Solving Linear ODEs

- ***ode45*** Numerical solution of ordinary differential equation *y' = f(t, y)*
- ***[t, y] = ode45('filename', t, y0)***
- ***t*** row vector with discrete time values
- ***y*** matrix with solutions for y and y' in first and second columns
- ***filename*** name of m-file containing *y' = f(t, y)*
- ***y0*** row vector with initial conditions y(0) and y'(0) in first and second columns

Example: Van der Pol's ODE

- **Given**:
  - Van der Pol's differential equation:
    $d^2x/dt^2 - m\,(1 - x^2)\,dx/dt + x = 0$
  - Initial conditions: $x(0) = 1$, $dx/dt(0) = 0$
- **Find**:
  - Numerical solution using **ode45** function
  - Time plot and phase plane plot

Example: Van der Pol's ODE (cont.)

- **Solution**:
  - Convert 2nd order ODE into system of two 1st order ODEs

$$\frac{d^2x}{dt^2} - m(1-x^2)\frac{dx}{dt} + x = 0 \qquad \dot{x} = \frac{dx}{dt} \qquad \ddot{x} = \frac{d^2x}{dt^2} \qquad \ddot{x} - m(1-x^2)\dot{x} + x = 0$$

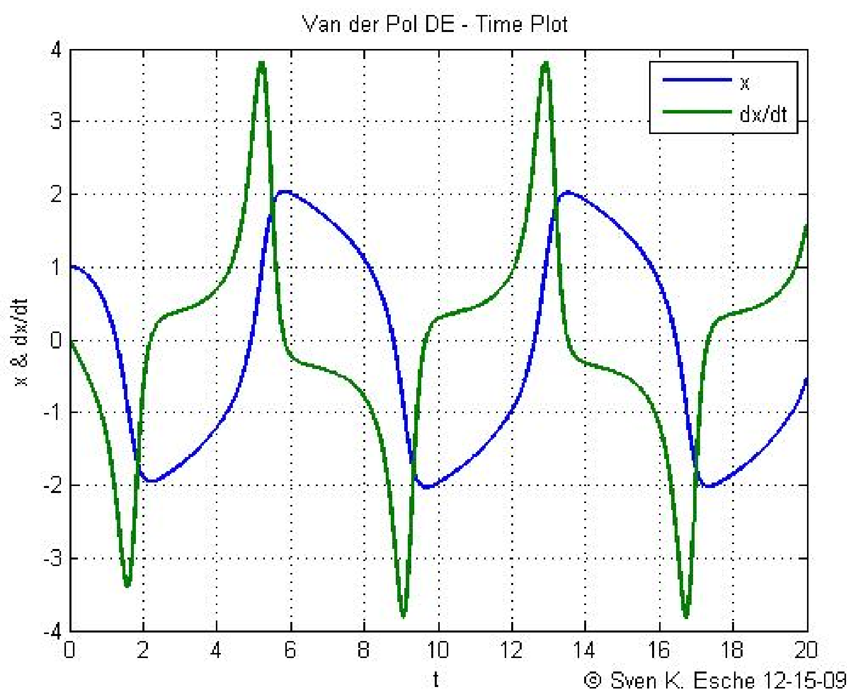
$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \qquad \dot{\mathbf{y}} = \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} y_2 \\ m(1-y_1^{\,2})y_2 - y_1 \end{bmatrix}$$

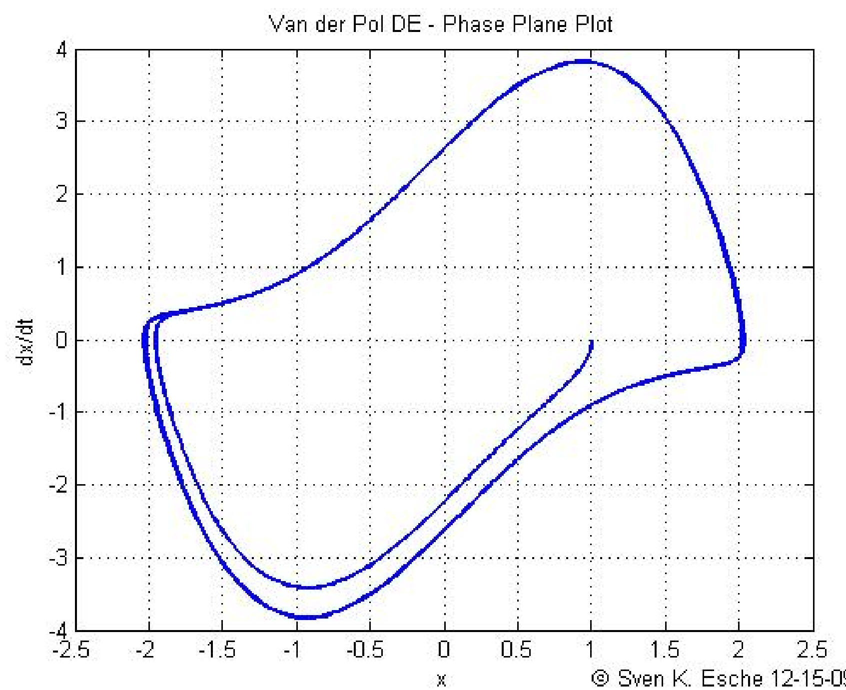
  - Define time vector **t** and initial condition vector $\mathbf{y}_0$

