Joint blind channel estimation and interference suppression with efficient implementation for OFDM systems

K. Amleh, H. Li and R. Wang

A joint blind channel estimation and interference suppression scheme with efficient implementation for orthogonal frequency-division multiplexing (OFDM) systems is presented. In particular, an equalising filterbank is designed and the channel estimate from a lower bound of the filterbank output power is derived. The proposed scheme and exhibits low sensitivity to unknown narrowband interference. It compares favourably with a subspace blind channel estimator.

Introduction: Channel estimation for OFDM systems has been investigated recently in numerous studies. Existing channel estimation schemes rely on either explicit training (e.g. [1]) or some inherent structure of the received signal such as the subspace structure exploited by [2, 3]. In this Letter, we consider the problem of joint blind channel estimation and interference suppression for OFDM. A generalised multichannel minimum variance principle (e.g. [4]) is invoked to design an equalising filterbank that preserves desired signal components and suppresses the overall interference. To avoid multidimensional search in the parameter space, we utilise an asymptotically (in SNR) tight lower bound of the filterbank output power [5] for channel estimation, which reduces the problem to a quadratic minimisation. Because of the large dimensions of the matrices involved in the proposed method, a brute-force implementation is not applicable. We present a computational analysis that exploits the structure of these matrices for efficient implementation. Through numerical examples, we show that the proposed scheme and exhibits low sensitivity to unknown narrowband interference. In addition, it compares favourably with the subspace blind channel estimator [2] in the presence of unknown narrowband interference.

Problem formulation: Consider an OFDM system in which a serial of information symbols are blocked into $K\times 1$ vectors $\mathrm{s}(n)=[s(nK),\ldots,\ s(nK+K-1)]^T$, which are linearly transformed into $\boldsymbol{u}(n)=\boldsymbol{F}\boldsymbol{s}(n)$ by $J\times K$ matrix $\boldsymbol{F}\triangleq[\bar{\boldsymbol{F}}_1^T,\ \bar{\boldsymbol{F}}^T]^T$, where $\bar{F}$ denotes the $K\times K$ IDFT matrix, and $\bar{F}_1$ is formed from the last $\mu\triangleq J-K$ rows of $\bar{\boldsymbol{F}}$, where μ is the length of the cyclic prefix.

In what follows, we will process a block of $N\geq 1$ OFDM symbols simultaneously. Let $\mathrm{s}_N(n)=[\mathrm{s}^T(nN),\ldots,\ \mathrm{s}^T(nN+N-1)]^T_{KN\times 1}$ and $\boldsymbol{u}_N(n)=[\boldsymbol{u}^T(nN),\ldots,\boldsymbol{u}^T(nN+N-1)]^T_{JN\times 1}=(\boldsymbol{I}_N\otimes\boldsymbol{F})\boldsymbol{s}_N(n)$. The channel is modelled as an FIR filter $\boldsymbol{h}\triangleq[h(0), h(1),\ldots,h(L)]^T$, where L is the channel order. Hence, the overall received samples is $JN+L$. We discard the first and last L samples and form a $(JN-L)\times 1$ vector $\boldsymbol{y}_N(n)$ [2]:

$$\boldsymbol{y}_N(n)=\boldsymbol{H}(\boldsymbol{I}_N\otimes\boldsymbol{F})\boldsymbol{s}_N(n)+\boldsymbol{w}_N(n)+\boldsymbol{e}_N(n) \tag{1}$$

where $\boldsymbol{w}_N(n)$ and $\boldsymbol{e}_N(n)$ denote the interference and channel noise vectors, respectively, and $\boldsymbol{H}$ is a $(JN-L)\times JN$ Toeplitz matrix with first row $[h(L),\ \ldots,\ h(0),\ \boldsymbol{0}_{1\times(JN-L-1)}]$ and first column $[h(L),\ \boldsymbol{0}_{1\times(JN-L-1)}]^T$. Note that $\boldsymbol{H}(\boldsymbol{I}_N\otimes\boldsymbol{F})$ has full column rank if $N\geq L/(U-K)$.

The problem of interest is to estimate the channel coefficients $\{h(n)\}_{n=0}^{L}$ from the observed data without any knowledge of the transmitted symbols.

