Blind channel estimation, equalisation and CRB for OFDM with unmodelled interference

K. Amleh, H. Li and R. Wang

Abstract: A blind channel estimation and equalisation scheme is given for orthogonal frequency-division multiplexing systems with unmodelled interference. The approach uses a generalised multichannel minimum variance principle to design an equalising filterbank that preserves the desired signal components and suppresses the overall interference. A channel estimate is then obtained by deriving an asymptotically tight lower bound of the filterbank output power, which reduces the problem to a quadratic minimisation. By imposing a special structure on the received signal, the performance is shown to be significantly increased. To assess the performance of the proposed scheme, an unconditional Cramér–Rao bound (CRB) is derived which, similar to the proposed blind channel estimator, does not assume knowledge of the transmitted information symbols. The CRB serves as a lower bound for all unbiased blind channel estimation schemes. Numerical examples show that the proposed scheme approaches the CRB as the SNR increases. It also exhibits low sensitivity to unknown narrowband interference and compares favourably with a subspace blind channel estimator.

1 Introduction

Orthogonal frequency division multiplexing (OFDM) is a multicarrier (MC) digital modulation technique that allows high data-rate transmission such as digital TV broadcasting and high-speed telephone line communication. The implementation relies on very high-speed digital signal processing which has only recently become available, making OFDM a competitive technology for future broadband wireless applications. In OFDM the transmitted information is transformed by the inverse fast Fourier transform (IFFT) into parallel blocks. When the channel is dispersive, inter-block interference (IBI) between successive blocks occurs. To eliminate the IBI a cyclic prefix (CP) is inserted at the beginning of each transmitted data block. By choosing the length of the CP to be greater than the channel impulse response, successive blocks will not interfere and can be reliably recovered at the receiver's end.

Numerous channel estimation schemes have been investigated recently. These schemes rely on either explicit training (e.g. [1]) or some inherent structure (e.g. subspace [2]) of the transmitted signal. Although the training-assisted schemes perform quite well, they reduce the spectral efficiency. Moreover, to track channel variations, training symbols have to be retransmitted periodically, leading to throughput reductions. Blind schemes, on the other hand, do not suffer from such drawbacks. Well-known blind channel estimation schemes for MC are the subspace-based methods proposed in ([2, 3] for example). However, when there is insufficient information about the interference so that prewhitening cannot be performed, subspace channel estimation is in general inaccurate.

In this work we consider the problem of blind channel estimation and equalisation for OFDM systems with unmodelled interference. Such unmodelled interference may be caused by time/frequency synchronisation errors, overlay with narrowband systems, among others. A generalised multichannel minimum variance principle (e.g. [4]) is invoked to design an equalising filterbank that preserves desired signal components and suppresses the overall interference. Even though multidimensional nonlinear search methods can be applied to find channel estimates by directly maximising the filterbank output power, such an approach is computationally prohibitive and suffers local convergence. To overcome the difficulty associated with multidimensional search methods, we derive an asymptotically (in SNR) tight lower bound of the filterbank output power and use it for channel estimation, which reduces the problem to a quadratic minimisation.

We also derive an unconditional Cramér–Rao bound (CRB) which, similar to the proposed blind channel estimator, does not assume knowledge of the transmitted information symbols. The CRB serves as a lower bound for all unbiased blind channel estimation schemes. By imposing a unique structure on the received data symbols, significant performance improvement can be obtained as we show through numerical examples in Section 6. Furthermore, we show that the proposed scheme exhibits low sensitivity to unknown narrowband interference and compares favorably with a subspace blind channel estimator.

Notation: Vectors (matrices) are denoted by boldface italic lower (upper) case letters; all vectors are column vectors; superscripts $(\cdot)^T$, $(\cdot)^*$ and $(\cdot)^H$ denote the transpose, conjugate and conjugate transpose, respectively; $\boldsymbol{I}_N$ denotes the $N \times N$ identity matrix; $\mathbf{0}$ denotes an all-zero matrix or vector; $\mathrm{tr}\{\cdot\}$ denotes the trace; $\mathrm{vec}(\cdot)$ stacks the columns of its matrix argument on top of one another; $E\{\cdot\}$ denotes the statistical expectation; the Matlab notation $A(i_1{:}\ i_2,$

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j_1: j_2) denotes $(i_2 - i_1 + 1) \times (j_2 - j_1 + 1)$ matrix formed from rows $i_1, \ldots, i_2$ and columns $j_1, \ldots, j_2$ of matrix A and finally, $\otimes$ denotes the matrix Kronecker product.

2 Problem formulation

Consider an OFDM system where a serial of information symbols are blocked into $K \times 1$ vectors $\boldsymbol{s}(n) = [s(nK), \ldots, s(nK + K - 1)]^T$, which are linearly transformed into $\boldsymbol{u}(n) = \boldsymbol{F}\boldsymbol{s}(n)$ by $J \times K$ matrix $\boldsymbol{F} \triangleq [\bar{\boldsymbol{F}}_1^T, \bar{\boldsymbol{F}}^T]^T$, where $\bar{\boldsymbol{F}}$ denotes the $K \times K$ IDFT unitary matrix, and $\bar{\boldsymbol{F}}_1 \in \mathbb{C}^{(J-K) \times K}$ is formed from the last $\mu \triangleq J - K$ rows of $\bar{\boldsymbol{F}}$, where μ is the length of the cyclic-prefix. To avoid multipath-induced interblock interference (IBI), transmission redundancy is introduced by choosing $\mu \geq L$, where L is the channel order. In the following, we process a block of $N \geq 1$ OFDM symbols simultaneously. Let

$$\boldsymbol{s}_N(n) = \begin{bmatrix} \boldsymbol{s}(nN) \\ \vdots \\ \boldsymbol{s}(nN + N - 1) \end{bmatrix}_{KN \times 1}$$
$$\boldsymbol{u}_N(n) = \begin{bmatrix} \boldsymbol{u}(nN) \\ \vdots \\ \boldsymbol{u}(nN + N - 1) \end{bmatrix}_{JN \times 1} = (\boldsymbol{I}_N \otimes \boldsymbol{F})\boldsymbol{s}_N(n) \quad (1)$$

