Modeling and Simulation for Failure Analysis

Failure Theories

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ME 345

(adapted from materials prepared by Dr. Kishore Pochiraju)

Performance Concerns

- Does it fail under operating conditions?
  - Define Failure mode
    - Static failure
    - Fatigue failure
    - Wear, Ageing or other durability issues
- Static Failure
  - Yield or Break (Maximum Stress < Yield or Failure stress)
  - Deform beyond limit.
    - (Maximum deflection < Specified δ^{max})

Failure Theories

MAXIMUM STRESS < MATERIAL STRENGTH

Problems: Failure theories compare two scalar quantities

Stress is a tensor, has six components! Failure also needs to be coordinate independent!

Most Failure Theories make assumptions…

Note: most failure theories are *phenomenological* - not based on atomic level detail

The popular ones are …

Failure of Ductile Materials

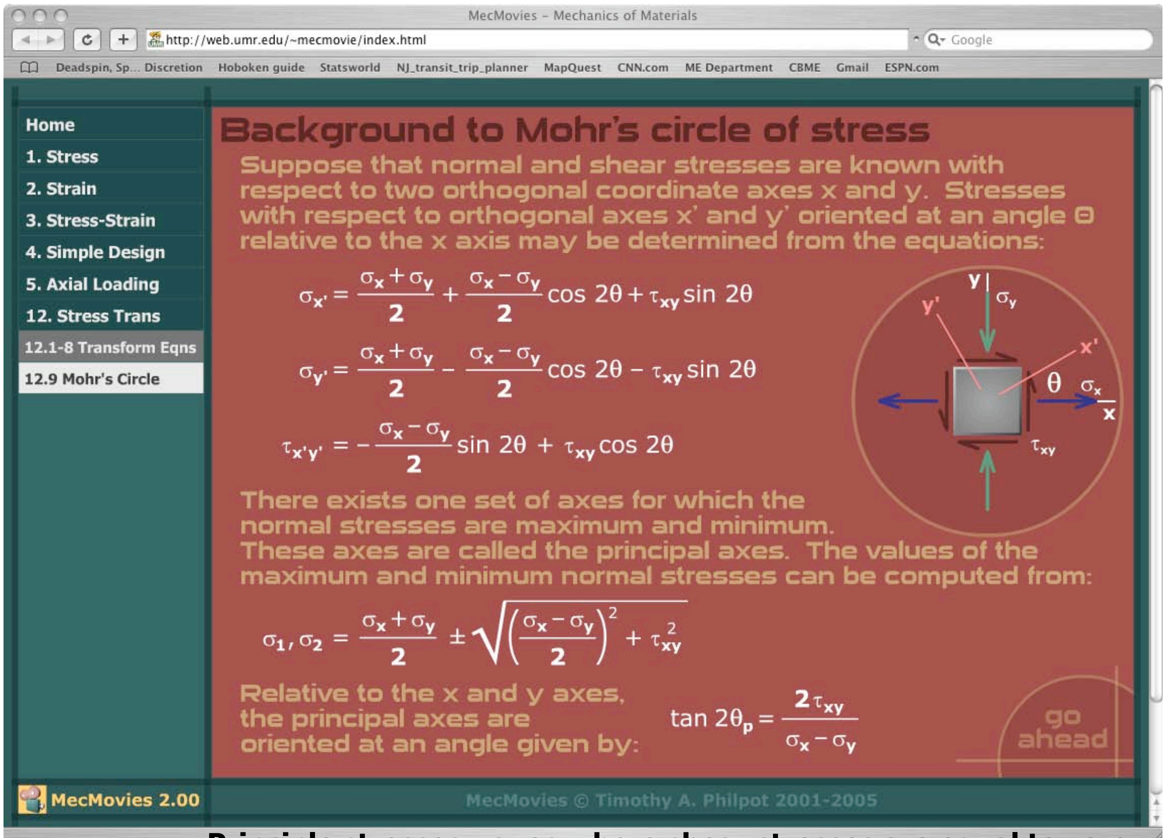
1. The ***maximum normal stress*** theory - BRITTLE MATERIALS ONLY

2. The ***maximum shear stress*** theory (also called the Tresca theory) - Good for ductile materials; satisfactory results, easy to use

3. The ***maximum strain energy*** theory - similar to Von Mises criterion below; better results than above, need Poisson ratio

4. ***Von Mises*** theory (also called Von Mises - Hencky theory, shear energy theory, and the maximum distortion energy theory) - BEST FOR DUCTILE MATERIALS

From Spotts, Shoup, and Hornberger, *Design of Machine Elements (ME 361)*



Principle stresses - axes where shear stresses are equal to zero

1. Maximum Normal (or principle) Stress

- Assumption - failure occurs when any principle stress exceeds the ultimate yield/failure stress from a simple 1D (tensile or compressive) test
- If S_{yc} is the yield strength in compression, S_{yt} yield strength in tension

$$S_{yc} \leq S_1, S_2, S_3 \leq S_{yt}$$

- To account for safety factor

$$\frac{S_{yc}}{N_{fs}} \leq S_1, S_2, S_3 \leq \frac{S_{yt}}{N_{fs}}$$

2. Maximum Shear Stress theory (Tresca)

- Assumption - failure occurs when any principle shear stress exceeds the ultimate yield/failure stress from a simple 1D (tensile or compressive) test
- If S_y is the yield strength

$$-S_y \le (S_1 - S_2),(S_2 - S_3),(S_1 - S_3) \le S_y$$

- To account for safety factor

$$\frac{-S_y}{N_{fs}} \le (S_1 - S_2),(S_2 - S_3),(S_1 - S_3) \le \frac{S_y}{N_{fs}}$$

3. Maximum strain energy theory

- Failure occurs when strain energy per unit volume exceeds that for a simple uniaxial test at failure
- Derivation in ME358 text - see page 130
- Usually not used as Von Mises failure is simpler and typically more accurate

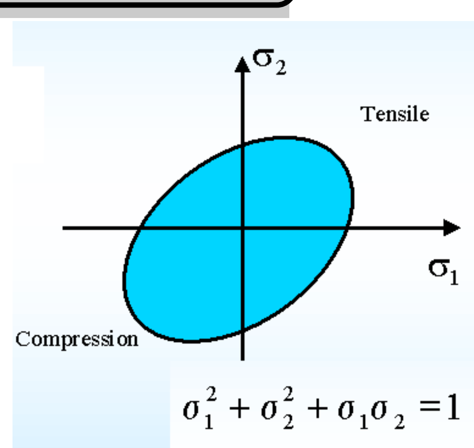
$$\frac{S_y}{N_{fs}} = \sqrt{\sigma_1{}^2 + \sigma_2{}^2 + \sigma_3{}^2 - 2\upsilon(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)}$$

4. Von Mises theory

- Distortion energy per unit volume in the part is equal to that of a tensile test specimen at failure
- If S_y is the yield strength

$$\frac{S_y}{N_{fs}} = \boxed{\sqrt{\sigma_1{}^2 + \sigma_2{}^2 + \sigma_3{}^2 - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)}}$$

Von Mises Effective Stress



How they fare!