

Modeling and Simulation for Failure Analysis

Failure Theories

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ME 345

(adapted from materials prepared by Dr. Kishore Pochiraju)

Performance Concerns

- Does it fail under operating conditions?
 - Define Failure mode
 - Static failure
 - Fatigue failure
 - Wear, Ageing or other durability issues
- Static Failure
 - Yield or Break (Maximum Stress < Yield or Failure stress)
 - Deform beyond limit.
 - (Maximum deflection < Specified δ^{\max})

Failure Theories

MAXIMUM STRESS < MATERIAL STRENGTH

Problems: Failure theories compare two scalar quantities

Stress is a tensor, has six components! Failure also needs to be coordinate independent!

Most Failure Theories make assumptions...

Note: most failure theories are *phenomenological* - not based on atomic level detail

The popular ones are ...

Failure of Ductile Materials

1. The **maximum normal stress** theory - BRITTLE MATERIALS ONLY
2. The **maximum shear stress** theory (also called the Tresca theory) - Good for ductile materials; satisfactory results, easy to use
3. The **maximum strain energy** theory - similar to Von Mises criterion below; better results than above, need Poisson ratio
4. **Von Mises** theory (also called Von Mises - Hencky theory, shear energy theory, and the maximum distortion energy theory) - BEST FOR DUCTILE MATERIALS

MecMovies - Mechanics of Materials

http://web.umd.edu/~mecmovie/index.html

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1. Stress

2. Strain

3. Stress-Strain

4. Simple Design

5. Axial Loading

12. Stress Trans

12.1-8 Transform Eqns

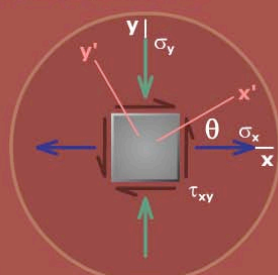
12.9 Mohr's Circle

Background to Mohr's circle of stress

Suppose that normal and shear stresses are known with respect to two orthogonal coordinate axes x and y . Stresses with respect to orthogonal axes x' and y' oriented at an angle θ relative to the x axis may be determined from the equations:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$


There exists one set of axes for which the normal stresses are maximum and minimum. These axes are called the principal axes. The values of the maximum and minimum normal stresses can be computed from:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Relative to the x and y axes, the principal axes are oriented at an angle given by:

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

go ahead

MecMovies 2.00

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Principle stresses - axes where shear stresses are equal to zero

1. Maximum Normal (or principle) Stress

- Assumption - failure occurs when any principle stress exceeds the ultimate yield/failure stress from a simple 1D (tensile or compressive) test
- If S_{yc} is the yield strength in compression, S_{yt} yield strength in tension

$$S_{yc} \leq S_1, S_2, S_3 \leq S_{yt}$$

- To account for safety factor

$$\frac{S_{yc}}{N_{fs}} \leq S_1, S_2, S_3 \leq \frac{S_{yt}}{N_{fs}}$$

2. Maximum Shear Stress theory (Tresca)

- Assumption - failure occurs when any principle shear stress exceeds the ultimate yield/failure stress from a simple 1D (tensile or compressive) test
- If S_y is the yield strength

$$-S_y \leq (S_1 - S_2), (S_2 - S_3), (S_1 - S_3) \leq S_y$$

- To account for safety factor

$$\frac{-S_y}{N_{fs}} \leq (S_1 - S_2), (S_2 - S_3), (S_1 - S_3) \leq \frac{S_y}{N_{fs}}$$

3. Maximum strain energy theory

- Failure occurs when strain energy per unit volume exceeds that for a simple uniaxial test at failure
- Derivation in ME358 text - see page 130
- Usually not used as Von Mises failure is simpler and typically more accurate

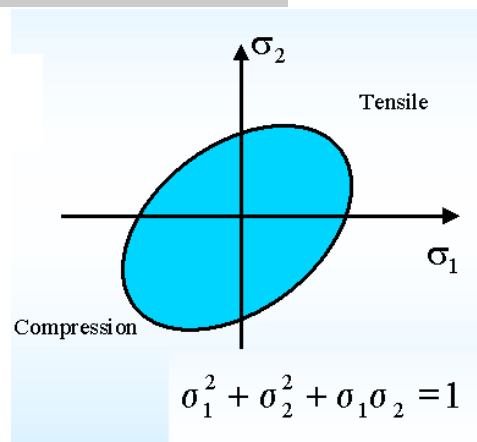
$$\frac{S_y}{N_{fs}} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)}$$

4. Von Mises theory

- Distortion energy per unit volume in the part is equal to that of a tensile test specimen at failure
- If S_y is the yield strength

$$\frac{S_y}{N_{fs}} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)}$$

Von Mises Effective Stress



How they fare!

