ME345 – Modeling and Simulation
Professor Frank Fisher
Spring 2006
May 3, 2006

NAME: KEY

"I pledge my honor that I have abided by the Stevens Honor Code."

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NOTE 1: For the final you may use up to five (5) 8 1/2'' by 11'' sheets of notes that you have prepared in reviewing for this test. You MAY NOT use review sheets that have been prepared by others in the class.

NOTE 2: This quiz counts for 20% of your grade for the course. Other components of the final grade are the three Case Studies (20% each), and 20% for homework and class participation.

PROBLEM 1. (15 points)

a) __X__ In accord with the Stevens Honor code, I will complete (or have already completed) the online course evaluation for ME 345 at http://www.stevens.edu/assess. (Check to confirm.) (3 points)

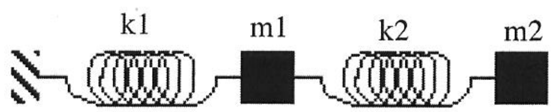
3 points whether checked or not.

all = 6
1/2 = 4

b) Briefly discuss the utility of analyzing the maximum von Mises stress in the context of a stress analysis. (6 points)

Max. VM stress gives a scalar stress measure to compare to, for instance, the yield stress of a material. Useful for analysis of 2D and 3D stress states.

c) For a particular two mass - two spring system, the state equations for the velocities of the masses as a function of time were found to be:



$$\underline{v_m} = \begin{Bmatrix} v_{m1}(t) \\ v_{m2}(t) \end{Bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}\left(a_1\cos(t) + b_1\sin(t)\right) + \begin{bmatrix} 1 \\ -1/2 \end{bmatrix}\left(a_2\cos(\sqrt{6}t) + b_2\sin(\sqrt{6}t)\right)$$

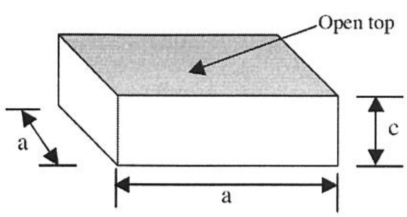
Find one set of values for a_2 and b_2 that will result in a normal mode behavior of the system. In this case, if the velocity of mass 1 is –5m/s (i.e. v_{m1}=-5m/s), what will the velocity of mass 2 be? (6 points)

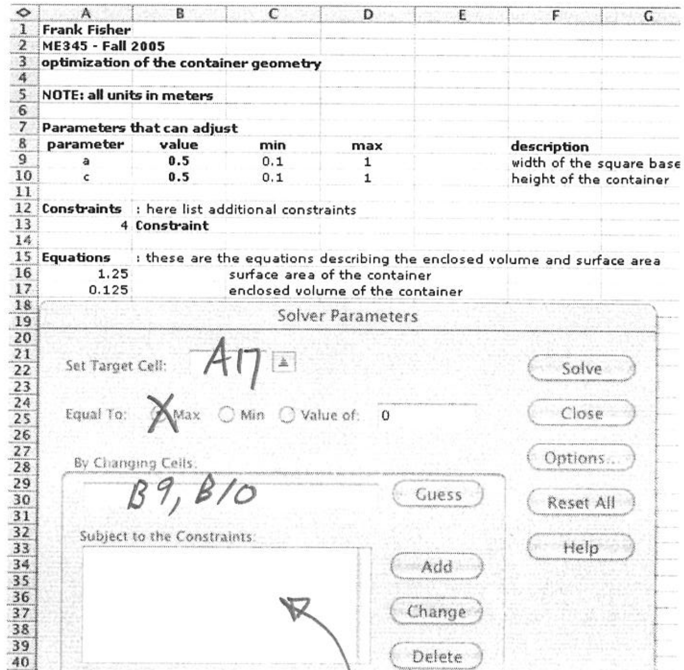
$a_2 = 0,\ b_2 = 0 \Rightarrow \underset{\sim}{v_m} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}(a_1\cos t + b_1 \sin t)$

FOR ALL t, in this mode $v_{m2} = 2v_{m1}$. If $v_{m1} = -5\,m/s$, then $v_{m2} = -10\,m/s$.

PROBLEM 2. (18 points)

The geometry of the *open-top, square-base* container (shown in the Figure) with a base dimension of a and height c is to be optimized to provide **maximum volume** while having a surface area less than or equal to 4 m^2. The range of variables for a and c are to be between 0.1 and 1 m.





a) Identify and derive any equations necessary to solve this optimization problem using the Excel solver tool. (6 points)

$SA = 4(a)(c) + a^2$

$V = a^2 c$

b) Clearly describe in words (or alternatively using the Figure on the right) how to solve this multivariate problem using the Excel solver tool. Clearly identify where how the equations above are used within the spreadsheet. (6 points)

A16 → put eqn. for SA above

A17 → put eqn. for V above

For rest, see figure

B9>C9
B9<D9
B10>C10
B10<D10
A16<A13

(note: EXCEL SOLUTION: $a=1, c=.75, V=.75$)

c) Neglecting the constraints on the values for a and c above, one **could** solve analytically for the value of a that satisfies the optimization problem (do <u>NOT</u> solve). Will the optimized volume in this case (found analytically) be greater than, less than, or equal to that found through the Excel solution in Part B above? Why? (6 points)

3

totally about a

The constraints on "a" and "c" mean that the optimal volume in EXCEL will be $\leq V_{nc}$ (no constraints).