The discrete-time baseband equivalent channel, which includes the transmitter/receiver filter and the physical channel, is modelled as an FIR filter $\boldsymbol{h} \triangleq [h(0), h(1), \ldots, h(L)]^T$. Hence the overall received samples resulting from the transmission of $\boldsymbol{u}_N(n)$ is $JN + L$. We discard the first and last L samples and form a $(JN - L) \times 1$ vector $\boldsymbol{y}_N(n)$, which can be expressed as [2]

$$\boldsymbol{y}_N(n) = \boldsymbol{H}(\boldsymbol{I}_N \otimes \boldsymbol{F})\boldsymbol{s}_N(n) + \boldsymbol{w}_N(n) + \boldsymbol{e}_N(n) \quad (2)$$

where $\boldsymbol{w}_N(n)$ and $\boldsymbol{e}_N(n)$ denote interference and channel noise vectors, respectively, and $\boldsymbol{H}$ is an $(JN - L) \times JN$ Toeplitz matrix defined as

$$\boldsymbol{H} = \begin{bmatrix} h(L) & \ldots & h(0) & 0 & \ldots & \ldots & 0 \\ 0 & h(L) & \ldots & h(0) & 0 & \ldots & 0 \\ \vdots & & & & & & \vdots \\ 0 & \ldots & \ldots & 0 & h(L) & \ldots & h(0) \end{bmatrix} \quad (3)$$

It is seen that $\boldsymbol{H}(\boldsymbol{I}_N \otimes \boldsymbol{F})$ is a tall matrix with full column rank if $JN - L \geq KN$ or equivalently, $N \geq L/(J - K)$. If the length of the cyclic prefix is chosen as $J - K = L$, then the minimum value of N that is needed is equal to one.

The problem of interest is to estimate the channel coefficients $\{h(n)\}_{n=0}^L$ from the observed data without any knowledge of the transmitted symbols.

3 Proposed scheme

Due to the presence of $\boldsymbol{w}_N(n)$ and $\boldsymbol{e}_N(n)$, the observed signal $\boldsymbol{y}_N(n)$ is noisy. Hence, instead of directly using the raw data, we propose to first pass $\boldsymbol{y}_N(n)$ through a bank of filters that are designed to enhance the useful signals and suppress the interference/noise, and then derive the channel estimates from the filtered data. Equation (2) represents a multiple-input multiple-output (MIMO) system with KN inputs and $JN - L$ outputs. The mixing matrix $\boldsymbol{H}(\boldsymbol{I}_N \otimes \boldsymbol{F})$ is partially known since $\boldsymbol{H}$ has a known Toeplitz structure and $\boldsymbol{F}$ is also known to the receiver. We can exploit this knowledge to design a bank of KN FIR filters $\boldsymbol{G} \in \mathbb{C}^{(JN-L) \times KN}$, each passing one symbol with unit-gain, completely annihilating the other $KN - 1$ interfering symbols, meanwhile suppressing interference $\boldsymbol{w}_N(n)$ as much as possible. While alternative design schemes may exist, we choose $\boldsymbol{G}$ based on the following idea: if $\boldsymbol{G}$ is effective in canceling the interference/noise, the average power of the filterbank output should be small; meanwhile, to avoid the trivial solution $\boldsymbol{G} = 0$ and to prevent signal cancellation, we should enforce certain constraint on $\boldsymbol{G}$ such that it will pass the desired signals with little distortion. In particular, we design an equalising filterbank according to the following minimum variance criterion

$$\boldsymbol{G} = \underset{\boldsymbol{G} \in \mathbb{C}^{(JN-L) \times KN}}{\operatorname{argmin}} \operatorname{tr}\{\boldsymbol{G}^H \boldsymbol{R} \boldsymbol{G}\}$$
$$\text{subject to } \boldsymbol{G}^H \boldsymbol{H}(\boldsymbol{I}_N \otimes \boldsymbol{F}) = \boldsymbol{I}_{KN} \quad (4)$$

where $\boldsymbol{R} \triangleq E\{\boldsymbol{y}_N(n) \times \boldsymbol{y}_N^H(n)\}$ denotes the covariance matrix, and the constraint $\boldsymbol{G}^H \boldsymbol{H}(\boldsymbol{I}_N \otimes \boldsymbol{F}) = \boldsymbol{I}_{KN}$ ensures that each filter (i.e. one column of $\boldsymbol{G}$) will pass only one signal component [corresponding to one column of $\boldsymbol{H}(\boldsymbol{I}_N \otimes \boldsymbol{F})$] undistorted with unit-gain, while completely eliminating inter-symbol interference (ISI) caused by the other columns of $\boldsymbol{H}(\boldsymbol{I}_N \otimes \boldsymbol{F})$. Using the Lagrange multiplier the solution to the constrained quadratic minimisation problem is given by (see, for example [5, p. 283])

$$\boldsymbol{G} = \boldsymbol{R}^{-1}\boldsymbol{H}(\boldsymbol{I}_N \otimes \boldsymbol{F})[(\boldsymbol{I}_N \otimes \boldsymbol{F})^H \boldsymbol{H}^H \boldsymbol{R}^{-1} \boldsymbol{H}(\boldsymbol{I}_N \otimes \boldsymbol{F})]^{-1} \quad (5)$$

