An Algebraic Approach to Blind Carrier Offset and Code Timing Estimation for DS-CDMA Systems

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***Abstract*—This letter presents a blind method for joint carrier offset and code timing estimation in direct-sequence code-division multiple-access systems. The proposed method is arrived at by exploiting the subspace structure of the observed signal along with analytical tools of polynomial matrices. It yields an algebraic solution which avoids a multidimensional search over the parameter space, a procedure that is computationally involved and often suffers local convergence. The proposed method is multiaccess interference resistant. Furthermore, it can be used in time-varying, multipath fading channels.**

***Index Terms*—Carrier offset, code division multiple access (CDMA), code timing, joint code and frequency synchronization, time-varying multipath fading.**

I. Introduction

INITIAL code and carrier synchronization that precedes symbol detection is a challenging problem in direct-sequence (DS) code-division multiple-access (CDMA) systems [1]. A conventional technique is to search serially through all potential code phases and frequencies for the desired user, meanwhile treating the multiaccess interference (MAI) as noise [1, ch. 5]. This approach, although easy to implement, suffers the MAI, particularly in a near-far environment [2], [3]. A number of MAI-resistant synchronization schemes have been introduced recently (e.g., [2]–[4] and references therein). Most of these schemes, however, consider only code synchronization, assuming that carrier synchronization has been achieved at a prior stage. One exception is a joint carrier offset and code timing estimator proposed in [5], where frequency-flat and time-invariant channels are considered. This estimator involves a multidimensional (MD) search over the parameter space, which is computationally involved and requires accurate initial parameter estimates that are often difficult to obtain.

We present herein an algebraic method for joint carrier offset and code timing estimation in CDMA systems. Our method is reminiscent of a technique proposed in [6] for joint channel and carrier offset estimation. However, there are notable distinctions. One is that [6] assumes a strictly *quasi-synchronous* system in which intersymbol interference (ISI)-free chip samples are available. Also assumed there is a finite-duration impulse response (FIR) channel model with *time-invariant and congruent* channel taps. We relax these assumptions to allow for channel variations and adjacent-symbol ISI (no ISI-free chips are available); the path delays are otherwise unrestricted.

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Notation

Vectors (matrices) are denoted by boldface lower (upper) case letters; all vectors are column vectors; superscripts $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$ denote the complex conjugate, transpose, and conjugate transpose, respectively; $\mathbf{I}_N$ denotes the $N \times N$ identity matrix; $\mathbf{0}$ denotes an all-zero vector/matrix; $\|\cdot\|$ denotes the vector 2-norm; $E\{\cdot\}$ denotes the statistical expectation; and $\mathrm{diag}\{\cdot\}$ denotes a diagonal matrix.

II. Data Model

Consider a baseband asynchronous K-user DS-CDMA system. The transmitted signal for user k is given by $s_k(t) = \sum_{m=0}^{M-1} d_k(m)\psi_k(t - mT_s)$, where M is the number of symbols considered for synchronization; $d_k(m)$ and $\psi_k(t)$ denote the mth data symbol and spreading waveform, respectively, for user k; and $T_s = NT_c$ denotes the symbol interval, with T_c and N being the chip interval and spreading gain, respectively. Signal $s_k(t)$ passes through a baseband time-varying frequency-selective channel. The received signal is given by

$$y(t) = \sum_{k=1}^{K}\sum_{l=1}^{L_k} \alpha_{k,l}(t) s_k(t - \tau_{k,l}) e^{j\Omega_k(t-\tau_{k,l})} + n(t) \quad (1)$$

where L_k denotes the number of paths of user k, Ω_k denotes the carrier frequency offset, $\alpha_{k,l}(t)$ and $\tau_{k,l}$ denote the time-varying fading coefficient and code timing, respectively, associated with path l of user k, and $n(t)$ denotes the noise. We assume that the delays of a particular user are distinct and remain (approximately) unchanged during acquisition. We also assume that the delay spread is within one symbol interval, i.e., $\tau_{k,l} < T_s$. This could be the case when the cell size is small relative to the transmission rate, or due to a prior coarse synchronization which pulls the timing uncertainty to within a symbol interval [3]. The receiver front-end is a chip-matched filter (CMF) which outputs samples $y(l) = y(t)|_{t=lT_i}$, where $T_i = T_c/Q$ is the sampling interval, with $Q \geq 1$ denoting the *oversampling factor* (an integer). It is convenient to write $\tau_{k,l}$ as $\tau_{k,l} = (p_{k,l} + \mu_{k,l})T_i$, where $p_{k,l}$ denotes an integer between 0 and $NQ-1$ and $\mu_{k,l} \in [0, 1)$ denotes the fractional delay.

