# EXAMPLE 2

~~EXAMPLE 3~~


1) CL $\quad f_d = b v_d$

2) GC $\quad v_d = v_m$

3) 


$\overset{+}{\rightarrow}\ \sum F_x = m a_m$

$-f_d = m a_m$

4) SV : $v_m$

5) $v_m' = a_m = -\frac{f_d}{m} = \frac{-b v_d}{m} = \frac{-b v_m}{m}$

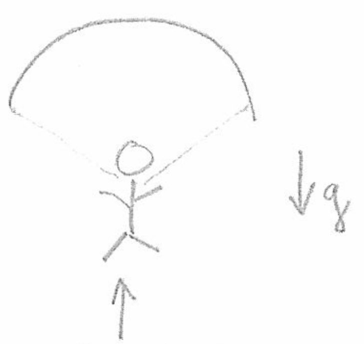
$$\boxed{v_m' = -\frac{b}{m} v_m}$$

Analytical solution : Guess $v_m(t) = Ae^{rt}$.
Follow procedure from before

$$v_m(t) = A e^{-\frac{b}{m} t}$$ ← general solution.

# EXAMPLE 3

## ~~EXAMPLE 2~~ (Parachute problem)

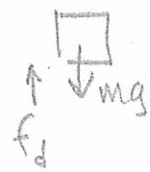



$f_d = b v_d$
(air resistence)

1] CL $\quad f_d = b\, v_d$

2] GC $\quad v_d = v_m$

3]




$\overset{+}{\downarrow} \Sigma F_y = m a_m$

$mg - f_d = m a_m$

4] SV: $\quad v_m$

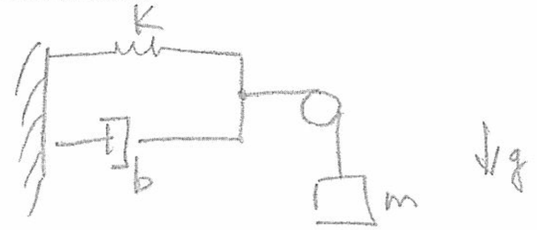
5] State equation(s):

$$v_m' = a_m = \frac{1}{m}(mg - f_d) = \frac{1}{m}(mg - b v_d)$$

$$\boxed{v_m' = \frac{1}{m}(mg - b v_m)}$$ ← appropriate state equation

# EXAMPLE 4

~~Example 4~~




1] CL: $f_s = K x_s$ $\quad f_d = b v_d$

2] GC $\quad v_s = v_d$

$v_s = v_m$

3] FBD



$\downarrow \Sigma F = ma$ $\qquad \overset{+}{\rightarrow} \Sigma F = \cancel{ma}^{=0}$ (massless connection)

$-f_{rope} + mg = m a_m$ $\qquad f_{rope} - f_s - f_d = 0$

4] SV: $x_s, v_m$ (here we have two state variables ... need two state equations)

5] Solve for each state equation separately:

$\underline{x_s' = v_s = v_m}$ ✓

$v_m' = a_m = \frac{1}{m}(-f_{rope} + mg) = \frac{1}{m}[-(f_s + f_d) + mg]$

$= \frac{1}{m}[-K x_s - b v_d + mg]$

$\underline{v_m' = \frac{1}{m}[-K x_s - b v_m + mg]}$ ✓

Thus the state equations are:

$$\begin{cases} x_s' = v_m \\ v_m' = \frac{1}{m}(mg - kx_s - bv_m) \end{cases}$$

Can write in matrix form as

$$\begin{Bmatrix} x_s' \\ v_m' \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/m & -b/m \end{bmatrix} \begin{Bmatrix} x_s \\ v_m \end{Bmatrix} + \begin{Bmatrix} 0 \\ g \end{Bmatrix}$$

Can also write in SECOND ORDER form:

$$x_s'' = ?$$

$$x_s'' = \frac{d}{dt}(x_s') = \frac{d}{dt}(v_m) = v_m'$$

$$x_s'' = v_m' = \frac{1}{m}(mg - kx_s - bv_m)$$

now $v_m$ is NOT a SV!

$$x_s'' = \frac{1}{m}(mg - kx_s - bx_s')$$

this is OK; lower order derivative

Can re-write in standard form:

$$\boxed{x_s'' + \frac{b}{m}x_s' + \frac{k}{m}x_s = g}$$

in math terms, this is "non-homogeneous"

# EXAMPLE 5

**PROBLEM 3. (22 points TOTAL)**

Consider the one dimensional problem shown in the Figure to the right, consisting of two springs, a damper, and a mass. **Assume gravity acts in the +y direction as shown.**

Note that the damper and the spring *k2* are in series.

Derive the **first order** state equations describing the system behavior. Clear and legible work will be eligible for partial credit. (22 points)

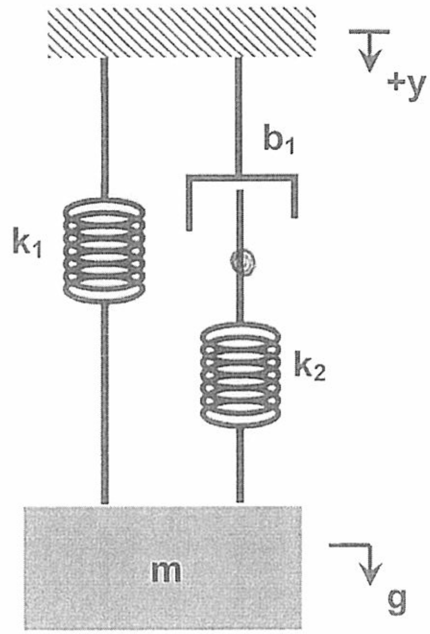



**Figure. Schematic for Problem 3.**

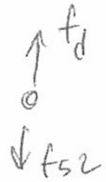
1) CL (3) $f_{s1} = k_1 x_{s1}$ $\quad f_{b1} = b_1 v_{d1}$

$f_{s2} = k_2 x_{s2}$

2) GC (3) $v_{s1} = v_m$ $\quad v_d + v_{s2} = v_m$

3) FBD

(3)

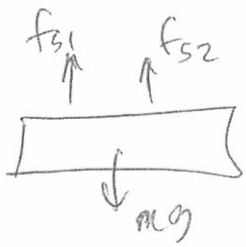



$\Sigma F_y = m\cancel{a_y}^{\,0}$

$\boxed{f_d = f_{s2}}$

(also, because in series)

(3)




$+\downarrow \Sigma F_y = m a_y$

$\boxed{mg - f_{s1} - f_{s2} = m a_m}$

(2) 4) SV's : $x_{s1}, x_{s2}, v_m$

5) (2) $x_{s1}' = v_{s1} = \underline{v_m}$ ✓

(3) $x_{s2}' = v_{s2} = v_m - v_d = v_m - \frac{f_d}{b} = v_m - \frac{f_{s2}}{b} = \underline{v_m - \frac{k_2 x_{s2}}{b}}$ ✓

(3) $v_m' = a_m = \frac{1}{m}(mg - f_{s1} - f_{s2}) = \underline{\frac{1}{m}(mg - k_1 x_{s1} - k_2 x_{s2})}$