Substituting (5) into (4), the minimised average power of the filterbank output is given by

$$V_1(\boldsymbol{h}) = \operatorname{tr}\{[(\boldsymbol{I}_N \otimes \boldsymbol{F})^H \boldsymbol{H}^H \boldsymbol{R}^{-1} \boldsymbol{H}(\boldsymbol{I}_N \otimes \boldsymbol{F})]^{-1}\}$$
$$= \operatorname{tr}\{[\boldsymbol{H}^H \boldsymbol{R}^{-1} \boldsymbol{H}(\boldsymbol{I}_N \otimes \boldsymbol{F}\boldsymbol{F}^H)]^{-1}\} \quad (6)$$

where we used the fact that $\operatorname{tr}(AB) = \operatorname{tr}(BA)$ for any A and B with compatible size. To find an estimate for $\boldsymbol{h}$, we want to maximise $V_1(\boldsymbol{h})$ with respect to $\boldsymbol{h}$, so that $\boldsymbol{G}$ will maximally preserve the signal power. Because of the nonlinear nature of $V_1(\boldsymbol{h})$, this approach is computationally involved and suffers local convergence. Instead, we maximise an asymptotic lower bound of $V_1(\boldsymbol{h})$. Using the Schwartz inequality (see [6], eq. 5), $V_1(\boldsymbol{h}) \geq K^2/\operatorname{tr}\{\boldsymbol{H}^H \boldsymbol{R}^{-1} \boldsymbol{H}[\boldsymbol{I}_N \otimes (\boldsymbol{F}\boldsymbol{F}^H)]\}$, where the equality is achieved asymptotically. So maximising $V_1(\boldsymbol{h})$ w.r.t. (with respect to) $\boldsymbol{h}$ is equivalent to minimising the following asymptotic lower bound

$$V_2(\boldsymbol{h}) = \operatorname{tr}\{\boldsymbol{H}^H \boldsymbol{R}^{-1} \boldsymbol{H}[\boldsymbol{I}_N \otimes (\boldsymbol{F}\boldsymbol{F}^H)]\}$$
$$= \operatorname{vec}^T(\boldsymbol{H}^*)\{[\boldsymbol{I}_N \otimes (\boldsymbol{F}\boldsymbol{F}^H)] \otimes \boldsymbol{R}^{-1}\}\operatorname{vec}(\boldsymbol{H}) \quad (7)$$

which becomes a quadratic minimisation problem. Next, we express $\operatorname{vec}(\boldsymbol{H})$ explicitly as a linear function in $\boldsymbol{h}$. In particular, we can write

$$\operatorname{vec}(\boldsymbol{H}) = \boldsymbol{S}\boldsymbol{h} \quad (8)$$

where $\boldsymbol{S}$ is a $(JN - L)JN \times (L + 1)$ matrix formed by elements 0 and 1 only. It is seen that $\boldsymbol{S}$ is full column rank since the mapping is one-to-one. For example, if $L = 2$, that is three-tap FIR filter/channel, then $\boldsymbol{S}$ can be expressed as

$$
\boldsymbol{S} = \left|\begin{array}{ccc} 0 & 0 & 1 \\ & \boldsymbol{0}_{(JN-L-1)\times(L+1)} & \\ \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ & \boldsymbol{0}_{(JN-L-2)\times(L+1)} & \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ & \boldsymbol{0}_{(JN-L-3)\times(L+1)} & \\ & \vdots & \\ & \boldsymbol{0}_{(JN-L-2)\times(L+1)} & \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ \\ & \boldsymbol{0}_{(JN-L-1)\times(L+1)} & \\ 1 & 0 & 0 \end{array}\right| \begin{array}{l} \}\text{block } 1 \\ \\ \\ \\ \}\text{block } 2 \\ \\ \\ \}\text{block } 3 \\ \\ \\ \vdots \\ \\ \\ \}\text{block } JN-1 \\ \\ \\ \}\text{block } JN \end{array} \tag{9}
$$

Remarks:

(i) There are JN blocks, each block is of size $(JN-L)\times(L+1)$.
(ii) Block 1: 1st row is formed by last row of $\boldsymbol{I}_{L+1}$; zeros elsewhere.
(iii) Block 2: first two rows formed by last two rows of $\boldsymbol{I}_{L+1}$; zeros elsewhere.
(iv) Block number $L+1$: first $L+1$ rows from $\boldsymbol{I}_{L+1}$; zeros elsewhere.
(v) Block number $JN-L$: last $L+1$ rows from $\boldsymbol{I}_{L+1}$; zeros elsewhere.
(vi) Block number $JN-1$: last two rows formed by first two rows of $\boldsymbol{I}_{L+1}$; zeros elsewhere.
(vii) Block number JN: last row formed by first row of $\boldsymbol{I}_{L+1}$; zeros elsewhere.
(viii) Explicit expression of the lth block of $\boldsymbol{S}$ is given by

$$
\boldsymbol{S}_l = \begin{cases} \sum_{j=1}^{l} \boldsymbol{E}_{l-j+1,L+2-j} & l = 1,2,\ldots,L \\ \sum_{j=1}^{L+1} \boldsymbol{E}_{l-j+1,L+2-j} & l = L+1, L+2, \ldots, JN-L \\ \sum_{j=1}^{JN-l+1} \boldsymbol{E}_{j+l-L-1,j} & l = JN-L+1, \\ & \quad JN-L+2,\ldots,JN \end{cases} \tag{10}
$$

where $\boldsymbol{E}_{ij}$ is a $(JN-L)\times(L+1)$ elementary matrix with unit element at the (i,j)th location and zeros elsewhere.

