

# Robust approach to channel estimation and detection for OFDM systems

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A robust channel estimation and detection scheme has been developed that explicitly accounts for channel estimation error by optimising the worst-case performance over a properly selected bounded uncertainty set. Numerical results show improved performance using the proposed robust approach over the one that ignores the prior estimation errors.

**Introduction:** Orthogonal frequency division multiplexing (OFDM) is a popular modulation technique for wideband wireless communications owing to its various benefits (e.g. bandwidth and computational efficiency). Many OFDM receiver structures require knowledge of the wireless channel, which is typically estimated at the receiver using training or some other channel estimation technique. The conventional practice is to treat the channel estimate as if it contained no estimation error, and directly use it for demodulation. However, estimation error is ubiquitous in all known OFDM channel estimation methods [1]. In this Letter, rather than ignoring the channel estimation error, we explicitly take it into account and introduce a robust approach that leads to improved estimation and detection performance.

Consider an OFDM system where a series of information symbols are blocked into  $K \times 1$  vectors  $\mathbf{s}(n) = [s(nK), \dots, s(nK + K - 1)]^T$ . At the receiver, the  $n$ th block of the data symbols can be expressed as (after DFT):

$$\mathbf{y}(n) = \mathbf{G}(\mathbf{h})\mathbf{s}(n) + \mathbf{e}(n) \quad (1)$$

where  $\mathbf{h}$  is an  $L$ th order channel vector,  $\mathbf{G}(\mathbf{h}) = \text{diag}(\mathbf{g})$ , with  $\mathbf{g} = \sqrt{K}\mathcal{F}\mathbf{h}$ , where  $\mathcal{F}$  stands for the first  $L + 1$  columns of a  $K \times K$  DFT matrix, and  $\mathbf{e}(n)$  is the frequency domain channel noise vector. It follows that  $\mathbf{y}(n)$  in (1) can be expressed as:

$$\begin{aligned} \mathbf{y}(n) &= \text{diag}(\mathbf{s}(n))\sqrt{K}\mathcal{F}\mathbf{h} + \mathbf{e}(n) \\ &= \mathbf{A}(n)\mathbf{h} + \mathbf{e}(n) \end{aligned} \quad (2)$$

**Proposed scheme:** To assist in determining the boundaries of the uncertainty set, we first analyse the estimation error of the least-squares (LS) based channel estimator using one OFDM training symbol. By dropping the index  $n$  in (2), the initial channel estimate is given as  $\hat{\mathbf{h}} = (\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H\mathbf{y}$ . It follows that channel estimation error associated with  $\hat{\mathbf{h}}$  is  $\Delta\mathbf{h} = \hat{\mathbf{h}} - \mathbf{h} = (\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H\mathbf{e}$ . Under the assumption of unit energy constellations and a zero-mean white Gaussian noise with  $E\{\mathbf{e}\mathbf{e}^H\} = \sigma^2\mathbf{I}_K$ ,  $\Delta\mathbf{h}$  will be a zero-mean Gaussian with covariance matrix

$$\text{cov}\{\Delta\mathbf{h}\} = \sigma^2(\mathbf{A}^H\mathbf{A})^{-1} = \frac{\sigma^2}{K}\mathbf{I}_{L+1} \quad (3)$$

Let  $\beta \triangleq \|\Delta\mathbf{h}\|^2$ , then  $\beta$  is  $\chi^2$  distribution with  $2(L + 1)$  degrees of freedom, with mean  $\mu_\beta = \sigma^2(L + 1)/K$  and variance  $\sigma_\beta^2 = \sigma^4(L + 1)/K^2$ . These calculations will help to choose the size of the uncertainty set.

**Bounding channel estimation error:** We now discuss how to bound the above estimation error. Since  $\beta \triangleq \|\Delta\mathbf{h}\|^2$ , is  $\chi^2$  distributed and therefore unbounded, we use the Chebyshev inequality, where the unbounded channel estimation error is bounded in probability. For any given positive number  $\delta_\beta$

$$P_\beta(|\beta - \mu_\beta| > \delta_\beta) \leq \frac{\sigma_\beta^2}{\delta_\beta^2} \quad (4)$$

For large  $\delta_\beta$ , we ignore the unmodelled channel estimation error, and consider a bounded set  $P_\beta(\beta \leq \mu_\beta + \delta_\beta) \geq 1 - \sigma_\beta^2/\delta_\beta^2$ .

