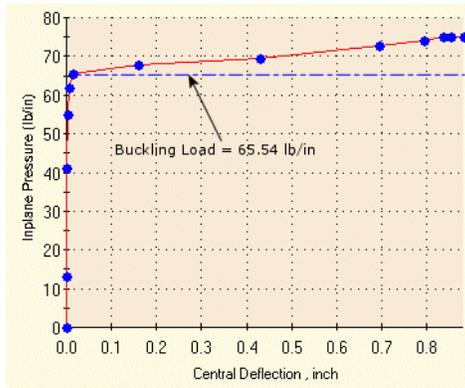
Restraints and Loads: The transverse force and the in-plane pressure are applied as shown in the figures below. In-plane rotations are set to zero and symmetry restraints are applied.



Results

	Theory	COSMOSWorks
Buckling Load (lb/in)	67.78	65.54

The figure below depicts the in-plane pressure versus the central deflection. The post buckling behavior matches the analytical solution.



NOTE: The above graph was not generated directly in COSMOSWorks. Results have been processed to generate the graph externally.

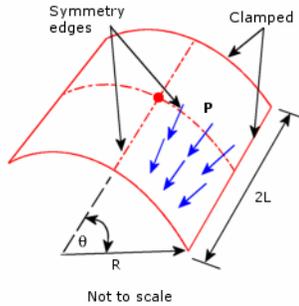
Reference

Timoshenko, S., and Woinosky-Krieger, "Theory of Shells and Plates," McGraw-Hill Book Co., 2nd Ed., pp. 389.

Large Displacement of a Cylindrical Shell - Pressure

Description

Determine the response of a cylindrical shell under normal uniform pressure P . All edges are fixed. The shell has the following dimensions: $L = 254$ mm, $\theta = 0.1$ rad, $R = 2,540$ mm.



File Name: <install_dir>\Examples\Verification\Nonlinear_Static_9.sldprt

Study Type: Nonlinear.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 3.175 mm - thin shell formulation.

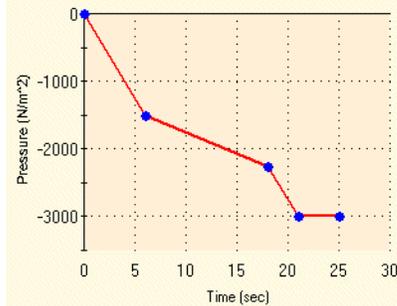
Meshing Options: Draft, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use a Global Size of 0.6".

Material Model: Linear elastic isotropic.

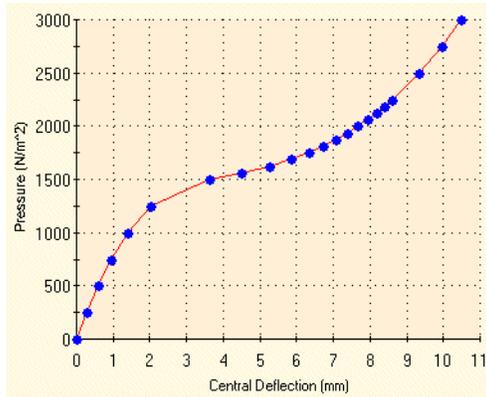
Material Properties: Modulus of elasticity = 3102.75 N/mm², Poisson's ratio = 0.3.

Restraints and Loads: The outer edges of the plate are fixed and symmetry conditions are applied to the edges of symmetry. The pressure is applied incrementally as follows:



Results

The pressure versus central deflection curve matches the analytical results.



NOTE: The above graph was not generated directly in COSMOSWorks. Results have been processed to generate the graph externally.

