

PROBLEM 3. (22 points TOTAL)

Consider the one dimensional problem shown in the Figure to the right, consisting of two springs, a damper, and a mass. Assume gravity acts in the +y direction as shown.

Note that the damper and the spring k_2 are in series.

Derive the **first order** state equations describing the system behavior. Clear and legible work will be eligible for partial credit. (22 points)

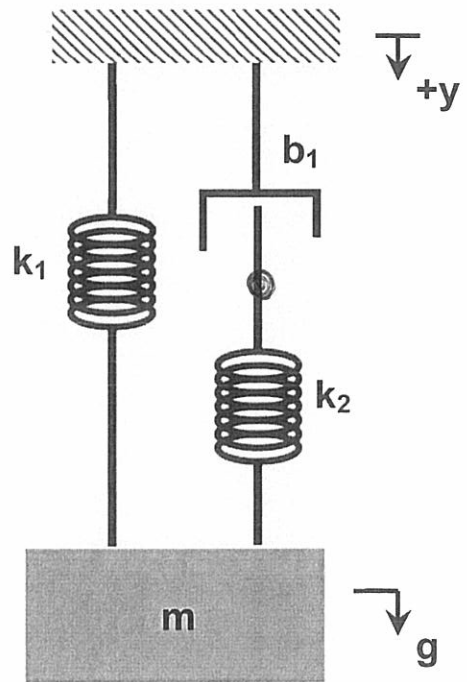


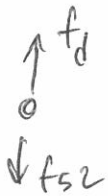
Figure. Schematic for Problem 3.

1) CL $f_{s1} = k_1 x_{s1}$ $f_{b1} = b_1 v_{d1}$

$f_{s2} = k_2 x_{s2}$

2) GC $v_{s1} = v_m$ $v_d + v_{s2} = v_m$

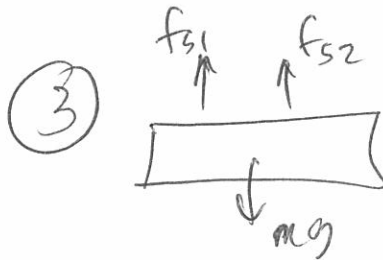
3) FBD



$\sum F_y = m a_y^0$

$f_d = f_{s2}$

(also, because in series)



$\sum F_y = m a_y$

$mg - f_{s1} - f_{s2} = m a_m$

4) SU'S : x_{s1}, x_{s2}, v_m

5) ② $x_{s1}' = v_{s1} = v_m$ ✓

③ $x_{s2}' = v_{s2} = v_m - v_d = v_m - \frac{f_d}{b} = v_m - \frac{f_{s2}}{b} = v_m - \frac{k_2 x_{s2}}{b}$ ✓

③ $v_m' = a_m = \frac{1}{m} (mg - f_{s1} - f_{s2}) = \frac{1}{m} (mg - k_1 x_{s1} - k_2 x_{s2})$