Using $\mathrm{vec}(\boldsymbol{H}) = \boldsymbol{Sh}$ from (8) back in $V_2(\boldsymbol{h})$ (7), we have

$$
V_2(\boldsymbol{h}) = \boldsymbol{h}^H \boldsymbol{S}^T\{[\boldsymbol{I}_N \otimes (\boldsymbol{FF}^H)] \otimes \boldsymbol{R}^{-1}\}\boldsymbol{Sh} \triangleq \boldsymbol{h}^H \boldsymbol{\Phi h} \tag{11}
$$

where

$$
\boldsymbol{\Phi}_{(L+1)\times(L+1)} \triangleq \boldsymbol{S}^T\{[\boldsymbol{I}_N \otimes (\boldsymbol{FF}^H)] \otimes \boldsymbol{R}^{-1}\}\boldsymbol{S} \tag{12}
$$

The solution $\hat{\boldsymbol{h}}$ that minimises $V_2(\boldsymbol{h})$ is given by the eigenvector of $\boldsymbol{\Phi}$ associated with the smallest eigenvalue. Note that for implementation, $\boldsymbol{R}$ has to be replaced by some covariance matrix estimate, for example the sample covariance matrix $\hat{\boldsymbol{R}} = P^{-1}\sum_{n=0}^{P-1} \boldsymbol{y}_N(n)\boldsymbol{y}_N^H(n)$ or some adaptive estimate of $\boldsymbol{R}$. It can be shown (e.g. [4]) that $\hat{\boldsymbol{h}}$ converges to the true channel $\boldsymbol{h}$ (up to a scalar factor) as the interference and noise vanish. For finite SNR and in the presence of interference, we evaluate the accuracy of $\hat{\boldsymbol{h}}$ via simulations in Section 6. Finally, like all other blind schemes, the channel estimate $\hat{\boldsymbol{h}}$ has a scalar ambiguity, which can be resolved either by differential coding or by transmitting a *few* pilot symbols.

4 Implementation

We note that the calculation of $\boldsymbol{\Phi}$ has to be performed carefully because of the large dimensions of the matrices involved: $\boldsymbol{FF}^H$ is $J\times J$, $\boldsymbol{I}_N \otimes (\boldsymbol{FF}^H)$ is $JN\times JN$, $\boldsymbol{S}$ is $(JN-L)JN\times(L+1)$ and $[\boldsymbol{I}_N \otimes (\boldsymbol{FF}^H)] \otimes \boldsymbol{R}^{-1}$ is a $(JN-L)JN \times (JN-L)JN$ matrix (e.g. $14\,336 \times 14\,336$ for $N=2$, $J=64$ and $L=16$). Hence, brute-force computation is impractical/inefficient except for small values. The spars/special structure of the matrices involved has to be exploited for efficient implementation. Let

$$
\boldsymbol{FF}^H = \begin{bmatrix} \boldsymbol{I}_\mu & \bar{\boldsymbol{F}}_1\bar{\boldsymbol{F}}^H \\ \bar{\boldsymbol{F}}\bar{\boldsymbol{F}}_1^H & \boldsymbol{I}_K \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_\mu & & \boldsymbol{I}_\mu \\ & \boldsymbol{I}_{K-\mu} & \\ \boldsymbol{I}_\mu & & \boldsymbol{I}_\mu \end{bmatrix} \tag{13}
$$

Then

$$
(\boldsymbol{FF}^H) \otimes \boldsymbol{R}^{-1} = \left[\begin{array}{c|c|c} \boldsymbol{I}_\mu \otimes \boldsymbol{R}^{-1} & & \boldsymbol{I}_\mu \otimes \boldsymbol{R}^{-1} \\ \hline & \boldsymbol{I}_{K-\mu} \otimes \boldsymbol{R}^{-1} & \\ \hline \boldsymbol{I}_\mu \otimes \boldsymbol{R}^{-1} & & \boldsymbol{I}_\mu \otimes \boldsymbol{R}^{-1} \end{array}\right] \tag{14}
$$

Let

$$
\boldsymbol{S} = \begin{bmatrix} \boldsymbol{S}_1 \\ \vdots \\ \boldsymbol{S}_N \end{bmatrix}_{(JN-L)JN\times(L+1)}
\qquad
\boldsymbol{S}_n = \begin{bmatrix} \boldsymbol{S}_{n,1} \\ \vdots \\ \boldsymbol{S}_{n,J} \end{bmatrix}_{(JN-L)J\times(L+1)} \tag{15}
$$

where $\boldsymbol{S}_{n,j}$ is $(JN-L)\times(L+1)$ and $n = 1,\ldots,N$. It follows that $\boldsymbol{\Phi}$ can be expressed as [cf. (14)]

$$
\begin{aligned} \boldsymbol{\Phi} &= \sum_{n=1}^{N} \boldsymbol{S}_n^T[(\boldsymbol{FF}^H) \otimes \boldsymbol{R}^{-1}]\boldsymbol{S}_n \\ &= \sum_{n=1}^{N}\left[\sum_{j=1}^{J} \boldsymbol{S}_{n,j}^T \boldsymbol{R}^{-1}\boldsymbol{S}_{n,j} \right. \\ &\quad \left. + \sum_{j=1}^{\mu}(\boldsymbol{S}_{n,j}^T\boldsymbol{R}^{-1}\boldsymbol{S}_{n,K+j} + \boldsymbol{S}_{n,K+j}^T\boldsymbol{R}^{-1}\boldsymbol{S}_{n,j})\right] \end{aligned} \tag{16}
$$

