



Figure 6: Modeling of the suspension system of a motorcycle

# Demonstration of the exponential decay law using beer froth

**A Leike**

Ludwig–Maximilians-Universität, Sektion Physik, Theresienstr. 37, D-80333 München, Germany

E-mail: leike@theorie.physik.uni-muenchen.de

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## Abstract

The volume of beer froth decays exponentially with time. This property is used to demonstrate the exponential decay law in the classroom. The decay constant depends on the type of beer and can be used to differentiate between different beers. The analysis shows in a transparent way the techniques of data analysis commonly used in science—consistency checks of theoretical models with the data, parameter estimation and determination of confidence intervals.

Exponential laws are common to many physical phenomena. Examples are the amplitude of an oscillator subject to linear friction, the discharge of a capacitor, cooling processes or radioactive decays. The demonstration described here has the advantages that it is cheap, clear and motivating because it investigates an everyday phenomenon. It can easily be repeated by the students elsewhere.

The decay of beer froth is mentioned as a very short notice in [1]. It is described in several German textbooks of mathematics. Recently, it also attracted the attention of Bavarian pupils [2].

The data analysis proposed in this paper has much in common with real science—see, for example, the determination of the Higgs mass by the LEP collaborations [3]. The techniques involved are of great practical importance but are often poorly understood by students [4].

Exponential decay can be demonstrated using beer froth, the volume of which reduces exponentially with time [1]. The exponential law can readily be derived from the assumption that the volume of froth  $dV$  disappearing in the time between  $t$  and  $t + dt$  is proportional to the volume  $V$  present at the time  $t$ ,  $dV = -(V/\tau) dt$ . In a cylindrical beer mug with an area  $A$ , the volume is proportional to the height,  $dV = A dh$ . The phenomenological theory of exponential decay predicts the height as a function of time

$$h^{th}(t) = h(0) \exp\left(-\frac{t}{\tau}\right). \quad (1)$$

The constant  $\tau$  is a free parameter of the theory. It defines how fast the froth decays; during the time  $\tau$  the amount  $1 - 1/e \approx 63\%$  of the froth disappears. Different kinds of beer have, in general, different parameters  $\tau$ .