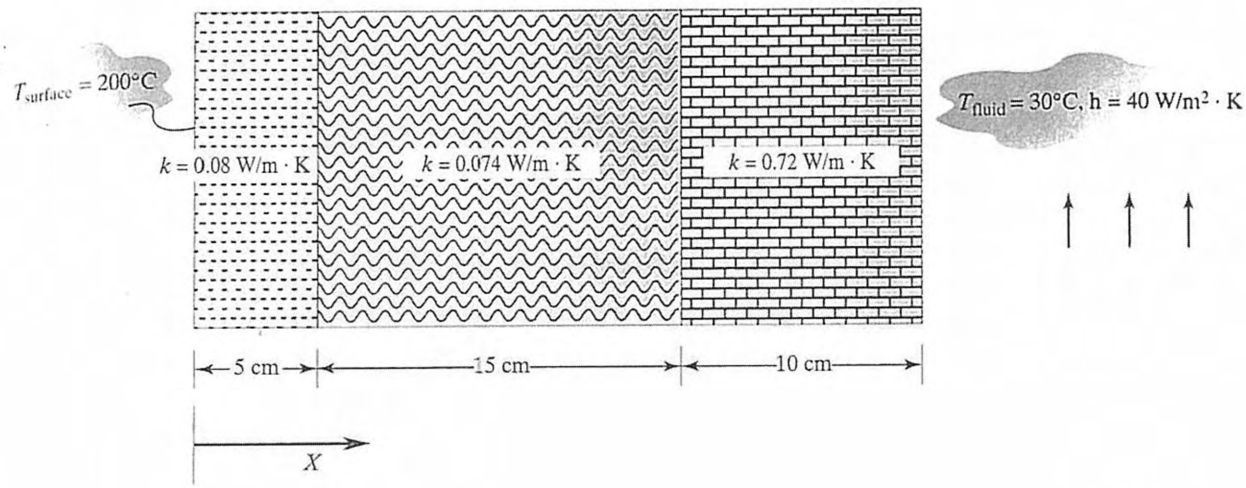
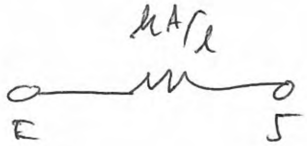
ME 345: Example heat conduction/convection problem via FEM

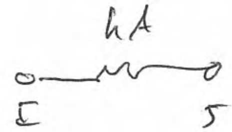
Given the problem below, and assuming a 1D heat transfer problem (i.e. with no heat losses perpendicular to the X direction), calculate the temperatures at X = 5 cm, 15 cm, and 25 cm (at the surface of the wall).



Recall these elements

Solution



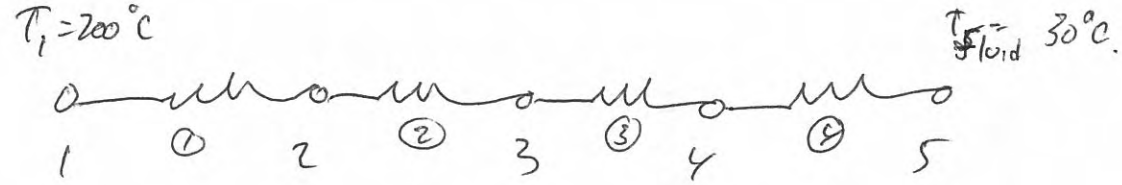


$$\begin{Bmatrix} q_i \\ q_j \end{Bmatrix} = \frac{kA}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} T_i \\ T_j \end{Bmatrix}$$

Conduction

$$\begin{Bmatrix} q_i \\ q_j \end{Bmatrix} = hA\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} T_i \\ T_j \end{Bmatrix}$$

Convection



assume 1

Element 1: conduction $\frac{k_1 A_1}{l_1} = \frac{(.08\ W/mK)(A)}{.05\ m} = 1.6\ W/K$

Element 2: conduction $\frac{k_2 A_2}{l_2} = \frac{(.074\ W/mK)(A)}{.15\ m} = .493\ W/K$

Element 3: conduction $\frac{k_3 A_3}{l_3} = \frac{(.72\ W/mK)(A)}{.1\ m} = 7.2\ W/K$

Element 4: Convection $h_4 A_4 = 40\ W/K$

Note: setting $A = 1\ m^2$ is equivalent to looking at heat flux (heat per unit area)

$$
\begin{bmatrix} Q_1 = ? \\ Q_2 = 0 \\ Q_3 = 0 \\ Q_4 = 0 \\ Q_5 = ? \end{bmatrix}
=
\begin{bmatrix}
1.6 & -1.6 & 0 & 0 & 0 \\
-1.6 & 1.6 + .493 & -.493 & 0 & 0 \\
0 & -.493 & .493 + 7.2 & -7.2 & 0 \\
0 & 0 & -7.2 & 7.2 + 40 & -40 \\
0 & 0 & 0 & -40 & 40
\end{bmatrix}
\begin{bmatrix} T_1 = 200^\circ C \\ T_2 \\ T_3 \\ T_4 \\ T_5 = 30^\circ C \end{bmatrix}
$$

Five equations, five unknowns: Q_1, Q_5, T_2, T_3, T_4.

Can reduce by "moving over" the BC's...

$$
\begin{bmatrix} Q_2 = 0 + (200)(1.6) \\ Q_3 = 0 \\ Q_4 = 0 + (40)(30) \end{bmatrix}
=
\begin{bmatrix}
2.093 & -.493 & 0 \\
-.493 & 7.693 & -7.2 \\
0 & -7.2 & 47.200
\end{bmatrix}
\begin{bmatrix} T_2 \\ T_3 \\ T_4 \end{bmatrix}
$$

$$
\begin{bmatrix} T_2 \\ T_3 \\ T_4 \end{bmatrix}
=
\begin{bmatrix} 162.3^\circ C \\ 39.9^\circ C \\ 31.5^\circ C \end{bmatrix}
$$

NOTE 1: To check, put these T's back into large matrix and multiply, will get $Q_1 = -Q_5$ (i.e. steady state)

Note 2: does °C or °K matter? (Ans: not if consistent!)