Let  $\varepsilon \triangleq \mu_\beta + \delta_\beta$ , denote a boundary of  $\beta$ , then  $P_\beta(\|\Delta\mathbf{h}\|^2 \leq \varepsilon) \geq 1 - \sigma_\beta^2/\delta_\beta^2$ , where  $P_\beta(\|\Delta\mathbf{h}\|^2 \leq \varepsilon)$  is the Chebyshev bounding probability. For a given probability  $P_\beta$ , the boundary  $\varepsilon$  is given by

$$\varepsilon = \mu_\beta + \sqrt{\frac{\sigma^2}{1 - P_\beta}} = \frac{\sigma^2}{K} \left( L + 1 + \sqrt{\frac{L + 1}{1 - P_\beta}} \right) \quad (5)$$

**Robust channel estimation and detection:** Following the minimum variance (MV) criterion (e.g. [2, 3])

$$\mathbf{W}_{\text{MV}} = \arg \min_{\mathbf{W} \in \mathbb{C}^{K \times K}} \text{tr}\{\mathbf{W}^H \mathbf{R}_y \mathbf{W}\}, \quad \text{subject to } \mathbf{W}^H \mathbf{G}(\mathbf{h}) = \mathbf{I}_K \quad (6)$$

where  $\mathbf{R}_y \triangleq E\{\mathbf{y}(n)\mathbf{y}^H(n)\}$  denotes the covariance matrix of  $\mathbf{y}(n)$ . Using the Lagrange multiplier, the solution to (6) is given by [4]:

$$\mathbf{W}_{\text{MV}} = \mathbf{R}_y^{-1} \mathbf{G}(\mathbf{h})(\mathbf{G}^H(\mathbf{h})\mathbf{R}_y^{-1}\mathbf{G}(\mathbf{h}))^{-1} \quad (7)$$

Substituting (7) into (6), the minimised average output power of  $\mathbf{W}_{\text{MV}}$  is given by

$$V_1(\mathbf{h}) = \text{tr}\left\{[\mathbf{G}^H(\mathbf{h})\mathbf{R}_y^{-1}\mathbf{G}(\mathbf{h})]^{-1}\right\} \quad (8)$$

Since the MV detector is sensitive to signal mismatch owing to errors in  $\hat{\mathbf{h}}$ , we consider robust channel estimation by maximising the output  $V_1(\mathbf{h})$  so that  $\mathbf{W}_{\text{MV}}$  will maximally preserve the signal power. Using the Schwartz inequality [3], it follows that maximising  $V_1(\mathbf{h})$  is equivalent to minimising  $\text{tr}\{\mathbf{G}^H(\mathbf{h})\mathbf{R}_y^{-1}\mathbf{G}(\mathbf{h})\}$ . The robust channel estimation can be obtained as:

$$\tilde{\mathbf{h}} = \arg \min_{\mathbf{h}} \text{tr}\{\mathbf{G}^H(\mathbf{h})\mathbf{R}_y^{-1}\mathbf{G}(\mathbf{h})\}, \quad \text{s.t. } \|\hat{\mathbf{h}} - \mathbf{h}\|^2 \geq \varepsilon \quad (9)$$

To proceed, we simplify the cost function in (9). It follows that  $\text{tr}\{\mathbf{G}^H(\mathbf{h})\mathbf{R}_y^{-1}\mathbf{G}(\mathbf{h})\} = \mathbf{h}^H[K\sum_{k=1}^K \mathbf{R}_y^{-1}(k, k)\mathcal{F}_k^H\mathcal{F}_k]\mathbf{h} = \mathbf{h}^H\Phi\mathbf{h}$ , where  $\mathcal{F}_k$  stands for the  $k$ th row of matrix  $\mathcal{F}$ . Since the solution of (9) will occur on the boundary of the uncertainty set (i.e. the worst case) [5], we have

$$\tilde{\mathbf{h}} = \arg \min_{\mathbf{h}} \mathbf{h}^H\Phi\mathbf{h}, \quad \text{subject to } \|\hat{\mathbf{h}} - \mathbf{h}\|^2 = \varepsilon \quad (10)$$

Using the Lagrange multiplier, the solution to (10) can be solved in a manner similar to [6]:

$$\tilde{\mathbf{h}} = \hat{\mathbf{h}} - (\mathbf{I}_{L+1} + \lambda\Phi^{-1})^{-1}\hat{\mathbf{h}} \quad (11)$$

With knowledge of  $\tilde{\mathbf{h}}$ ,  $\mathbf{g}$  can be updated as  $\tilde{\mathbf{g}} = \sqrt{K}\mathcal{F}\tilde{\mathbf{h}}$ , and  $\tilde{\mathbf{G}}(\mathbf{h}) = \text{diag}(\tilde{\mathbf{g}})$ . The robust MV detector is given by:

$$\tilde{\mathbf{W}}_{\text{robust MV}} = \mathbf{R}_y^{-1}\tilde{\mathbf{G}}(\mathbf{h})(\tilde{\mathbf{G}}(\mathbf{h})^H\mathbf{R}_y^{-1}\tilde{\mathbf{G}}(\mathbf{h}))^{-1} \quad (12)$$

**Numerical results:** We consider an OFDM system with  $K = 48$  and  $L = 3$ . We compare the proposed robust MV detector in (12), with the standard MV detector in (7). Fig. 1 depicts the receiver output signal-to-interference-and-noise ratio (SINR) against the normalised  $\varepsilon/E\{\|\mathbf{h}\|^2\}$  when  $\sigma^2$  [cf. (3)] is fixed and SNR = 10 dB. Since the conventional detector ignores the prior estimation error, it is independent of  $\varepsilon$ . While the robust detector requires a choice of  $\varepsilon$ , it is insensitive to the choice. Compared with the standard detector, the robust detector shows a notable improvement in SINR. Figs. 2 and 3, respectively, show the SINR and the average bit error rate (BER) performance against SNR, when  $\varepsilon/E\{\|\mathbf{h}\|^2\} = 0.1$ . It is seen that the robust detector outperforms the standard nonrobust detector. Moreover, the conventional detector has an irreducible error floor owing to poor initial channel estimates. For the case considered in the examples, the receiver behaviour is dominated by the poor channel estimates, and increasing SNR helps only very little.

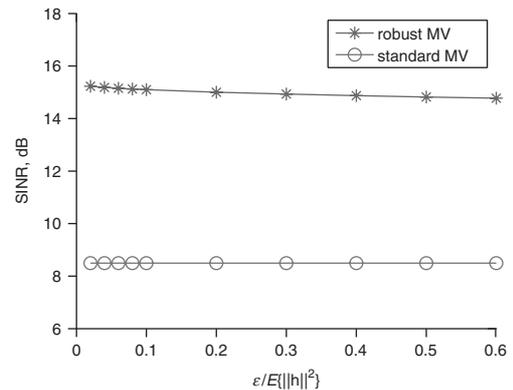


Fig. 1 SINR against normalised channel uncertainty

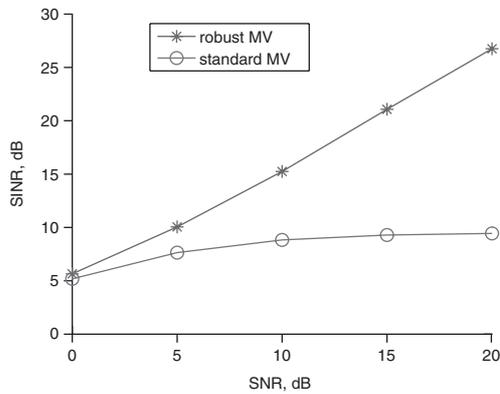


Fig. 2 SINR against SNR when  $\varepsilon/E\{\|h\|^2\} = 0.1$

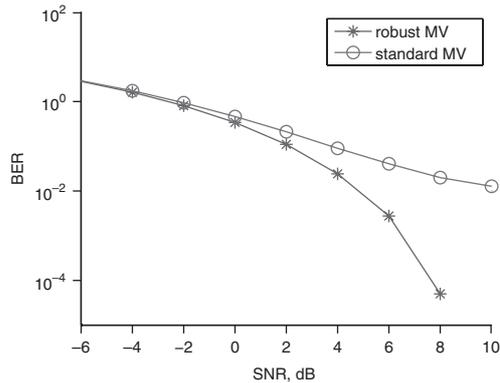


Fig. 3 Average BER against input SNR when  $\varepsilon/E\{\|h\|^2\} = 0.1$

*Conclusions:* A robust detector has been obtained by optimising the worst-case performance over a bounded uncertainty set pertaining to the prior estimation error in the initial channel estimate.

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#### References

- 1 Ozdemir, M.K., and Arslan, H.: 'Channel estimation for wireless OFDM systems', *IEEE Commun. Surv. Tutor.*, 2007, **9**, (2), pp. 18–48
- 2 Schodorf, J.B., and Williams, D.B.: 'A constrained optimization approach to multiuser detection', *IEEE Trans. Signal Process.*, 1997, **45**, (1), pp. 258–262
- 3 Li, H.: 'Blind channel estimation for multicarrier systems with narrowband interference suppression', *IEEE Commun. Lett.*, 2003, **7**, (7), pp. 326–328
- 4 Stoica, P., and Moses, R.L.: 'Introduction to spectral analysis' (Prentice Hall, Upper Saddle River, NJ, 1997)
- 5 Boyd, S., and Vandenberghe, L.: 'Convex optimization' (Cambridge University Press, 2004)
- 6 Li, J., Stoica, P., and Wang, Z.: 'On robust Capon beamforming and diagonal loading', *IEEE Trans. on Signal Processing*, 2003, **51**, (7), pp. 1702–1715