To compute $S_{n,j}$ for all n and j, we can reform $S_{n,j}$ as follows

$$\begin{aligned} S_{n,j} &= [\mathbf{0}_{(JN-L)\times L} \quad I_{JN-L} \quad \mathbf{0}_{(JN-L)\times L}] \\ &\quad \times \begin{bmatrix} \mathbf{0}_{[(n-1)J+j-1]\times(L+1)} \\ I_{L+1} \\ \mathbf{0}_{[(JN-(n-1)J-j]\times(L+1)} \end{bmatrix} \\ &\triangleq [\mathbf{0}_{(JN-L)\times L} \quad I_{JN-L} \quad \mathbf{0}_{(JN-L)\times L}]\tilde{S}_{n,j} \end{aligned} \tag{17}$$

Following this reformulation, one can see that the first item in the second equality of (16), that is $S_{n,j}^T R^{-1} S_{n,j}$, can be simplified as

$$\begin{aligned} S_{n,j}^T R^{-1} S_{n,j} &= \tilde{S}_{n,j}^T \begin{bmatrix} \mathbf{0}_{L\times(JN-L)} \\ I_{JN-L} \\ \mathbf{0}_{L\times(JN-L)} \end{bmatrix} \\ &\quad \times R^{-1} [\mathbf{0}_{(JN-L)\times L} \quad I_{JN-L} \quad \mathbf{0}_{(JN-L)\times L}]\tilde{S}_{n,j} \\ &= \tilde{S}_{n,j}^T \begin{bmatrix} \mathbf{0}_{L\times L} & \mathbf{0}_{L\times(JN-L)} & \mathbf{0}_{L\times L} \\ \mathbf{0}_{(JN-L)\times L} & R^{-1} & \mathbf{0}_{(JN-L)\times L} \\ \mathbf{0}_{L\times L} & \mathbf{0}_{L\times(JN-L)} & \mathbf{0}_{L\times L} \end{bmatrix} \tilde{S}_{n,j} \\ &\triangleq \tilde{S}_{n,j}^T \mathcal{R} \tilde{S}_{n,j} \end{aligned} \tag{18}$$

Note that $\mathcal{R}$ is independent of n or j. Additionally, $\tilde{S}_{n,j}$ is essentially performing a matrix truncating operation. As such, we have

$$\begin{aligned} S_{n,j}^T R^{-1} S_{n,j} &= \tilde{S}_{n,j}^T \mathcal{R} \tilde{S}_{n,j} = \mathcal{R}(i_1 : i_1 + L, i_1 : i_1 + L) \\ &\quad \text{for} \quad n = 1, \ldots, N; \; j = 1, \ldots, J \end{aligned} \tag{19}$$

where $i_1 = (n-1)J + j$. Similarly, for the second term in (16), we have

$$\begin{aligned} S_{n,j}^T R^{-1} S_{n,K+j} &= \tilde{S}_{n,j}^T \mathcal{R} \tilde{S}_{n,K+j} \\ &= \mathcal{R}(i_1 : i_1 + L, i_1 + K : i_1 + K + L) \\ S_{n,K+j}^T R^{-1} S_{n,j} &= (S_{n,j}^T R^{-1} S_{n,K+j})^T \\ &\quad \text{for} \quad n = 1, \ldots, N; \; j = 1, \ldots, \mu \end{aligned} \tag{20}$$

We summarise the implementation steps of $\boldsymbol{\Phi}$ along with the computational complexity in terms of number of flops as follows:

- *Step 1:* Set $\boldsymbol{\Phi} = \mathbf{0}_{(L+1)\times(L+1)}$. $\Rightarrow$ total $O(0)$ flops
- *Step 2:* Form $\mathcal{R}$ using (18). $\Rightarrow$ total $O(0)$ flops
- *Step 3:* Partition $\mathcal{R}$ and sum them using (19) and (20). (In the following loops $i_1 = (n-1)J + j$.)

```
for n = 1 : N
  for j = 1 : J
    Φ = Φ + R(i1 : i1 + L, i1 : i1 + L);
  end ⇒ total O(JN(L + 1)^2) flops
  for j = 1 : μ
    Φ = Φ + R(i1 : i1 + L, i1 + K : i1 + K + L)
          + R(i1: i1 + L, i1 + K : i1 + K + L)^T;
  end ⇒ total O(μJ(L + 1)^2) flops
end
```

With direct implementation, the number of operations (flops) needed to calculate $\boldsymbol{\Phi}$ is given by $O((L+1)((JN-L)JN)^2)$, or approximately $O((L+1)J^4N^4)$ when $JN \gg L$ (i.e. $JN - L \simeq JN$).

5 Cramér–Rao bound

We derive the *unconditional* Cramér–Rao Bound (CRB) that is averaged over the unknown user symbols. The CRB provides a suitable lower bound for all unbiased blind estimators, which do not assume knowledge of the user symbols and channel coefficients. Conditional CRBs (i.e. CRBs which are conditioned on the information symbols) for various blind channel identification problems have been investigated in the literature; see [7–9] and references therein.

Referring to (2), the information symbols are assumed to be IID and drawn from some unit-energy constellation, that is $E\{s_N(n)s_N(n)^H\} = I_{KN}$. Additionally, we assume that $e_N(n)$ is temporally and spatially white normally distributed random vector with zero mean and autocovariance given by $\sigma^2 I_{(JN-L)}$ and that the information symbols and noise samples are independent of each other.