Let $\mathbf{y}(m) \triangleq [y(mNQ), \ldots, y(mNQ + NQ - 1)]^T$, $\mathbf{c}_k \triangleq [c_k(0), \ldots, c_k(NQ - 1)]^T$, where $c_k(n) =$

$1/(T_i)\int_{(n-1)T_i}^{nT_i}\psi_k(t)\mathrm{d}t$. Due to asynchronism, two adjacent symbols in each path contribute to $\mathbf{y}(m)$ [2]–[5]

$$\mathbf{y}(m)=\sum_{k=1}^{K}\mathbf{\Sigma}_k(\omega_k)\mathbf{A}_k(\tau_k)\boldsymbol{\beta}_k(m)+\mathbf{n}(m),\quad m=0,1,\ldots,M-1 \tag{2}$$

where $\omega_k \triangleq \Omega_k T_i$ denotes the *normalized carrier offset*, $\boldsymbol{\tau}_k \triangleq [\tau_{k,1},\cdots,\tau_{k,L_k}]^T$

$$\begin{aligned}\mathbf{\Sigma}_k(\omega_k) &\triangleq \mathrm{diag}\{1,e^{j\omega_k},\ldots,e^{j\omega_k(NQ-1)}\}\\ \mathbf{A}_k(\boldsymbol{\tau}_k) &\triangleq [\mathbf{a}_k(\tau_{k,1}),\bar{\mathbf{a}}_k(\tau_{k,1}),\ldots,\mathbf{a}_k(\tau_{k,L_k}),\bar{\mathbf{a}}_k(\tau_{k,L_k})]\\ \boldsymbol{\beta}_k(m) &\triangleq [\beta_{k,1}(m),\bar{\beta}_{k,1}(m),\ldots,\beta_{k,L_k}(m),\bar{\beta}_{k,L_k}(m)]^T\\ \beta_{k,l}(m) &\triangleq \alpha_{k,l}(m)d_k(m-1)e^{j\omega_k mNQ}\\ \bar{\beta}_{k,1}(m) &\triangleq \alpha_{k,l}(m)d_k(m)e^{j\omega_k mNQ}\end{aligned}$$

and $\mathbf{n}(m)$ denotes the $NQ\times 1$ noise vector. Furthermore, we have [4]

$$\begin{aligned}\mathbf{a}_k(\tau_{k,l}) &= \mathbf{F}_k(p_{k,l})\boldsymbol{\mu}_{k,l}\\ \bar{\mathbf{a}}_k(\tau_{k,l}) &= \bar{\mathbf{F}}_k(p_{k,l})\boldsymbol{\mu}_{k,l}\\ \boldsymbol{\mu}_{k,l} &\triangleq [1-\mu_{k,l},\mu_{k,l}]^T\end{aligned} \tag{3}$$

where $\mathbf{F}_k(p_{k,l})$ [respectively, $\bar{\mathbf{F}}_k(p_{k,l})$] are $NQ\times 2$ matrices with the first and second column consisting of the *acyclic left shift* (respectively, *acyclic right shift*) of $\mathbf{c}_k$ by $p_{k,l}$ and $p_{k,l}+1$ samples, respectively; see [4, eqs. (38) and (39)] for exact expressions of $\mathbf{F}_k(p_{k,l})$ and $\bar{\mathbf{F}}_k(p_{k,l})$.

The problem of interest is to estimate the code timing $\{\boldsymbol{\tau}_k\}_{k=1}^K$, and the carrier offset $\{\omega_k\}_{k=1}^K$ from the received data $\{\mathbf{y}(m)\}_{m=0}^{M-1}$, without any knowledge of the transmitted information symbols.