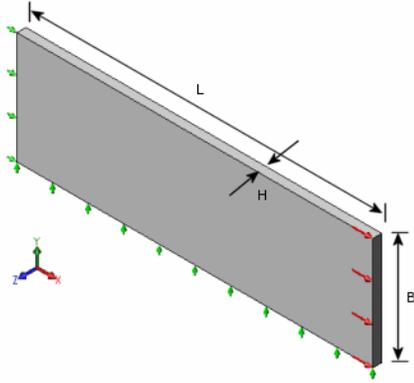
Reference

Sabir, A. B., and Lock, A. C., "The Application of Finite Elements to the Large Deflection Geometrically Nonlinear Behavior of Cylindrical Shells," Variational Methods in Engineering, South Hampton University Press (1973).

Viscoelastic of a Bar Under Constant Axial Load

Description

A bar made of a viscoelastic material is fixed at one end and a constant axial pressure is suddenly applied at the other end. Compare the axial strain to analytical results. The bar dimensions are as follows: $L = 10$ ", $B = 3.14159$ ", $H = 0.25$ ".



File Name: <install_dir>\Examples\Verification\Nonlinear_Static_10.sldprt

Study Type: Nonlinear.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 0.25" - thick shell formulation.

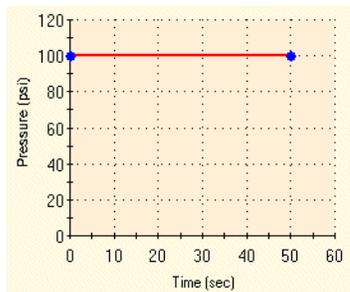
Meshing Options: Draft, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use a Global Size of 5".

Material Model: Viscoelastic material model.

Material Properties: Modulus of elasticity = 10,000 psi, $(\text{NUXY})_0 = 0.4833$, Shear relaxation modulus = 0.9010, Time value for the shear relaxation modulus = 0.9899 sec.

Restraints and Loads: Restraints are illustrated on the figure above. The pressure is applied suddenly as follows:

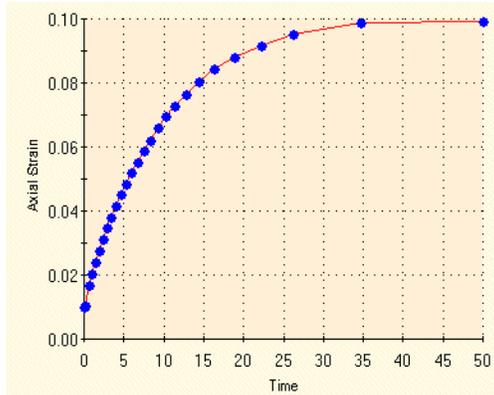


Results

The following tables summarizes the results for the instantaneous and long term behaviors:

	Instantaneous Behavior (t = 0.001 sec)	Long Term Behavior (t = 50 sec)
Theory	$(NUXY)_0 = 0.4833$	$(NUXY) = 0.4983$
COSMOSWorks	$(NUXY)_0 = -EPSY/EPSX = 0.4833$	$(NUXY) = -EPSY/EPSX = 0.4983$

The time history of the axial strain from COSMOSWorks is in good agreement with the analytical solution:



NOTE: The above graph was not generated directly in COSMOSWorks. Results have been processed to generate the graph externally.

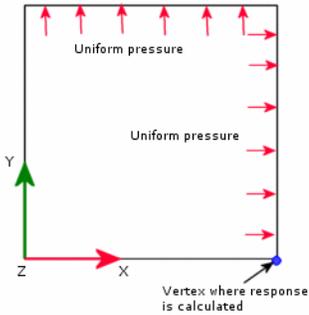
Reference

R. M. Christensen, "Theory of Viscoelasticity – an Introduction," 2nd Ed., 1982, pp. 1-76.

Uniformly Loaded Elastoplastic Plate

Description

A square elastoplastic plate of side 1" and thickness 0.001" is subjected to non-proportional stressing. Use kinematic hardening to study its behavior.



File Name: <install_dir>\Examples\Verification\Nonlinear_Static_11.sldprt

Study Type: Nonlinear.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 0.001" - thick shell formulation.