Proposed scheme: We design an equalising filterbank of KN FIR filters $G\in\mathbb{C}^{((JN-L)\times KN)}$ according to the following generalised minimum variance criterion:

$$\begin{aligned}\boldsymbol{G}&=\arg_{\boldsymbol{G}\in\mathbb{C}}\min_{(JN-L)\times KN}\mathrm{tr}\{\boldsymbol{G}^H\boldsymbol{R}\boldsymbol{G}\}\\ &\text{subject to }\boldsymbol{G}^H\boldsymbol{H}(\boldsymbol{I}_N\otimes\boldsymbol{F})=\boldsymbol{I}_{KN}\end{aligned} \tag{2}$$

where $\boldsymbol{R}\triangleq E\{\boldsymbol{y}_N(n)\boldsymbol{y}_N^H(n)\}$, and the constraint ensures that each filter (i.e. one column of $\boldsymbol{G}$) will pass only one signal undistorted. Using the Lagrange multiplier, the solution is

$$\boldsymbol{G}=\boldsymbol{R}^{-1}\boldsymbol{H}(\boldsymbol{I}_N\otimes\boldsymbol{F})[(\boldsymbol{I}_N\otimes\boldsymbol{F})^H\boldsymbol{H}^H\boldsymbol{R}^{-1}\boldsymbol{H}(\boldsymbol{I}_N\otimes\boldsymbol{F})]^{-1} \tag{3}$$

Substituting (3) into (2), the minimised average power of the filterbank output is given by

$$V_1(\boldsymbol{h})=\mathrm{tr}\{[\boldsymbol{H}^H\boldsymbol{R}^{-1}\boldsymbol{H}(\boldsymbol{I}_N\otimes\boldsymbol{F}\boldsymbol{F}^H)]^{-1}\} \tag{4}$$

Maximising $V_1(\boldsymbol{h})$ with respect to $\boldsymbol{h}$ is asymptotically (in SNR) equivalent to minimising the following lower bound [5]:

$$\begin{aligned}V_2(\boldsymbol{h})&=\mathrm{tr}\{\boldsymbol{H}^H\boldsymbol{R}^{-1}\boldsymbol{H}[\boldsymbol{I}_N\otimes(\boldsymbol{F}\boldsymbol{F}^H)]\}\\ &=\mathrm{vec}^T(\boldsymbol{H}^*)\{[\boldsymbol{I}_N\otimes(\boldsymbol{F}\boldsymbol{F}^H)]\otimes\boldsymbol{R}^{-1}\}\,\mathrm{vec}\,(\boldsymbol{H})\end{aligned} \tag{5}$$

which becomes a quadratic minimisation problem. Note that $\boldsymbol{H}$ is a linear matrix in $\boldsymbol{h}$. That is, we have $\mathrm{vec}(\boldsymbol{H})=\boldsymbol{S}\boldsymbol{h}$, where $\boldsymbol{S}$ is a $(JN-L)JN\times(L+1)$ matrix formed by elements 0 and 1. Explicit expression of $\boldsymbol{S}$ can be obtained by using elementary matrix $\boldsymbol{E}_{ij}$: a $(JN-L)\times(L+1)$ matrix with unit element at the ijth location and zeros elsewhere. For example, the first L blocks may be expressed as $\boldsymbol{S}_l=\sum_{j=1}^{l}\boldsymbol{E}_{l-j+1,L+2-j}$, $l=1,\ 2,\ \ldots,\ L$. The structure of $\boldsymbol{S}$ is further explored in the following 'Implementation' Section.

Using $\mathrm{vec}(\boldsymbol{H})=\boldsymbol{S}\boldsymbol{h}$ back in (5) yields

$$V_2(\boldsymbol{h})=\boldsymbol{h}^H\boldsymbol{S}^T\{[\boldsymbol{I}_N\otimes(\boldsymbol{F}\boldsymbol{F}^H)]\otimes\boldsymbol{R}^{-1}\}\boldsymbol{S}\boldsymbol{h}\triangleq\boldsymbol{h}^H\boldsymbol{\Phi}\boldsymbol{h} \tag{6}$$

The solution $\hat{\boldsymbol{h}}$ is given by the eigenvector of $\boldsymbol{\Phi}$ associated with the smallest eigenvalue. $\hat{\boldsymbol{h}}$ converges to the true channel $\boldsymbol{h}$ up to a scalar factor (e.g. see [4]). The scalar ambiguity can be resolved either by differential coding or by transmitting a few pilot symbols.

