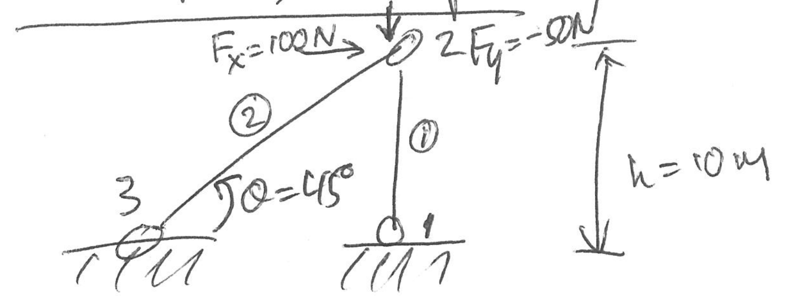
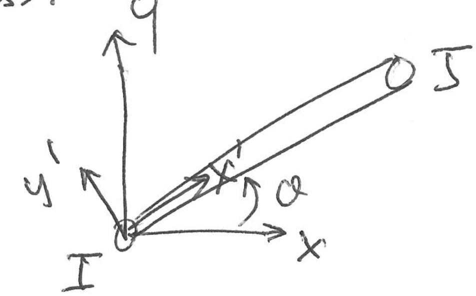
2D Truss example



$A_1E_1 = A_2E_2 = 10^6$ N

$F_{2x} = 100$ N, $F_{2y} = -50$N

NOTE: will need to use the 2D Element Stiffness matrix for a truss.



node I: d_{Ix}, d_{Iy}
f_{Ix}, f_{Iy}

node J: d_{Jx}, d_{JI}
f_{Jx}, f_{JY}

$$\begin{Bmatrix} f_{Ix} \\ f_{Iy} \\ f_{Jx} \\ f_{Jy} \end{Bmatrix} = \frac{AE}{L} \left[\begin{array}{cc|cc} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ \hline -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{array}\right] \begin{Bmatrix} d_{Ix} \\ d_{Iy} \\ d_{Jx} \\ d_{Jy} \end{Bmatrix}$$

$c = \cos\theta$, $s = \sin\theta$

NOTES:

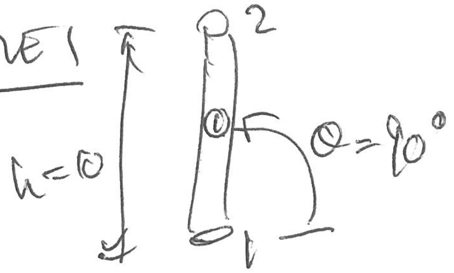
1) need to be consistent → keep the nodes and directions together!

2) this will double the size of the global stiffness matrix

3) θ needs to be determined for each element!

BACK TO THE PROBLEM...

ELE 1



$$\frac{A_1E_1}{L_1} = \frac{10^6\ N}{10\ m} = 10^5\ N/m.$$

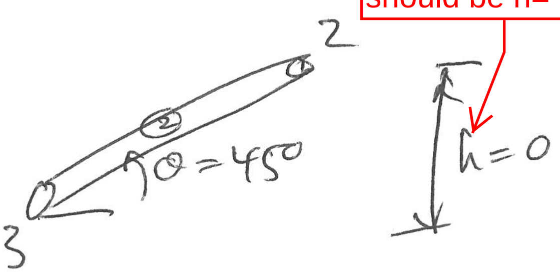
$$\sin 90° = 0, \quad \cos 90° = 1.$$

plug into the truss element stiffness matrix

matrix should be...
[0 0 0 0
0 1 0 -1
0 0 0 0
0 -1 0 1]

$$\begin{Bmatrix} f_{1x} \\ f_{1y} \\ f_{2x} \\ f_{2y} \end{Bmatrix} = 10^5 \begin{bmatrix} 0 & 1 & 0 & -1 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} d_{1x} \\ d_{1y} \\ d_{2x} \\ d_{2y} \end{Bmatrix}$$

ELE 2



should be h= 10 m

$$\sin 45° = \frac{h}{L_2} = \frac{10\ m}{L_2}$$

$$L_2 = 10\sqrt{2}\ m$$

Be careful how the nodes are numbered! Here $I=3$, $J=2$!

$$\frac{A_2E_2}{L_2} = \frac{10^6\ N}{10\sqrt{2}\ m} = .707\times 10^5\ \frac{N}{m}.$$

$$\sin 45° = \frac{\sqrt{2}}{2}, \quad \cos 45° = \frac{\sqrt{2}}{2}$$

NOTE: In this case, all c^2, s^2, cs, etc terms will be $\pm\frac{1}{2}$.

$$\begin{Bmatrix} f_{3x} \\ f_{3y} \\ f_{2x} \\ f_{2y} \end{Bmatrix} = .707\times10^{5} \begin{bmatrix} 1/2 & 1/2 & -1/2 & -1/2 \\ 1/2 & 1/2 & -1/2 & -1/2 \\ -1/2 & -1/2 & 1/2 & 1/2 \\ -1/2 & -1/2 & 1/2 & 1/2 \end{bmatrix} \begin{Bmatrix} d_{3x} \\ d_{3y} \\ d_{2x} \\ d_{2y} \end{Bmatrix}$$

NOW DO THE ASSEMBLY FOR THE GLOBAL STIFFNESS MATRIX. BE SURE TO PUT THINGS IN THE CORRECT LOCATION!

<u>NOTE</u>: $.707\times10^{5} \times \pm\frac{1}{2} = \underline{\pm .354\times10^{5}\ N/m}$.

$$\begin{Bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ F_{3x} \\ F_{3y} \end{Bmatrix} = 10^{5} \begin{bmatrix} & & & & & \\ & 1 & & -1 & & \\ & & .354 & .354 & -.354 & -.354 \\ & -1 & .354 & 1+.354 & -.354 & -.354 \\ & & -.354 & -.354 & .354 & .354 \\ & & -.354 & -.354 & .354 & .354 \end{bmatrix} \begin{Bmatrix} d_{1x} \\ d_{1y} \\ d_{2x} \\ d_{2y} \\ d_{3x} \\ d_{3y} \end{Bmatrix}$$

ALL BLANK TERMS ARE ZERO!

NOW APPLY DISPLACEMENT BCs: $d_{1x} = d_{1y} = d_{3x} = d_{3y} = 0$!

THIS LEAVES ME WITH...

$$\begin{Bmatrix} \overset{100\,N}{F_{2x}} \\ \overset{-50\,N}{F_{2y}} \end{Bmatrix} = 10^5 \begin{bmatrix} .354 & .354 \\ .354 & 1.354 \end{bmatrix} \begin{Bmatrix} d_{2x} \\ d_{2y} \end{Bmatrix}$$

$$\boxed{\begin{aligned} d_{2x} &= .0043\ m \\ d_{2y} &= -.0015\ m. \end{aligned}}$$

NOTE: - makes sense from directions
- would also expect to move more in x-direction

(IF WANT TO PRACTICE ANOTHER PROBLEM, IF $F_{2x} = -500N$ and $F_{2y} = 380\ N$, $d_{2x} = -.0229\ m$, $d_{2y} = .0088\ m$)

See attached Matlab code for how to solve using the TRUSS PROGRAM.