Blind-Channel Estimation and Interference Suppression for Single-Carrier and Multicarrier Block Transmission Systems

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***Abstract*—Block transmission has recently been considered as an alternative to the conventional continuous transmission technique. In particular, block transmission techniques with zero padding (ZP) and cyclic prefix (CP) are becoming attractive procedures for their ability to eliminate both intersymbol interference (ISI) and interblock interference (IBI). In this paper, we present a unified approach to blind-channel estimation and interference suppression for block transmission using ZP or CP in both single-carrier (SC) and multicarrier (MC) systems. Our approach uses a generalized multichannel minimum variance principle to design an equalizing filterbank. The channel estimate is then obtained from an asymptotically tight lower bound of the filterbank output power. Through an asymptotic analysis of the subspace of the received signal, we determine an upper bound for the number of interfering tones that can be handled by the proposed schemes. As a performance measure, we derive an unconditional Cramér–Rao bound (CRB) that, similar to the proposed blind channel estimators, does not assume knowledge of the transmitted information symbols. Numerical examples show that the proposed schemes approach the CRB as the signal-to-noise ratio (SNR) increases. Additionally, they exhibit low sensitivity to unknown narrowband interference and favorably compare with subspace blind-channel estimators.**

***Index Terms*—Blind-channel estimation, equalization, interference suppression, multicarrier (MC) systems, single-carrier (SC) systems.**

I. Introduction

BROADBAND digital wireless communication systems that rely on continuous transmission may suffer from serious intersymbol interference (ISI). To combat this interference and effectively restore the transmitted signal, appropriate equalization with high complexity needs to be done at the receiver. As the first multicarrier (MC) block transmission technique, orthogonal frequency-division multiplexing (OFDM) was proposed as an alternative to conventional continuous transmission schemes and has been adapted as a standard for high-speed data transmission in indoor and outdoor communication systems. Meanwhile, single-carrier (SC) block transmission techniques were shown to offer performance similar to that of OFDM while maintaining low-complexity efficient equalization without suffering from several problems of OFDM (e.g., sensitivity to carrier offset, peak-to-average power ratio problem, etc.).

When block transmission is used over a dispersive channel, interblock interference (IBI) between successive blocks occurs. To eliminate IBI, a guard interval of length μ greater than or equal to the order of the channel impulse response is inserted in each block in the form of cyclic prefix (CP) or zero padding (ZP). In block transmission with CP, we concatenate on top of each transmitted block its last μ elements and have them removed at the receiver. This process converts the channel effect from a linear into a circular convolution, where ISI can be efficiently removed using fast Fourier transform. In the ZP case, trailing zeros of length μ are amended at the end of each transmitted data block. As a result, successive blocks will not interfere with each other and can reliably be recovered at the receiver's end. A key advantage of ZP over CP is that symbol recovery is always guaranteed for ZP, regardless of the channel zero locations [1], [2], whereas this is not the case for CP-based transmission.

In this paper, we consider the problem of blind-channel estimation and equalization for SC and MC systems that employ either the ZP or CP techniques. Numerous channel estimation schemes have recently been investigated. These schemes rely on either *explicit training* (e.g., [3]) or some *inherent structure* (e.g., [4] and [5]) of the transmitted signal. Although the training-assisted schemes perform quite well, they reduce the spectral efficiency. Moreover, to track channel variations, training symbols have to periodically be retransmitted, leading to throughput reduction. Blind schemes, on the other hand, do not suffer from such drawbacks. A well-known class of blind-channel estimation schemes for MC are the subspace-based methods (e.g., [5] and [6]). Subspace-based methods generally assume absence of interference; otherwise, some information of the interference should be known so that the interference can be canceled prior to subspace estimation. However, when there is insufficient information about the interference so that prewhitening cannot be performed, subspace channel estimation is, in general, inaccurate.

Recently, numerous studies on the comparison between OFDM and SC were made. It was shown in [7] that block SC transmission has similar equalization complexity and coded performance to that of OFDM. The power efficiency for SC and

Manuscript received August 26, 2006; revised June 4, 2007, September 17, 2007, and December 10, 2007. This work was supported in part by the National Science Foundation under Grant CCF-0514938 and in part by the National Natural Science Foundation under Grant 60502011. The review of this paper was coordinated by Prof. G. Såulnier.