Implementation: The calculation of $\boldsymbol{\Phi}$, which is a $(JN-L)JN\times(JN-L)JN$ matrix (e.g. 10098×10098 for $N=2$, $J=51$ and $L=3$), has to be performed carefully. A brute-force implementation is clearly impractical. The structure of the matrices involved has to be exploited for efficient implementation. Specifically, note that

$$\begin{aligned}\boldsymbol{F}\boldsymbol{F}^H&=\begin{bmatrix}\boldsymbol{I}_\mu & \bar{\boldsymbol{F}}_1\bar{\boldsymbol{F}}^H\\ \bar{\boldsymbol{F}}\bar{\boldsymbol{F}}_1^H & \boldsymbol{I}_K\end{bmatrix}=\begin{bmatrix}\boldsymbol{I}_\mu & & \boldsymbol{I}_\mu\\ & \boldsymbol{I}_{K-\mu} & \\ \boldsymbol{I}_\mu & & \boldsymbol{I}_\mu\end{bmatrix},\\ \boldsymbol{S}&=\begin{bmatrix}\boldsymbol{S}_1\\ \vdots\\ \boldsymbol{S}_N\end{bmatrix},\quad \boldsymbol{S}_n=\begin{bmatrix}\boldsymbol{S}_{n,1}\\ \vdots\\ \boldsymbol{S}_{n,J}\end{bmatrix}\end{aligned} \tag{7}$$

where $\boldsymbol{S}_{n,j}$ is $(JN-L)\times(L+1)$, $n=1,\ldots,N$. Then $\boldsymbol{\Phi}$ can be expressed as

$$\begin{aligned}\boldsymbol{\Phi}&=\sum_{n=1}^{N}\boldsymbol{S}_n^T[(\boldsymbol{F}\boldsymbol{F}^H)\otimes\boldsymbol{R}^{-1}]\boldsymbol{S}_n\\ &=\sum_{n=1}^{N}\left[\sum_{j=1}^{J}\boldsymbol{S}_{n,j}^T\boldsymbol{R}^{-1}\boldsymbol{S}_{n,j}\right.\\ &\qquad\left.+\sum_{j=1}^{\mu}(\boldsymbol{S}_{n,j}^T\boldsymbol{R}^{-1}\boldsymbol{S}_{n,K+j}+\boldsymbol{S}_{n,K+j}^T\boldsymbol{R}^{-1}\boldsymbol{S}_{n,j})\right]\end{aligned} \tag{8}$$

To compute $\boldsymbol{S}_{n,j}$ for all n and j, we can reform $\boldsymbol{S}_{n,j}$ as follows:

$$\begin{aligned}\boldsymbol{S}_{n,j}&=\begin{bmatrix}\boldsymbol{0}_{(JN-L)\times L} & \boldsymbol{I}_{JN-L} & \boldsymbol{0}_{(JN-L)\times L}\end{bmatrix}\\ &\quad\times\begin{bmatrix}\boldsymbol{0}_{[(n-1)J+j-1]\times(L+1)}\\ \boldsymbol{I}_{L+1}\\ \boldsymbol{0}_{[JN-(n-1)J-j]\times(L+1)}\end{bmatrix}\\ &\triangleq\begin{bmatrix}\boldsymbol{0}_{(JN-L)\times L} & \boldsymbol{I}_{JN-L} & \boldsymbol{0}_{(JN-L)\times L}\end{bmatrix}\tilde{\boldsymbol{S}}_{n,j}\end{aligned} \tag{9}$$

The first item in the second equality of (8), i.e. $\boldsymbol{S}_{n,j}^T\boldsymbol{R}^{-1}\boldsymbol{S}_{n,j}$, can be simplified as

$$\begin{aligned}\boldsymbol{S}_{n,j}^T\boldsymbol{R}^{-1}\boldsymbol{S}_{n,j}&=\tilde{\boldsymbol{S}}_{n,j}^T\begin{bmatrix}\boldsymbol{0}_{L\times L} & \boldsymbol{0}_{L\times(JN-L)} & \boldsymbol{0}_{L\times L}\\ \boldsymbol{0}_{(JN-L)\times L} & \boldsymbol{R}^{-1} & \boldsymbol{0}_{(JN-L)\times L}\\ \boldsymbol{0}_{L\times L} & \boldsymbol{0}_{L\times(JN-L)} & \boldsymbol{0}_{L\times L}\end{bmatrix}\tilde{\boldsymbol{S}}_{n,j}\\ &\triangleq\tilde{\boldsymbol{S}}_{n,j}^T\mathcal{R}\tilde{\boldsymbol{S}}_{n,j}\end{aligned} \tag{10}$$

$\tilde{\boldsymbol{S}}_{n,j}$ essentially performs matrix truncation. As such we have

$$\begin{aligned}&\boldsymbol{S}_{n,j}^T\boldsymbol{R}^{-1}\boldsymbol{S}_{n,j}=\tilde{\boldsymbol{S}}_{n,j}^T\mathcal{R}\tilde{\boldsymbol{S}}_{n,j}=\mathcal{R}(i_1:i_1+L,i_1:i_1+L),\\ &\text{for } n=1,\ldots,N;\ \ j=1,\ldots,J\end{aligned} \tag{11}$$

where $i_1=(n-1)J+j$. The second term in (8) can be evaluated in a similar fashion.