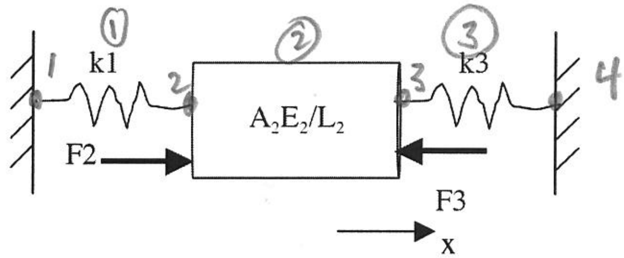
Analytical solution (not necessary):

$SA = 4 = 4ac + a^2 \Rightarrow c = \frac{a}{4}(4-a^2)$. Substitute into $V = a^2 c \Rightarrow V = a - \frac{a^3}{4}$. Set $\frac{\partial V}{\partial a} = 0 \Rightarrow a = \sqrt{4/3}$, $c = .577$.

PROBLEM 3. (26 points)

Consider the **1D problem below (x-direction)**, where gravity effects are negligible. Prior to the application of the external forces, springs 1 and 3 are at their unstretched (equilibrium) length. *Be consistent with units.*

In the figure below, k1 = 10 N/mm, k3 = 20 N/mm, and A_2E_2/L_2=50 N/mm, whereas the magnitudes of the forces F2 and F3 are 100 N and 90 N *in the directions shown in the Figure*, respectively.



Recall that the element stiffnesses for a spring element and a rod element are as given below:

spring	***rod***
$k\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} d_I \\ d_J \end{Bmatrix} = \begin{Bmatrix} F_I \\ F_J \end{Bmatrix}$	$\dfrac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} d_I \\ d_J \end{Bmatrix} = \begin{Bmatrix} F_I \\ F_J \end{Bmatrix}$

a. Clearly identify the nodes and elements on the Figure above. (1 point)

b. Assemble and <u>clearly label</u> the global stiffness matrix. (7 points)

Ele 1: $\begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} = k_1\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix}$ Ele 2: $\begin{Bmatrix} f_2 \\ f_3 \end{Bmatrix} = \dfrac{A_2E_2}{L_2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} d_2 \\ d_3 \end{Bmatrix}$

Ele 3: $\begin{Bmatrix} f_3 \\ f_4 \end{Bmatrix} = k_3\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} d_3 \\ d_4 \end{Bmatrix}$

Assemble for global stiffness:

$$\begin{Bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{Bmatrix} = \begin{bmatrix} k_1 & -k_1 & & \\ -k_1 & k_1 + \frac{A_2E_2}{L_2} & -\frac{A_2E_2}{L_2} & \\ & -\frac{A_2E_2}{L_2} & \frac{A_2E_2}{L_2} + k_3 & -k_3 \\ & & -k_3 & +k_3 \end{bmatrix}\begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{Bmatrix}$$

c. Solve for the unknown nodal displacements in the problem. (7 points)

From Part b, enforce boundary conditions to get

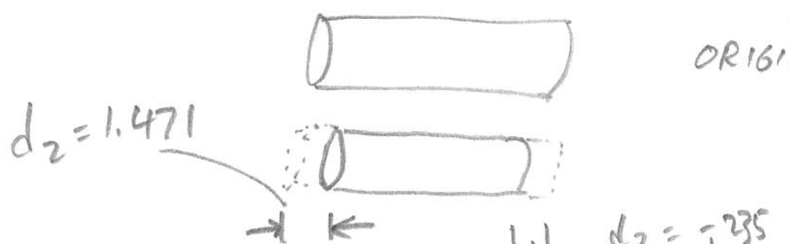
$$\begin{Bmatrix} \cancel{f_2}\ 100 \\ \cancel{f_3}\ -90 \end{Bmatrix} = \begin{bmatrix} 60 & -50 \\ -50 & 70 \end{bmatrix} \begin{Bmatrix} d_2 \\ d_3 \end{Bmatrix}$$

$$\begin{Bmatrix} d_2 \\ d_3 \end{Bmatrix} = \text{inv} \begin{pmatrix} 60 & -50 \\ -50 & 70 \end{pmatrix} \begin{Bmatrix} 100 \\ -90 \end{Bmatrix} \Rightarrow \boxed{\begin{Bmatrix} d_2 \\ d_3 \end{Bmatrix} = \begin{Bmatrix} 1.471 \\ -.235 \end{Bmatrix} \text{mm}}$$

d. Identify which, if any, elements are in compression. (4 points)

only element 2 (rod) in compression

e. What is the deformation of the rod due to the application of the applied forces? (4 points)



$$\delta = d_3 - d_2$$
$$= -.235 - 1.471$$
$$\boxed{\delta = -1.706 \text{ mm}}$$

f. What is the internal force in Spring 1 due to the application of the applied forces? (4 points)

$f_1 = k_1 X_1$ — elongation of spring 1.

$$f_1 = \left(110 \frac{N}{mm}\right)(1.471\,mm) = \boxed{14.71\ N}$$