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Digital Object Identifier 10.1109/TVT.2008.915497

OFDM was studied in [8], where it was shown that SC is more power efficient. Here, we study joint blind-channel estimation and interference suppression for block transmission by unifying ZP or CP for both SC and MC systems. In addition to ISI and IBI, we consider the presence of unmodeled interference that may be caused by time/frequency synchronization errors, overlay with narrowband systems, among others. In deriving the proposed schemes, a generalized multichannel minimum variance principle is invoked to design an equalizing filterbank that preserves the desired signal components and suppresses the overall interference. To overcome the convergence problem associated with multidimensional search-based methods, we derive an asymptotically (in signal-to-noise ratio or SNR) tight lower bound of the filterbank output power and use it for channel estimation, which reduces the problem to a quadratic minimization. By exploiting the asymptotic structure of a matrix that is related to the filterbank output power, we determine an upper bound for the number of interfering tones that the proposed schemes can handle. We also derive an unconditional Cramér–Rao bound (CRB), which, similar to the proposed blind-channel estimators, does not assume knowledge of the transmitted information symbols. The CRB serves as a lower bound for all unbiased blind-channel estimation schemes in the presence of interference. We show through numerical examples in Section VII that the proposed schemes exhibit lower sensitivity to unknown narrowband interference and favorably compare with the subspace blind-channel estimators. Additionally, the proposed schemes approach the CRB as the SNR increases.

The rest of this paper is organized as follows. In Section II, we formulate the problem of interest. We introduce the proposed algorithms in Section III followed by implementation schemes in Section IV. We present asymptotic analysis in Section V. The unconditional CRB is derived in Section VI. Section VII numerically illustrates the performance of the proposed as well as competing schemes. Finally, we draw conclusions in Section VIII.

Notation: Vectors (matrices) are denoted by boldface lowercase (uppercase) letters. All vectors are column vectors. Superscripts $(\cdot)^T$, $(\cdot)^*$, and $(\cdot)^H$ denote the transpose, conjugate, and conjugate transpose, respectively. $\mathbf{I}_N$ denotes the $N \times N$ identity matrix. $\mathbf{0}$ denotes an all-zero matrix or vector. $\mathrm{tr}\{\cdot\}$ denotes the trace. $\mathrm{vec}(\cdot)$ stacks the columns of its matrix argument on top of one another. $E\{\cdot\}$ denotes the statistical expectation. The Matlab notation $\mathbf{A}(i_1 : i_2, j_1 : j_2)$ denotes a $(i_2 - i_1 + 1) \times (j_2 - j_1 + 1)$ matrix formed from rows $i_1, \ldots, i_2$ and columns $j_1, \ldots, j_2$ of matrix $\mathbf{A}$. Finally, $\otimes$ denotes the matrix Kronecker product.

II. Problem Formulation

Consider a system where a serial of information symbols are blocked into $K \times 1$ vectors $\mathbf{s}(n) = [s(nK), \ldots, s(nK + K - 1)]^T$. To avoid multipath-induced IBI, a guard interval of length μ is inserted in each block, where the length μ is chosen to be greater than or equal to the channel order L. The discrete-time baseband equivalent channel, which includes the transmitter/receiver filters and the physical channel, is modeled as a finite-duration impulse response (FIR) filter $\mathbf{h} \stackrel{\triangle}{=} [h(0), h(1), \ldots, h(L)]^T$. In what follows, we introduce the system models for SC with ZP (SC-ZP), MC with ZP (MC-ZP), SC with CP (SC-CP), and MC with CP (MC-CP), respectively. We note here that MC-CP is simply the famous OFDM system. In ZP block transmission, trailing zeros of length μ are introduced at the end of each block before transmission. This can be done by multiplying $\mathbf{s}(n)$ with a $J \times K$ guard insertion matrix $\mathbf{T}_{zp} = [\mathbf{I}_K^T \quad \mathbf{0}_{\mu \times K}^T]^T$, where $\mathbf{I}_K$ is a $K \times K$ identity matrix, and the symbol $J = K + \mu$. The trailing zeros will guarantee the removal of IBI from the received signal. CP block transmission relies on staking on top of the transmitted vector $\mathbf{s}(n)$ its last μ elements by multiplying $\mathbf{s}(n)$ with a $J \times K$ matrix $\mathbf{T}_{cp} = [\mathbf{I}_{\mu \times K}^T \quad \mathbf{I}_K^T]^T$, where $\mathbf{I}_{\mu \times K}$ is formed from the last μ rows of the identity matrix $\mathbf{I}_K$.

To effectively suppress the interference (as shown in Section VII), we develop a general model by simultaneously processing a block of $N \geq 1$ symbols at the receiver side. Let

$$\begin{aligned} \mathbf{s}_N(n) &= \left[\mathbf{s}^T(nN) \cdots \mathbf{s}^T(nN + N - 1)\right]^T_{KN \times 1} \\ \mathbf{u}_N(n) &= \left[\mathbf{u}^T(nN) \cdots \mathbf{u}^T(nN + N - 1)\right]^T_{JN \times 1} \\ &= (\mathbf{I}_N \otimes \mathbf{T})\mathbf{s}_N(n) \end{aligned} \tag{1}$$

where $\mathbf{u}(n) \stackrel{\triangle}{=} \mathbf{T}\mathbf{s}(n)$, and $\mathbf{T}$ is the guard insertion matrix that equals $\mathbf{T}_{zp}$ for ZP and $\mathbf{T}_{cp}$ for CP block transmission, respectively, as we detailed before. Next, we discuss the received signals for the four different block-transmission schemes.

A. SC-ZP

The nth block of the received signal in the time domain can be expressed as

$$\mathbf{y}_{zp}(n) = \mathbf{H}(\mathbf{I}_N \otimes \mathbf{T}_{zp})\mathbf{s}_N(n) + \mathbf{w}_{zp}(n) + \mathbf{e}_{zp}(n) \tag{2}$$

where $\mathbf{H}$ is the $JN \times JN$ Toeplitz matrix with first row $[h(0), \mathbf{0}_{1\times(JN-1)}]$ and first column $[h(0), \ldots, h(L), \mathbf{0}_{1\times(JN-L-1)}]^T$, and $\mathbf{w}_{zp}(n)$ and $\mathbf{e}_{zp}(n)$ denote the $JN \times 1$ unmodeled interference and channel noise vectors, respectively. As mentioned before, the interference might be caused by time/frequency synchronization errors or overlay with narrowband systems, among others.

For later use and to help determining the rank, we express the composite matrix $\mathbf{H}(\mathbf{I}_N \otimes \mathbf{T}_{zp})$ as

$$\mathbf{H}(\mathbf{I}_N \otimes \mathbf{T}_{zp}) = \mathbf{I}_N \otimes \boldsymbol{\mathcal{H}} \tag{3}$$

where $\boldsymbol{\mathcal{H}}$ is a $J \times K$ full column rank matrix given by

$$\boldsymbol{\mathcal{H}} = \begin{bmatrix} h(0) & 0 & \cdots & 0 \\ \vdots & h(0) & \cdots & 0 \\ h(L) & \cdots & \ddots & \vdots \\ 0 & h(L) & \cdots & h(0) \\ \vdots & \cdots & \ddots & \vdots \\ 0 & \cdots & 0 & h(L) \\ & & \mathbf{0}_{(\mu-L)\times K} & \end{bmatrix}. \tag{4}$$

B. MC-ZP

In MC transmission, the data symbols $\mathbf{s}(n)$ are modulated with an inverse discrete Fourier transform (IDFT) unitary matrix $\mathbf{F} \in \mathbb{C}^{K\times K}$, whose (k,l)th element is given by $K^{-1/2}\exp\{j2\pi(k-1)(l-1)\}$. The received signal can be expressed as

$$\begin{aligned}\bar{\mathbf{y}}_{zp}(n) &= \mathbf{H}\left(\mathbf{I}_N \otimes (\mathbf{T}_{zp}\mathbf{F})\right)\mathbf{s}_N(n) + \mathbf{w}_{zp}(n) + \mathbf{e}_{zp}(n) \\ &= \left(\mathbf{I}_N \otimes (\boldsymbol{\mathcal{H}}\mathbf{F})\right)\mathbf{s}_N(n) + \mathbf{w}_{zp}(n) + \mathbf{e}_{zp}(n). \qquad (5)\end{aligned}$$