Meshing Options: Draft, Standard, 4 Points, and Smooth surface.

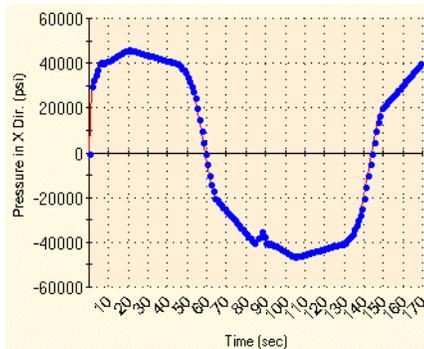
Material Model: Plasticity - von Mises.

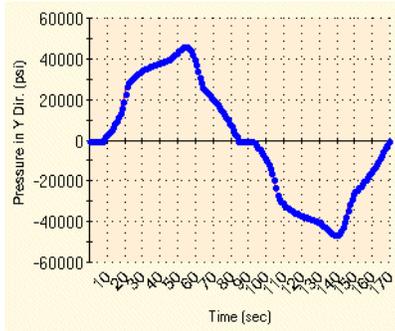
Material Properties: Modulus of elasticity = 3×10^7 psi, Poisson's ratio = 0.3, SIGYLD = 1×10^4 psi, Tangent modulus of elasticity = 5×10^6 psi, Hardening factor = 1 (full kinematic hardening).

Restraints and Loads: Two edges are simply supported and all rotations are set to zero. The plate is subjected to uniform pressures σ_x and σ_y as illustrated in the following table:

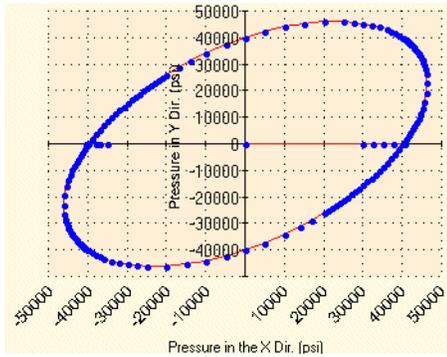
Loading Step	σ_x (psi)
1	30,000
2	32,500
3	35,000
4	37,500
5	40,000
6	40,500
7	40,000
7 through 84	$(\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y)^{1/2} = 40,000$

Note that the plate is loaded into the plastic range in uniaxial tension ($\sigma_y = 40,000$ psi). The load history is described by the equation in the table are shown in the following figures:



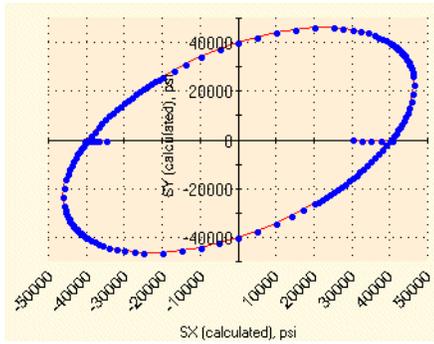


The following plot shows load input σ_y versus σ_x .

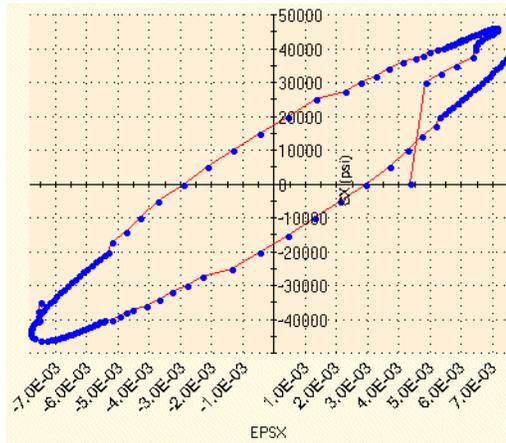


Results

The following plot shows σ_y versus σ_x plot as calculated by the program:



The σ_x versus ϵ_x plot, shown below, is in agreement with the analytical results.