III. Joint Carrier Offset and Code Timing Estimation

Let $\mathbf{R}_y \triangleq E\{\mathbf{y}(m)\mathbf{y}(m)^H\}$ denote the data covariance matrix, and $L\triangleq\sum_{k=1}^K L_k$ the total number of paths of all users. Assuming that $NQ>2L$, the eigendecomposition of $\mathbf{R}_y$ can be expressed as $\mathbf{R}_y=\mathbf{E}_s\Lambda_s\mathbf{E}_s^H+\sigma_n^2\mathbf{E}_n\mathbf{E}_n^H$, where Λ_s is a diagonal matrix made from the $2L$ largest eigenvalues associated with the eigenvectors that form $\mathbf{E}_s\in\mathbb{C}^{NQ\times 2L}$, and $\mathbf{E}_n\in\mathbb{C}^{NQ\times(NQ-2L)}$ contains the eigenvectors corresponding to the smallest eigenvalue σ_n^2 with multiplicity $NQ-2L$, with σ_n^2 denoting the variance of the channel noise. Since $\mathbf{E}_n$ spans the orthogonal complement of the signal subspace, we have

$$\mathbf{E}_n^H\mathbf{\Sigma}_k(\omega_k)\mathbf{A}_k(\boldsymbol{\tau}_k)=\mathbf{0},\qquad k=1,\cdots,K. \tag{4}$$

Estimates of ω_k and $\boldsymbol{\tau}_k$ can be obtained from the above nonlinear equation through a search over an (L_k+1)-dimensional parameter space, which is computationally involved and may suffer local convergence. Note that the scheme in [5] is a special case of the above approach when the channel is frequency-flat and time-invariant.

To seek an alternative solution, we invoke the theory of polynomial matrices (e.g., [7]). For the lth path of user k, (4) is equivalent to (hereafter, we drop the subscripts k and l for notational brevity): $\mathbf{E}_n^H\mathbf{\Sigma}(\omega)\mathbf{a}(\tau)=\mathbf{0}$ and $\mathbf{E}_n^H\mathbf{\Sigma}(\omega)\bar{\mathbf{a}}(\tau)=\mathbf{0}$. Let $z\triangleq e^{j\omega}$ (hence, $\mathbf{\Sigma}(z)=\mathrm{diag}\{1,z,\cdots,z^{NQ-1}\}$). We have [cf. (3)]

$$\mathbf{\Psi}(z)\boldsymbol{\mu}=\mathbf{0}\quad \bar{\mathbf{\Psi}}(z)\boldsymbol{\mu}=\mathbf{0} \tag{5}$$

where $\mathbf{\Psi}(z)\triangleq\mathbf{E}_n^H\mathbf{\Sigma}(z)\mathbf{F}(p)$ and $\bar{\mathbf{\Psi}}(z)\triangleq\mathbf{E}_n^H\mathbf{\Sigma}(z)\bar{\mathbf{F}}(p)$ are both $(NQ-2L)\times 2$ polynomial matrices in z of order $NQ-1$. Equation (5) indicates that $\boldsymbol{\mu}$, which is always nontrivial by construction [cf. (3)], is a null vector of both $\mathbf{\Psi}(z)$ and $\bar{\mathbf{\Psi}}(z)$. Therefore, the nullity of $\mathbf{\Psi}(z)$ and $\bar{\mathbf{\Psi}}(z)$ is at least one. Let $\mathbf{\Psi}(z)\triangleq[\boldsymbol{\psi}_1(z),\boldsymbol{\psi}_2(z)]$ and $\bar{\mathbf{\Psi}}(z)\triangleq[\bar{\boldsymbol{\psi}}_1(z),\bar{\boldsymbol{\psi}}_2(z)]$. The previous analysis also indicates that $\boldsymbol{\psi}_1(z)$ and $\boldsymbol{\psi}_2(z)$ [respectively, $\bar{\boldsymbol{\psi}}_1(z)$ and $\bar{\boldsymbol{\psi}}_2(z)$] are linearly dependent. Hence, we can construct projection matrices $\mathbf{P}_{\boldsymbol{\psi}_2}^{\perp}(z)$ and $\bar{\mathbf{P}}_{\bar{\boldsymbol{\psi}}_2}^{\perp}(z)$ that project to the orthogonal complement of vectors $\boldsymbol{\psi}_2$ and $\bar{\boldsymbol{\psi}}_2$, respectively, i.e.,

$$\mathbf{P}_{\boldsymbol{\psi}_2}^{\perp}(z)\boldsymbol{\psi}_1(z)=\mathbf{0}\qquad \bar{\mathbf{P}}_{\bar{\boldsymbol{\psi}}_2}^{\perp}(z)\bar{\boldsymbol{\psi}}_1(z)=\mathbf{0}. \tag{6}$$