Let the observation time consist of a total of P blocks where each block consists from N OFDM symbols. We collect all samples within this observation time into an $P(JN-L)\times 1$ vector y, defined as

$$y \triangleq [y_N^T(P), y_N^T(P-1), \ldots, y_N^T(1)]^T \tag{21}$$

With the aforementioned assumptions, it is clear that y follows a Gaussian distribution with zero mean and covariance matrix

$$\begin{aligned} R_P &= I_P \otimes [H(I_N \otimes (FF^H))H^H + R_w + \sigma^2 I_{(JN-L)}] \\ &= I_P \otimes R \end{aligned} \tag{22}$$

where

$$\begin{aligned} R_w &= E\{w_N(n)w_N^H(n)\} \\ &\quad \times \begin{bmatrix} r(0) & r^*(1) & \cdots & r^*(JN-L-1) \\ r(1) & r(0) & \cdots & r^*(JN-L-2) \\ \vdots & \vdots & \ddots & \vdots \\ r(JN-L-1) & r(JN-L-2) & \cdots & r(0) \end{bmatrix} \end{aligned} \tag{23}$$

and $r(k)$ denotes the autocorrelation of the interference samples for $k = 0, \ldots, JN-L-1$. Let $m = 2JN+1$ be the total number of unknown parameters and define the $m \times 1$ vector of unknown parameters as

$$\boldsymbol{\theta} = [h_r^T \quad h_i^T \quad r_i^T \quad r_i^T]^T \tag{24}$$

where $h_r = [h_r(0), \ldots, h_r(L)]$, $h_i = [h_i(0), \ldots, h_i(L)]$, $r_r = [r(0), r_r(1), \ldots, r_r(JN-L-1)]$, $r_i = [r_i(1), \ldots, r_i(JN-L-1)]$, and the subscripts r and i denote the real and imaginary parts, respectively. By the Slepian–Bangs formula, the $m \times m$ Fisher information matrix (FIM) is given, element-wise, by [5]

$$\begin{aligned} [J(\boldsymbol{\theta})]_{s,t} &= \mathrm{tr}\left[R_P^{-1} \frac{\partial R_P}{\partial[\boldsymbol{\theta}]_s} R_P^{-1} \frac{\partial R_P}{\partial[\boldsymbol{\theta}]_t}\right] \\ &= P \, \mathrm{tr}\left[R^{-1} \frac{\partial R}{\partial[\boldsymbol{\theta}]_s} R^{-1} \frac{\partial R}{\partial[\boldsymbol{\theta}]_t}\right] \end{aligned} \tag{25}$$

where $[\boldsymbol{J}(\boldsymbol{\theta})]_{s,t}$ denotes the (s, t)th element of the FIM matrix, $[\boldsymbol{\theta}]_s$ the sth element of $\boldsymbol{\theta}$, and the second equality is due to the block diagonal structure of the covariance matrix (22). We next calculate the partial differentiation w.r.t. each unknown parameter. First consider the partial differentiation w.r.t. the channel real and imaginary part parameters contained in $\boldsymbol{h}_r$ and $\boldsymbol{h}_i$, respectively

$$\frac{\partial \boldsymbol{R}}{\partial h_r(l)} = \boldsymbol{X}_l(\boldsymbol{I}_N \otimes (\boldsymbol{F}\boldsymbol{F}^H))\boldsymbol{H}^H + \boldsymbol{H}(\boldsymbol{I}_N \otimes (\boldsymbol{F}\boldsymbol{F}^H))\boldsymbol{X}_l^H \qquad (26)$$

$$\frac{\partial \boldsymbol{R}}{\partial h_i(l)} = j[\boldsymbol{X}_l(\boldsymbol{I}_N \otimes (\boldsymbol{F}\boldsymbol{F}^H))\boldsymbol{H}^H - \boldsymbol{H}(\boldsymbol{I}_N \otimes (\boldsymbol{F}\boldsymbol{F}^H))\boldsymbol{X}_l^H] \qquad (27)$$

where

$$\boldsymbol{X}_l \triangleq \begin{bmatrix} \boldsymbol{0}_{(JN-L)\times(L-l)} & \boldsymbol{I}_{JN-L} & \boldsymbol{0}_{(JN-L)\times l} \end{bmatrix} \; l = 0, \ldots, L \qquad (28)$$

Next we calculate the partial differentiation w.r.t. the interference real and imaginary part parameters contained in $\boldsymbol{r}_r$ and $\boldsymbol{r}_i$, respectively

$$\frac{\partial \boldsymbol{R}}{\partial [\boldsymbol{r}_r]_z} = \frac{\partial \boldsymbol{R}_w}{\partial [\boldsymbol{r}_r]_z} = \begin{cases} \boldsymbol{I}_{JN-L} & z = 1 \\ \boldsymbol{Q}_z + \boldsymbol{Q}_z^T & z = 2, \ldots, JN - L \end{cases} \qquad (29)$$

$$\frac{\partial \boldsymbol{R}}{\partial [\boldsymbol{r}_i]_z} = \frac{\partial \boldsymbol{R}_w}{\partial [\boldsymbol{r}_i]_z} = j(\boldsymbol{Q}_z - \boldsymbol{Q}_z^T) \quad z = 2, \ldots, JN - L \qquad (30)$$

where

$$\boldsymbol{Q}_z \triangleq \begin{bmatrix} \boldsymbol{0}_{(z-1)\times(JN-L-z+1)} & \boldsymbol{0}_{(z-1)\times(z-1)} \\ \boldsymbol{I}_{JN-L-z+1} & \boldsymbol{0}_{(JN-L-z+1)\times(z-1)} \end{bmatrix} \qquad (31)$$

Using (26), (27), (29) and (30), which are subsequently substituted into (25), the FIM can be computed entry by entry.