$$\mathbf{y}_0 = [x_o, \quad \dot{x}_o]^T = [1, \quad 0]^T$$

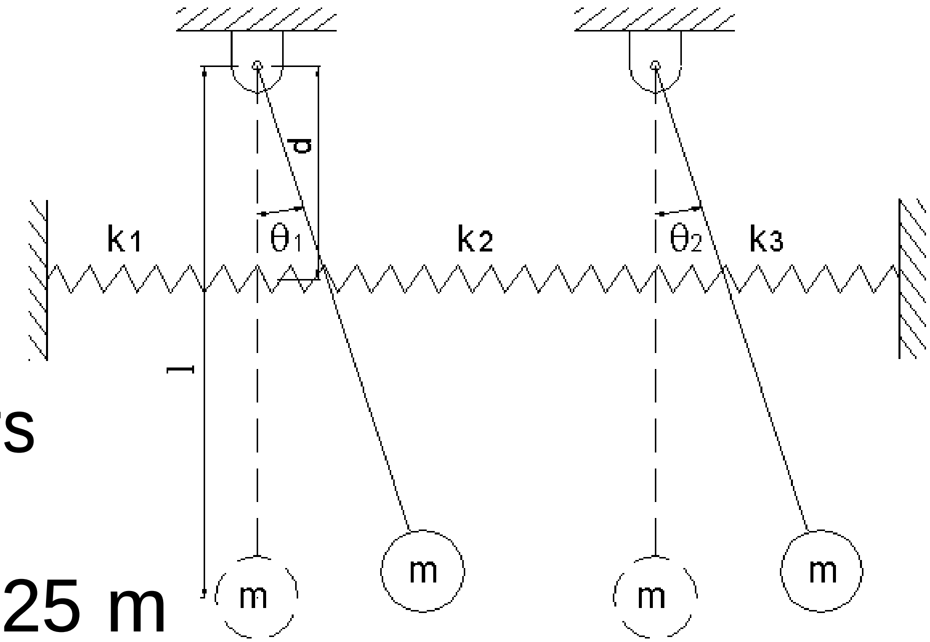
Example: Van der Pol's ODE (cont.)

- **Results**:



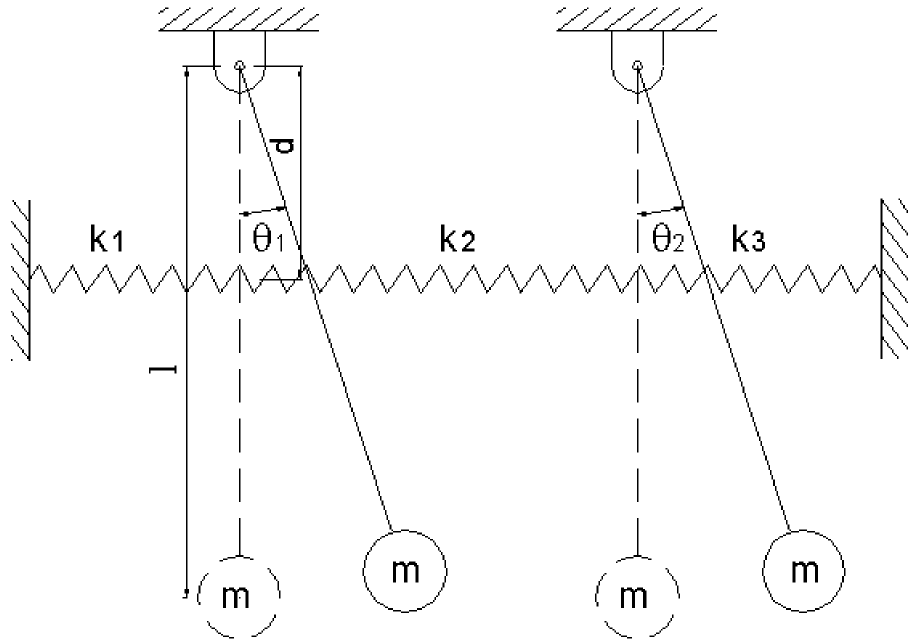


Example: Double Pendulum



- **Given**:
  - Spring-loaded double pendulum
  - System parameters
    $m_1 = m_2 = 0.1$ kg
    $d = 0.05$ m, $l = 0.25$ m
    $k_1 = k_3 = 200$ N/m, $k_2 = 20$ N/m
  - Initial conditions
    $\theta_1(0) = 0.1$ rad, $\theta_2(0) = 0.0$ rad
    $\omega_1 = \omega_2 = 0.0$ rad/s

Example: Double Pendulum (cont.)



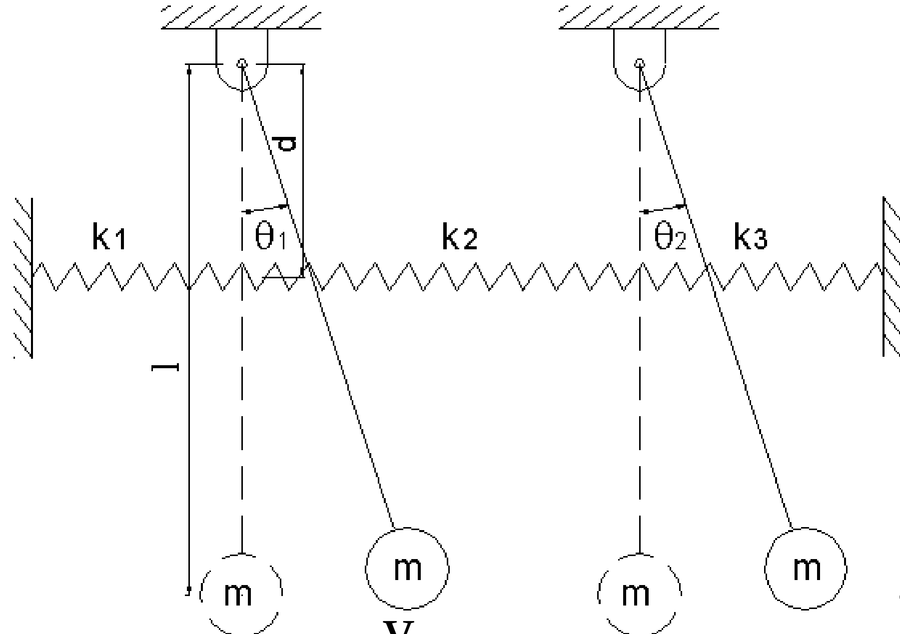
- **Find**:
  - Numerical solution using **ode45** function
  - Time plot
- **Solution:**
  - Governing equations (for small angles):

$$m_1 l^2 \ddot{\theta}_1 + \left[\left(k_1 + k_2 \right) d^2 + m_1 g l \right] \theta_1 - k_2 d^2 \theta_2 = 0$$

$$m_2 l^2 \ddot{\theta}_2 + \left[\left(k_2 + k_3 \right) d^2 + m_2 g l \right] \theta_2 - k_2 d^2 \theta_1 = 0$$

Example: Double Pendulum (cont.)

- Convert ODEs



$$\mathbf{y}=\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}=\begin{bmatrix} \theta_1 \\ \dot{\theta}_1 \\ \theta_2 \\ \dot{\theta}_2 \end{bmatrix}=\begin{bmatrix} \theta_1 \\ \omega_1 \\ \theta_2 \\ \omega_2 \end{bmatrix}$$

$$\dot{\mathbf{y}}=\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \end{bmatrix}=\begin{bmatrix} \dot{\theta}_1 \\ \ddot{\theta}_1 \\ \dot{\theta}_2 \\ \ddot{\theta}_2 \end{bmatrix}=\begin{bmatrix} \omega_1 \\ \alpha_1 \\ \omega_2 \\ \alpha_2 \end{bmatrix}=\begin{bmatrix} y_2 \\ \left\{k_2 d^2 y_3-\left[\left(k_1+k_2\right)d^2+m_1 g l\right]y_1\right\}/m_1 l^2 \\ y_4 \\ \left\{k_2 d^2 y_1-\left[\left(k_2+k_3\right)d^2+m_2 g l\right]y_3\right\}/m_2 l^2 \end{bmatrix}$$

$$\mathbf{y}_0=\begin{bmatrix} \theta_{10} & \omega_{10} & \theta_{20} & \omega_{20} \end{bmatrix}^T$$

Example: Double Pendulum (cont.)

- **Results**: