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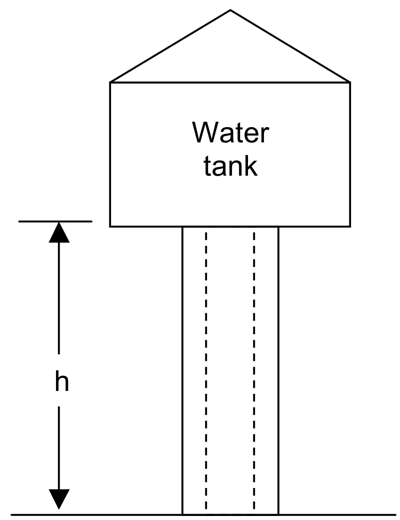
ME345 Modeling and Simulation
EXCEL Optimization Example: Water Tower

A hollow circular column is to be designed for the water tank shown below. The column should be 15 meters high (i.e. h = 15 m) and must support 50,000 kg of water in the tank. The cylindrical column is to be fabricated from steel with the following material properties:

- Density = 7827 kg/m^3
- Young's modulus (E) = 200 GPa
- Yield strength = 180 MPa
- Poisson's ratio (ν) = 0.27



Create an Excel spreadsheet that will determine the optimum outer diameter (D_o) and thickness (t) to minimize the mass of the column while being subjected to the following constraints:

1. The outer diameter of the column must be less than or equal to 0.5 m. (Of course, it must also be greater than zero!)

2. The (compressive) stress in the column must be less than the yield strength of the material (assume that the contribution of the mass of the column is negligible in this calculation).

3. The stress in the column must be less than the critical stress that would initiate Euler buckling according to Euler's buckling formula. For these end conditions, the critical Euler stress for buckling is given as

$$\sigma_{Euler} = \frac{\pi^2 E I}{4 A L^2}, \qquad \text{where } I = \frac{\pi}{64}\left(D_o^4 - D_i^4\right)$$

   where D_o and D_i are the outer and inner diameters of the hollow column, respectively, and A is the cross-sectional area.

For the last two constraints, a safety factor of 3.0 should be applied (and be a separate cell within Excel so that it can be varied if desired).

<u>**At this stage**</u>, you can find the optimal column outer diameter and thickness that minimizes the mass of the column while satisfying the above constraints.

- As initial guesses for the diameter and thickness, please use an outer diameter of 0.25 m and an inner diameter of 0.05 m. *To check your results, when I use these parameters I get that the mass of the column is* 5532 kg. (Note: if you do not have this value, your Excel solver may not be set up properly.)

- Note that there are multiple local minimum for this problem! Using the initial values above, I think that I found a global minimum. However, if I used initial conditions of OD = 0.1 m and ID = 0.098 m, upon using the solver I get a **local solution (namely, the mass is equal to 10774 kg)**. (Note that depending on your Excel Solver settings you may not get the local solution, but instead get the desired *global* solution as found above.)

SOLUTION: My solution to this problem was a mass of 2673 kg.

Part B. To complete the problem, now assume that to prevent catastrophic failure of the structure in the event of an earthquake, the natural frequency of the system should be greater than 2.5 Hz. (As an overly conservative estimate damping in this problem will be assumed negligible.)

Here one can estimate the natural frequency of the structure by using standard expressions for the appropriate equivalent mass and equivalent spring. In this case, the equivalent mass is that of a cantilever beam of uniformly distributed mass *m* carrying an end mass M, and the equivalent spring is that for a cantilever with an end load, i.e.,

$$\omega_{eq} = \sqrt{\frac{k_{eq}}{m_{eq}}} \qquad \text{where...} \qquad m_{eq} = M + 0.23m, \qquad k_{eq} = \frac{3EI}{h^3},$$

Adding this additional constraint, now find a suitable solution that satisfies all necessary constraints.

- As another check, when I use an outer diameter of 0.25 m and an inner diameter of 0.05 m, I calculate a natural frequency of 0.815 Hz. However, **this initial condition does not converge** to a suitable solution. (For my solution initial guesses of OD = 0.4 m and ID = 0.2 m converges… but using starting values of 0.5m and 0.1 m my solutions converges to a global minimum!)

SOLUTION: My solution to this extended problem was a mass of 8383 kg.