The FIR $\boldsymbol{J}(\boldsymbol{\theta})$, however, is singular due to the scalar ambiguity inherent in all blind channel identification problems [7–9]. To eliminate the ambiguity, various constraints can be enforced to regularise the estimation problem. In what follows, we present the CRB for parameters that satisfy the constraints

$$\boldsymbol{f}(\boldsymbol{\theta}) = \begin{bmatrix} h_r(0) - a_r \\ h_i(0) - a_i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad (32)$$

Then the $2 \times m$ gradient matrix for the constraints is given by

$$\boldsymbol{F}(\boldsymbol{\theta}) = \frac{\partial \boldsymbol{f}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}^T} = \begin{bmatrix} 1 & \boldsymbol{0}_{1\times 2JN} & \\ \boldsymbol{0}_{1\times(L+1)} & 1 & \boldsymbol{0}_{1\times(2JN-L-1)} \end{bmatrix} \qquad (33)$$

The gradient matrix $\boldsymbol{F}(\boldsymbol{\theta})$ has full row rank, therefore, there exists a matrix $\boldsymbol{U} \in \mathbb{C}^{m\times(m-2)}$ whose columns form basis for the null-space of $\boldsymbol{F}(\boldsymbol{\theta})$, that is

$$\boldsymbol{F}(\boldsymbol{\theta})\boldsymbol{U} = \boldsymbol{0} \qquad (34)$$

It follows that the constrained CRB is given by [8, 10]

$$\text{CRB}(\boldsymbol{\theta}; \boldsymbol{f}(\boldsymbol{\theta}) = \boldsymbol{0}) = \boldsymbol{U}(\boldsymbol{U}^T\boldsymbol{J}(\boldsymbol{\theta})\boldsymbol{U})^{-1}\boldsymbol{U}^T \qquad (35)$$

6 Numerical results

We present simulation results reflecting two different scenarios based on how the received signal is being processed. Precisely, we form blocks of N symbols each, and have the N-symbols arranged in overlapping and nonoverlapping fashion. To see this, let the total number of OFDM symbols M equal to an integer multiples of N, that is $M = NP$ for any integer P. As illustrated by Fig. 1, this will result in P and $M - N + 1$ nonoverlapping and overlapping blocks simultaneously. The mathematical expression for the received nonoverlapping symbols is given by (2). By forming an overlapping symbols, the received data will have similar expression as in (2) with the index n running from 1 to $M - N + 1$.

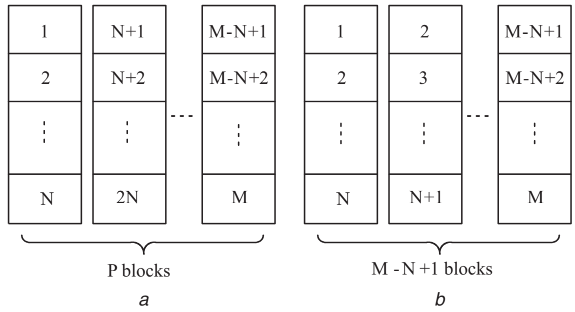

Fig. 1 *Orthogonal frequency-division multiplexing symbols*

a Nonoverlapping
b Overlapping

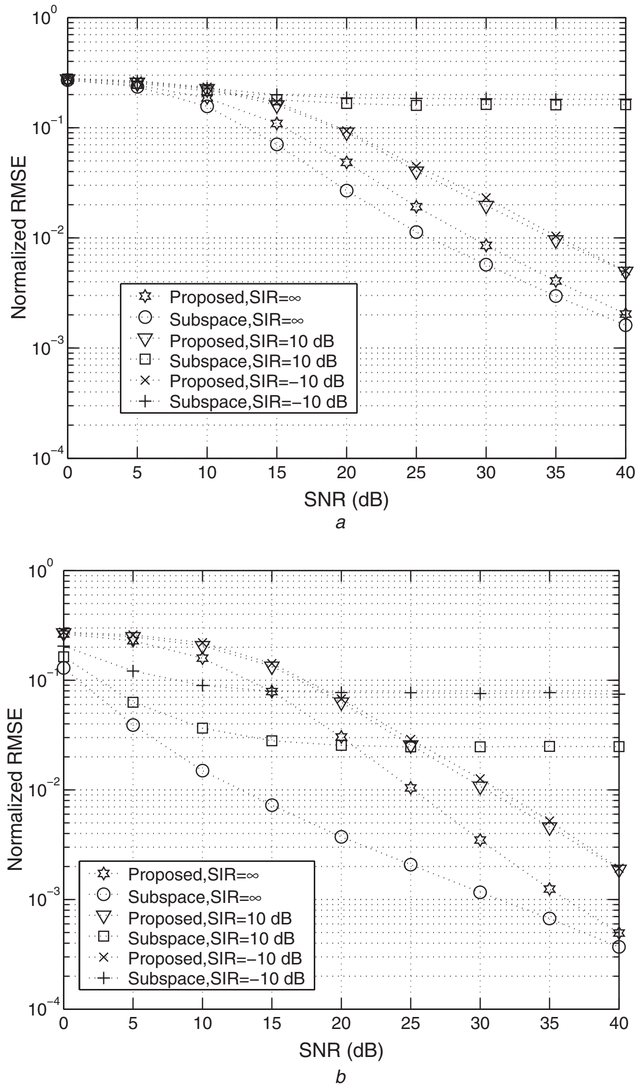

Fig. 2 *Normalised RMSE of proposed and subspace blind channel estimates against SNR and SIR, for K = 48, L = 3 and N = 2*