To construct $\mathbf{P}_{\boldsymbol{\psi}_2}^{\perp}(z)$ and $\bar{\mathbf{P}}_{\bar{\boldsymbol{\psi}}_2}^{\perp}(z)$, we note that by the Bezout identity [7, p. 379] (also see [6]), there exist $1\times(NQ-2L)$ polynomial vectors $\mathbf{g}^H(z)$ and $\bar{\mathbf{g}}^H(z)$ such that

$$\mathbf{g}^H(z)\boldsymbol{\psi}_2(z)=z^{-n_o}\qquad \bar{\mathbf{g}}^H(z)\bar{\boldsymbol{\psi}}_2(z)=z^{-\bar{n}_o} \tag{7}$$

for some appropriate delays n_o and $\bar{n}_o$. It follows that

$$\begin{aligned}\mathbf{P}_{\boldsymbol{\psi}_2}^{\perp}(z) &= z^{-n_0}\mathbf{I}_{NQ-2L}-\boldsymbol{\psi}_2(z)\mathbf{g}^H(z)\\ \bar{\mathbf{P}}_{\bar{\boldsymbol{\psi}}_2}^{\perp}(z) &= z^{-\bar{n}_0}\mathbf{I}_{NQ-2L}-\bar{\boldsymbol{\psi}}_2(z)\bar{\mathbf{g}}^H(z).\end{aligned} \tag{8}$$

Using (6) along with the projection matrices constructed in (8), an estimate of the carrier offset is obtained as

$$\hat{\omega}=\arg\min_{\omega}\{\|\mathbf{P}_{\boldsymbol{\psi}_2}^{\perp}(z)\boldsymbol{\psi}_1(z)\|^2+\|\bar{\mathbf{P}}_{\bar{\boldsymbol{\psi}}_2}^{\perp}(z)\bar{\boldsymbol{\psi}}_1(z)\|^2\} \tag{9}$$

which need be minimized for all possible values of p. This can be done by using polynomial rooting, similar to the root-MUSIC algorithm [8]. Once an estimate of ω is known, $\mathbf{\Psi}(z)$ and $\bar{\mathbf{\Psi}}(z)$ are parameterized by the integer delay p. Hence, we may write them as $\mathbf{\Psi}(p)$ and $\bar{\mathbf{\Psi}}(p)$. It follows from (5) that we can use the following criterion to estimate the integer and fractional delay:

$$\{\hat{p},\hat{\mu}\}=\arg\min_{p,\mu}\{\|\mathbf{\Psi}(p)\boldsymbol{\mu}\|^2+\|\bar{\mathbf{\Psi}}(p)\boldsymbol{\mu}\|^2\}. \tag{10}$$

In the multipath case, there are L_k solutions corresponding to the L_k paths of user k, all achieving identically the same minimum of the cost function, which is zero if $\mathbf{E}_n$ is known exactly (see discussions next). We remark that the above criterion is equivalent to the one employed in [2, eq. (23)] for code acquisition assuming no carrier offset. As shown there, it can be efficiently minimized by a sequence of polynomial rooting.

Remark 1: The ideal noise eigenvectors $\mathbf{E}_n$ have to be estimated from the observed data. We can use the sample eigenvector estimates obtained from the eigendecomposition of the sample covariance matrix $\hat{\mathbf{R}}_y\triangleq(1/M)\sum_{m=0}^{M-1}\mathbf{y}(m)\mathbf{y}^H(m)$. Alternatively, they may be computed adaptively via subspace tracking algorithms (e.g., [9]). Due to finite-sample errors in the noise eigenvector estimates, (6) holds only approximately in this case. Hence, we seek code timing estimates as those achieving the smallest minima of (10).

Remark 2: For the delays and carrier offsets to be uniquely identifiable, it is necessary for the matrices $\mathbf{P}_{\boldsymbol{\psi}_2}^{\perp}(z)$ and $\bar{\mathbf{P}}_{\bar{\boldsymbol{\psi}}_2}^{\perp}(z)$ in (6) as well as $\mathbf{\Psi}(z)$ and $\bar{\mathbf{\Psi}}(z)$ in (5) to have nullity exactly

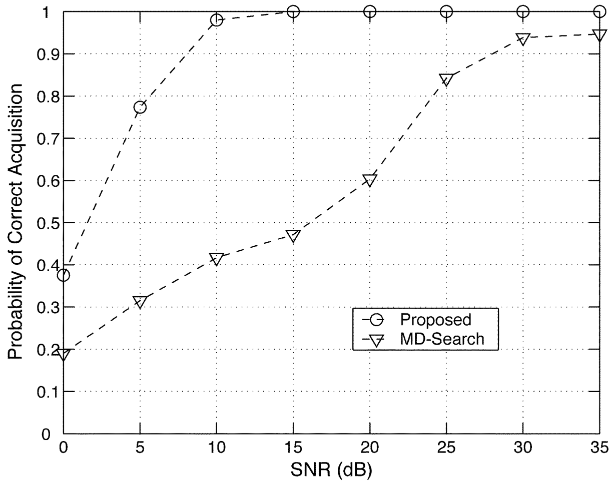

Fig. 1. Probability of correct acquisition in near-far time-varying multipath channels.