Numerical results: We present simulation results reflecting two scenarios based on how the received signal is being processed. We form blocks of N symbols each, and have the N symbols arranged in either overlapping or non-overlapping fashion. For M symbols, this will result in M/N (assume M/N is an integer) and $M-N+1$ non-overlapping and overlapping blocks, respectively. We compare the proposed method with the subspace channel estimator [2]. The system under study utilises BPSK with $K=48$, $L=3$, $N=2$, $M=200$. Two narrowband interfering signals are added with various values of signal-to-interference ratio (SIR). As a performance measure, we consider the normalised root mean-squared error (RMSE) of the channel estimates.

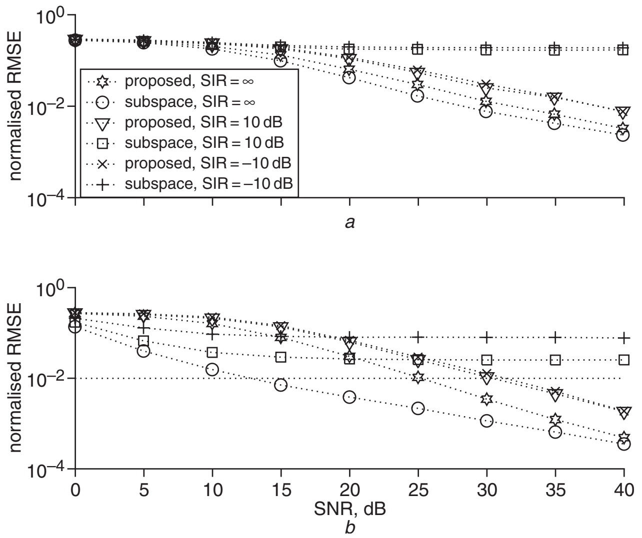

Fig. 1 *Normalised RMSE of proposed and subspace channel estimates against SNR and SIR*

a No overlapping
b Overlapping

Fig. 1 shows the performance against SNR and SIR for both non-overlapping (Fig. 1*a*) and overlapping (Fig. 1*b*) scenarios. In the absence of interference, the subspace estimator outperforms the proposed method. However, even with fairly weak interference, the subspace estimator degrades significantly and exhibits irreducible error. In addition, overlapping of the received signal helps improve data efficiency and, in turn, performance of the proposed method. For SIR = 10 dB.

Conclusion: A joint blind channel estimation and interference suppression method with efficient implementation for OFDM systems is presented. The proposed scheme compares favourably with a subspace blind channel estimator.

Acknowledgments: This work was supported in part by the NSF under grant number CCF-0514938, the Army Research Office under grant number DAAD19-03-1-0184 and by the Air Force Research Laboratory (AFRL) under grant number FA8750-05-2-0001.

27 April 2005
Electronics Letters online no: 20051533
doi: 10.1049/el:20051533

K. Amleh (*Engineering Department, Penn State University at Mont Alto, PA, 17237, USA*)

H. Li and R. Wang (*Department of Electrical and Computer Engineering, Stevens Institute of Technology, Hoboken, NJ 07030, USA*)

E-mail: hli@stevens.edu

References

1 Li, Y.G., Cimini, L.J. Jun., and Sollengerger, N.R.: 'Robust channel estimation for OFDM systems with rapid dispersive fading channels', *IEEE Trans. Commun.*, 1998, **46**, (7), pp. 902–915

2 Cai, X., and Akansu, A.N.: 'A subspace method for blind channel identification in OFDM systems'. Proc. IEEE Int. Conf. on Communications, 2000, pp. 929–933

3 Scaglione, A., Giannakis, G.B., and Barbarossa, S.: 'Redundant filter precoders and equalizers Part II: Blind channel estimation, synchronization direct equalization', *IEEE Trans. Signal Process.*, 1999, **47**, (7), pp. 2007–2032

4 Tsatsanis, M.K., and Xu, Z.: 'Performance analysis of minimum variance CDMA receiver', *IEEE Trans. Signal Process.*, 1998, **46**, (11), pp. 3014–3022

5 Li, H.: 'Blind channel estimation for multicarrier systems with narrowband interference suppression', *IEEE Commun. Lett.*, 2003, **7**, (7), pp. 326–328