Matlab programming exercises: New Game Show Simulator ME345 – Modeling and Simulation

(This question was originally posted as a brainteaser on The Riddler section of the FiveThirtyEight website; see http://fivethirtyeight.com/features/can-you-win-this-hot-new-game-show/. You'll note that some of the solutions clearly use Matlab to find the answer... can you?)

Two players go on a hot new game show called "Higher Number Wins." The two go into separate booths, and each presses a button, and a random number between zero and one appears on a screen. (At this point, neither knows the other's number, but they do know the numbers are chosen from a standard uniform distribution.) They can choose to keep that first number, or to press the button again to discard the first number and get a second random number, which they *must* keep. Then, they come out of their booths and see the final number for each player on the wall. The grand prize is awarded to the player who kept the higher number.

Which number is the optimal cutoff for players to discard their first number and choose another? Put another way, within which range should they choose to keep the first number, and within which range should they reject it and try their luck with a second number?

Hint: You may find it easier to simplify the problem at first and gradually build in layers of complexity. First, if you assume that the other player always keeps their first number, then your cutoff number from probability theory is 0.5 (i.e. if you number is larger than 0.5 you should keep it). Then add in complexity; clearly if the competitor's first number is very low (i.e. 0.1) they will choose a new number; this will increase their likelihood of having a better number and thus increase your final cutoff number to a slightly larger value.