a Nonoverlapping
b Overlapping

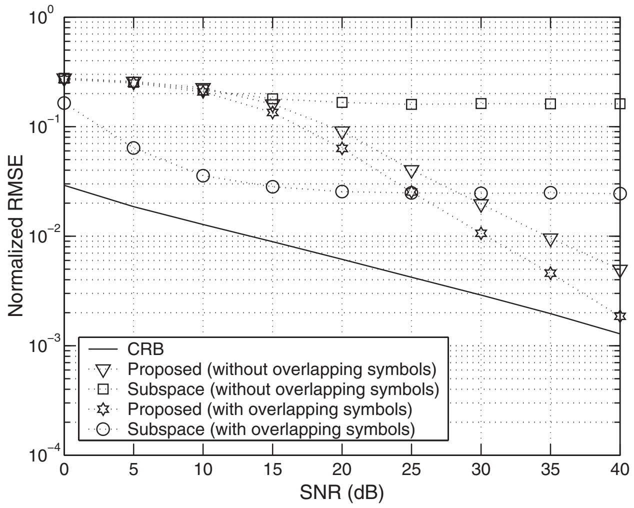

Fig. 3 *Normalised RMSE of CRB, proposed and subspace blind channel estimates against SNR, for K = 48, L = 3, N = 2 and SIR = 10 dB*

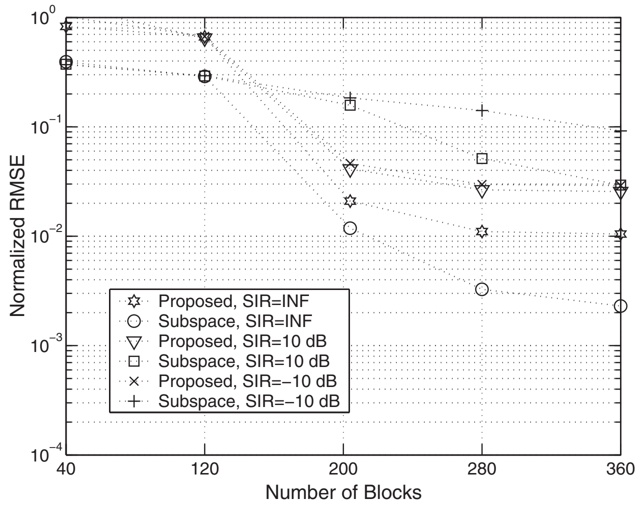

Fig. 4 *Normalised RMSE of the proposed and subspace blind channel estimates against number of blocks and SIR, for K = 48, L = 3, N = 2 and SNR = 25 dB*

We compare here the proposed method with the subspace blind channel estimators [2]. The system under study utilises the IDFT transform and a BPSK constellation with $K = 48$ and $N = 2$. Additionally, both estimators use a total of $M = 200$ OFDM symbols for channel estimation. The channel is a four-tap ($L = 3$) FIR channel. Two narrow-band interfering signals are added with various values of signal-to-interference ratio (SIR). As a performance measure, we consider here the normalised root mean-squared error (RMSE) defined as

$$\frac{1}{\|\boldsymbol{h}\|}\sqrt{\frac{1}{D(L+1)}\sum_{i}^{D}\left\|\hat{\boldsymbol{h}}_i - \boldsymbol{h}\right\|^2}$$

that is averaged over $D = 500$ Monte Carlo runs. For all examples, parameters are changed randomly from one trial to another.

Figs. 2*a* and *b* show the performance against SNR and SIR for both nonoverlapping and overlapping scenarios, respectively. In the absence of interference (i.e. SIR = ∞), the subspace estimator outperforms the proposed scheme slightly. However, even with fairly weak interference (i.e. SIR = 10 dB), the subspace estimator degrades significantly and exhibit irreducible error. By employing an overlapping structure to the received data symbols, a significant performance improvement can be obtained for both estimators as seen in Fig. 2*b*.

Fig. 3 depicts the RMSE of the proposed and subspace channel estimators along with the CRB for SIR = 10 dB. In this example we see that the proposed scheme, for both overlapping and non overlapping cases, approach the CRB as the SNR increases. Meanwhile the subspace scheme is suffering from the moderately increased interference level.

Fig. 4 shows the performance against number of data block and SIR for the nonoverlapping case when the SNR is 25 dB. Here we see that the proposed method can handle strong interference and maintain good performance as the number of data blocks is increased. As for the subspace estimator, it outperforms the proposed scheme in the absence of interference. However, even with large number of data blocks, the subspace estimator degrades as the interference is moderately increased. This is because subspace cannot handle unknown or unmodelled interference.

7 Conclusions

We have presented a blind channel estimation and equalisation scheme for orthogonal frequency-division multiplexing systems with unmodelled interference. A generalised multichannel minimum variance principle was invoked to design an equalising filterbank that preserves desired signal components and suppresses the overall interference. To overcome computational difficulty and local convergence problems that accompany multidimensional search methods we have derived an asymptotically (in SNR) tight lower bound of the filterbank output power and used it for channel estimation, which reduces the problem to a quadratic minimisation. To assess the performance of the proposed scheme an unconditional Cramér–Rao bound (CRB) was derived. Numerical examples show that the proposed scheme approaches the CRB as the SNR level is increased. Additionally, the proposed scheme compares favourably with a subspace blind channel estimator in the presence of unknown narrowband interference. By imposing an overlapping structure on the received data symbols the performance of both estimators was significantly improved.

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