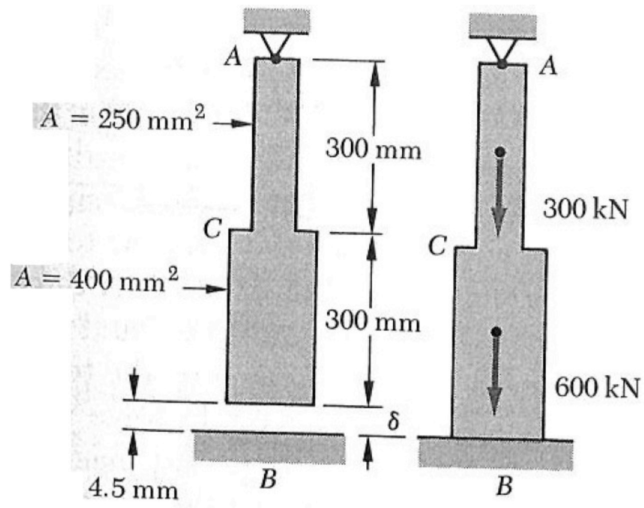
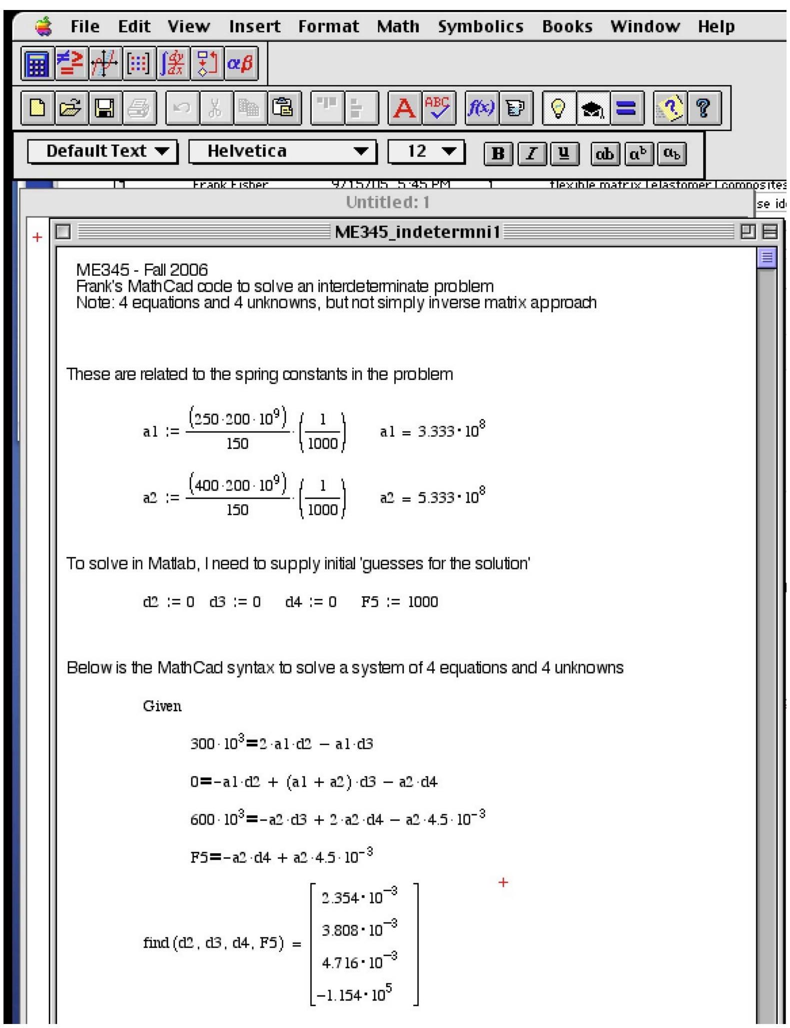
ME345 – Modeling and Simulation

Consider the problem at right. With no load applied there is a gap of 4.5mm between the stepped rod and the wall *B*. Find the nodal displacements and stresses within the stepped rod after the application of the forces shown on the right. (Prove that as shown in the figure on right there will be contact between the stepped rod and the wall *B*.) Modulus of each section is 200 GPa.



Solution. First, complete a FEM analysis of the problem by hand *neglecting* the presence of the wall *B*. You will find that the displacement of node 5 (bottom of the rod) is > 4.5mm, such that the wall will effect the solution. Since this is the case, re-do the problem, this time enforcing the presence of the wall by setting the displacement of node 5 to be equal to 4.5mm. (Typically in the past we've had zero displacement boundary conditions; this is an example of a non-zero displacement BC.) To solve the resulting set of simultaneous equations I've used the program *MathCad* as shown below.



Note: to solve via matrix multiplication (inverse), you can rewrite the equation such that all unknowns are on one side of the equation. Doing this here (and using the fact that d1 = 0, d5 = 4.5e-3 m, and a1 and a2 are the element stiffness terms for the thin and thick sections, respectively), you would get:

$$\begin{bmatrix} 0 \\ 300e3 \\ 0 \\ 600e3 + a2 * d5 \\ a2 * d5 \end{bmatrix} = \begin{bmatrix} -1 & -a1 & 0 & 0 & 0 \\ 0 & 2a1 & -a1 & 0 & 0 \\ 0 & -a1 & a1 + a2 & -a2 & 0 \\ 0 & 0 & -a2 & 2a2 & 0 \\ 0 & 0 & 0 & a2 & 1 \end{bmatrix} \begin{bmatrix} F1 \\ d2 \\ d3 \\ d4 \\ F5 \end{bmatrix}$$

Solving for the unknown values on the right you would find (here I used Matlab):

$$\begin{bmatrix} F1 \\ d2 \\ d3 \\ d4 \\ F5 \end{bmatrix} = \begin{bmatrix} -7.85e5\ N \\ 0.0024\ m \\ 0.0038\ m \\ 0.0047\ m \\ -1.154e5\ N \end{bmatrix}$$

Note that the two reaction forces F1 and F5 together equal 900 kN (as we would expect)!