NOTE: The above graph was not generated directly in COSMOSWorks. Results have been processed to generate the graph externally.

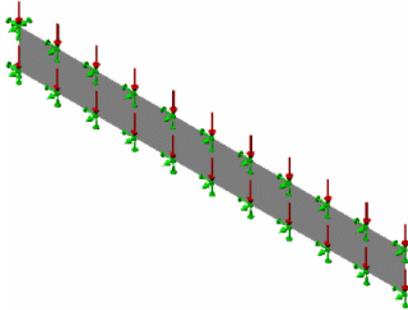
Reference

Foster Wheeler Corporation, "Intermediate Heat Exchanger for Fast Flux Test Facility: Evaluation of the Inelastic Computer Program," prepared for Westinghouse ARD, Livingston, N. J., 1972.

Large Displacement Analysis of a Cantilever Beam

Description

Investigate large displacements of a cantilever beam shown in the figure below. The cantilever beam has a length of 10" and a square cross section of side 1".



File Name: <install_dir>\ Examples\ Verification\ Nonlinear_Static_12.sldprt

Study Type: Nonlinear.

Mesh Type: Shell mesh using surfaces.

Shell Parameters: Shell thickness = 1" - Thin shell formulation.

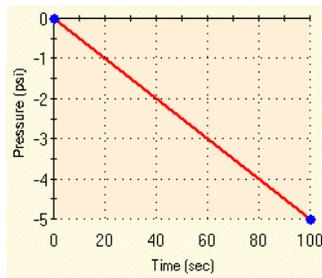
Meshing Options: High, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use 0.3 in Global Size.

Material Model: Linear elastic isotropic.

Material Properties: Modulus of elasticity = 12000 psi, Poisson's ratio = 0.2, Density = 0.2782 lb/in³.

Restraints and Loads: A fixed restraint is applied to one end of the cantilever and a pressure of magnitude 1 lb is applied incrementally (as shown in the figure) to the top and bottom edges. Translation in the normal direction and all rotations are set to zero on the side face of the cantilever.

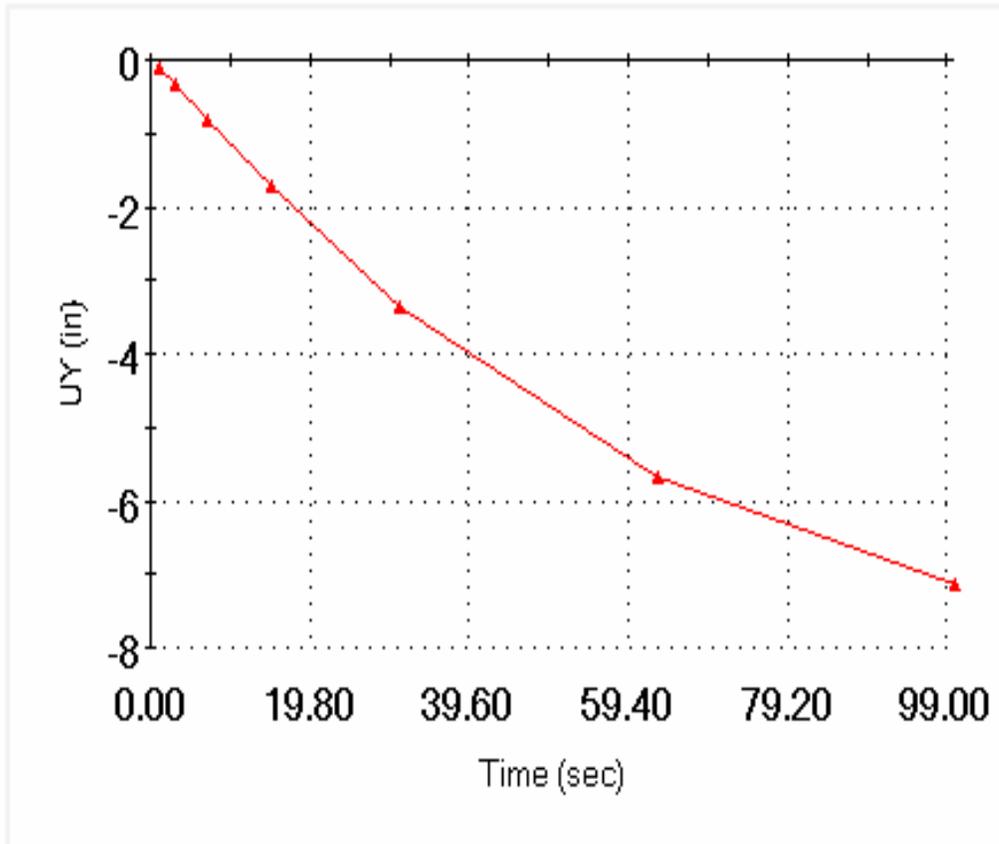


Results

	Theory	COSMOSWorks
Displacement at the free edge (UY), inch	7.1697	7.1402

The tip vertical displacement history is shown below:

Nonlinear Response



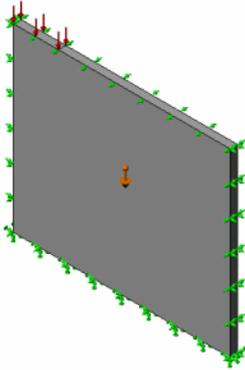
Reference

Holden, J. T., "On the Finite Deflections of Thin Beams," Int. J. Solid Structure, Vol. 8, pp. 1051-1055, 1972.

Bearing Capacity for a Strip Footing

Description

Find the maximum vertical displacement of a strip footing of width 120" sitting at the ground surface subjected to a uniform pressure. The elastic properties of the soil are given below (in the material properties section).



File Name: <install_dir>\Examples\ Verification\ Nonlinear_Static_13.sldprt

Study Type: Nonlinear.

Mesh Type: Solid mesh.

Meshing Options: Draft, Standard, 4 Points, and Smooth surface.

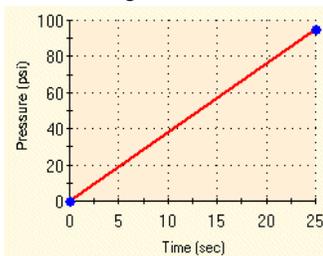
Meshing Parameters: Use the default Global Size.

Material Model: Drucker-Prager elastoplastic material model.

Material Properties: Elastic modulus = 3E4 psi, Poisson's ratio = 0.3, Cohesion strength = 1 psi, Friction angle = 27.27°, Density = 0.069337 lb/in³.

Modeling Hints: Due to symmetry, half of the soil is modeled. Because the bearing capacity of the foundations depends on the self-weight of the soil, an acceleration of gravity in the Y-direction is applied to simulate this effect. When the applied pressure load approaches the limit load (bearing capacity of the foundations), the soil bulging takes place adjacent to the footing.

Restraints and Loads: Symmetry is applied at the face of symmetry of the model. Gravitational acceleration is applied to the model in the -Y direction. The pressure is applied incrementally according to the following time curve:



Results

	Displacement in the Y-direction (UY), in
Theory	0.5992
COSMOSWorks	0.6005

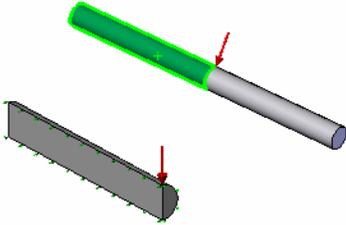
Reference

Joseph E. Bowels, "Foundation Analysis and Design," McGraw-Hill Book Co., 2nd Edition, 1977, pp. 113-124.