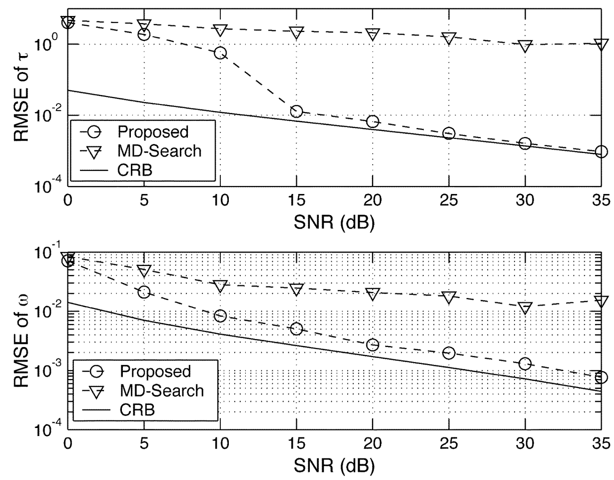

Fig. 2. RMSE and CRB of τ (top) and ω (bottom) in near-far time-varying multipath channels.

equal to one. The exact identification conditions and their implications are being investigated and will be reported elsewhere.

Remark 3: A necessary condition for this method to work is to have $NQ > 2L$ so that $\mathbf{E}_n$ is nontrivial. For large L (e.g., in overloaded systems and/or with large delay spread), the oversampling factor Q would have to be increased, which, in turn, would require excess bandwidth to avoid ill-conditioning problems. An interesting future subject is to compare both training-based and blind estimators with comparable bandwidth expansion, in order to determine which of these two different types of methods utilize bandwidth more efficiently.

IV. Numerical Results

We consider an asynchronous DS-CDMA system that uses $N = 15$ large Kasami codes and BPSK constellation. The simulated channel is frequency-selective, with two independent paths for each user, and time-varying, generated by the Jakes' model [10] with a normalized Doppler rate of 0.0067 (i.e., carrier frequency $=$ 900 MHz, symbol rate $=$ 10 kHz, and mobile speed $=$ 50 mi/h). In what follows, we consider near-far environments without enforcing stringent power control, where the total (from all paths) average power for the desired user is scaled so that $P_1 \triangleq \sum_{l=1}^{L_1} P_{1,l} = 1$, while the power for the $K-1$ interfering users follows a log normal distribution with a mean power $\bar{P}$ dB higher than that of the desired user. The near-far ratio (NFR) is defined as $\bar{P}$ (in decibels). The *normalized* carrier offset is set to $\omega_k = 0.1$ for the desired user. We compare the proposed estimator and the multidimensional search (MD-Search) based scheme (4). The latter is initialized by estimates obtained by the method in [2], assuming zero initial carrier offset, and then iterates using the Matlab nonlinear optimization routine `fminsearch` till convergence. One performance measure is the *probability of correct acquisition* (PCA), defined as the probability of the event that the code-timing estimation error is less than $T_c/2$. Another performance measure, at a finer scale, is the root mean-squared error (rmse) of the code timing and carrier offset estimates. Fig. 1 shows the PCA of the two methods as a function of the signal-to-noise ratio (SNR) when $K = 5$, $M = 200$ and $Q = 2$, and $\mathrm{NFR} = 10$ db. The MD-Search scheme is seen to suffer local convergence caused by poor initialization, yielding a lower PCA than the proposed method at low SNR. Fig. 2 depicts the rmse of the code timing (top) and carrier offset (bottom) estimates, along with the Cramér–Rao bound (CRB) that gives the best performance that can be achieved. Note that the rmse results are obtained by averaging *all* Monte Carlo runs (a total of 300 independent trials have been performed).

The MD-Search yields much higher rmse than the proposed scheme, and saturates at high SNR due to local convergence caused by poor initialization. It is seen that the rmse of the proposed scheme, for both carrier offset and code timing estimates, approaches the CRB as the SNR increases.

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