Three-Point Bending of a Nitinol Wire

Description

A three-point bending test is performed on a Nitinol wire of circular cross section with diameter $d = 1.49$ mm. The wire is 20 mm long and it is simply supported at both ends.



- Obtain the graph of the applied force versus deflection for the mid-span section of the wire.
- Verify that increasing the ultimate plastic strain for the material results in a closer match with experimental results.

Displacement control is used to solve this problem. The node for displacement control is selected to be same as the node where the force is applied. This node is displaced 5.2 mm in the direction of the force and then brought back to zero

File Name: <install_dir>\Examples\Verification\Nonlinear_Static_14.sldprt

Study Type: Nonlinear.

Mesh Type: Solid mesh.

Meshing Options: Draft, Standard, 4 Points, and Smooth surface.

Meshing Parameters: Use element size of 0.3 mm.

Material Model: Nitinol. Three studies are used (**Sample_1, Sample_2, and Sample_3**) with different

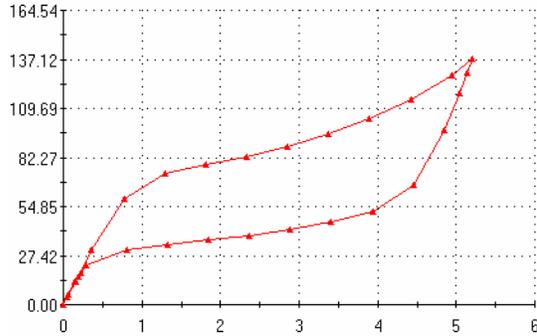
Ultimate plastic strain measure (Tension) of **0.092 mm/mm, 0.15 mm/mm, and 0.092 mm/mm.**

Sample_3 differs from Sample_1 by using **Exponential flow rule**. Refer to the material definition in each study for details.

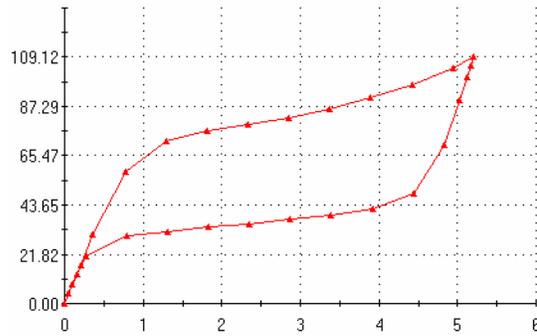
Results

The load factor versus UX displacement curve at the point of force application is shown for the three studies. Curves from study **Sample_1** and **Sample_2** show close agreement with the reference. The figure from study **Sample_3** shows closer results to experimental data presented in the same reference.

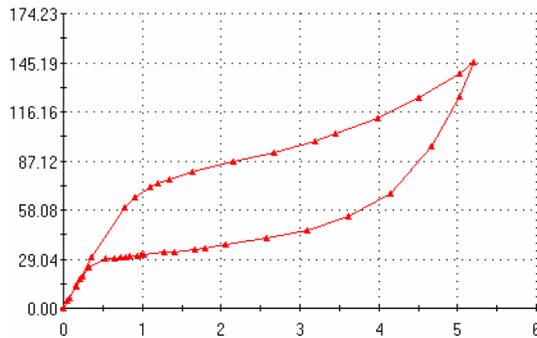
Load factor (vertical) versus **UX Displacement** in mm (horizontal) for study **Sample_1**.



Load factor (vertical) versus **UX Displacement** in mm (horizontal) for study **Sample_2**.



Load factor (vertical) versus **UX Displacement** in mm (horizontal) for study **Sample_3**.



Reference

Auricchio, F., Taylor, R.L., and Lubliner, J., "Shape-Memory-Alloys: Macromodeling and Numerical Simulations of the Superelastic Behavior," *Computer Methods in Applied Mechanics and Engineering*, vol. 146, pp. 281-312, 1997.