C. SC-CP

To eliminate IBI from the nth received block, the last L samples, which were introduced as a result of convolving with the channel, are removed. We also discard the first L samples, which contain the IBI, by using an appropriate truncation matrix $\Gamma = [\mathbf{0}_{(JN-L)\times L} \quad \mathbf{I}_{JN-L}]$ and form a $(JN-L)\times 1$ vector $\mathbf{y}_{cp}(n)$ as

$$\mathbf{y}_{cp}(n) = \Gamma\mathbf{H}(\mathbf{I}_N \otimes \mathbf{T}_{cp})\mathbf{s}_N(n) + \mathbf{w}_{cp}(n) + \mathbf{e}_{cp}(n) \qquad (6)$$

where $\mathbf{w}_{cp}(n)$ and $\mathbf{e}_{cp}(n)$ denote $(JN-L)\times 1$ interference and channel noise vectors, respectively. The $(JN-L)\times KN$ composite matrix $\Gamma\mathbf{H}(\mathbf{I}_N \otimes \mathbf{T}_{cp})$ has N blocks along its diagonal, where consecutive blocks are overlapping with L rows. The middle $N-2$ blocks are identical, where each block is a $(J+L)\times K$ matrix given by

$$\bar{\boldsymbol{\mathcal{H}}} = \begin{bmatrix} \mathbf{0}_{\mu\times(K-\mu)} & & h(0) & & \mathbf{0} \\ & & \vdots & \ddots & \\ h(0) & & h(L) & & h(0) \\ \vdots & \ddots & \mathbf{0}_{(K-L-1)} & \ddots & \vdots \\ h(L) & & \ddots & \ddots & h(L) \\ & \ddots & & \ddots & \mathbf{0}_{(K-L-1)} \\ & & \ddots & & h(0) \\ & & & \ddots & \vdots \\ \mathbf{0} & & & & h(L) \end{bmatrix}. \qquad (7)$$

The remaining first and last two blocks have the same structure as $\bar{\boldsymbol{\mathcal{H}}}$, with the first L rows removed from the first block and the last L rows removed from the last block. We note that $\Gamma\mathbf{H}(\mathbf{I}_N \otimes \mathbf{T}_{cp})$ is a tall matrix if $JN-L > KN$, or equivalently, $N > L/J - K$. If the length of the CP is chosen as $J-K=L$, then the minimum value of N that is needed is equal to 2.

D. MC-CP

Similar to MC-ZP, the transmitted signal $\mathbf{s}(n)$ needs to be converted by IDFT prior to transmission. The received signal can be expressed as

$$\bar{\mathbf{y}}_{cp}(n) = \Gamma\mathbf{H}\left(\mathbf{I}_N \otimes (\mathbf{T}_{cp}\mathbf{F})\right)\mathbf{s}_N(n) + \mathbf{w}_{cp}(n) + \mathbf{e}_{cp}(n). \qquad (8)$$

The composite matrix $\Gamma\mathbf{H}(\mathbf{I}_N \otimes (\mathbf{T}_{cp}\mathbf{F}))$ takes the same form as the one described for the SC-CP case with $\bar{\boldsymbol{\mathcal{H}}}$ replaced by $\bar{\boldsymbol{\mathcal{H}}}\mathbf{F}$.

The problem of interest is to find channel estimates $\hat{\mathbf{h}}$ for all of the mentioned schemes, namely, SC-ZP, MC-ZP, SC-CP, and MC-CP, without the knowledge of the transmitted data symbols.

III. Proposed Schemes

Due to the presence of narrowband interference and noise, the observed signals $\mathbf{y}_{zp}(n)$, $\bar{\mathbf{y}}_{zp}(n)$, $\mathbf{y}_{cp}(n)$, and $\bar{\mathbf{y}}_{cp}(n)$ are all noisy. Hence, instead of directly using raw data, we propose to first pass the received signals through a bank of filters, which are designed to enhance the useful signals and suppress the interference/noise, and then derive the channel estimates from the filtered data. The problem under study involves multichannel communication systems. In all of them, channel matrices are partially known because of their Toeplitz structure, and $\mathbf{F}$ (for the MC case) is also known to the receiver. We can exploit this knowledge to design a bank of FIR filters, where each passes one symbol with unit gain, completely annihilating the other interfering symbols and, meanwhile, suppressing narrowband interference as much as possible.

A. Channel Estimation for SC-ZP

We design an equalizing filterbank according to the following minimum variance criterion:

$$\mathbf{G}_{zp} = \arg\min_{\mathbf{G}_{zp}\in\mathbb{C}^{JN\times KN}} \operatorname{tr}\left\{\mathbf{G}_{zp}^H\mathbf{R}_{zp}\mathbf{G}_{zp}\right\} \quad \text{subject to } \mathbf{G}_{zp}^H\mathbf{H}(\mathbf{I}_N \otimes \mathbf{T}_{zp}) = \mathbf{I}_{KN} \qquad (9)$$

where $\mathbf{R}_{zp} \triangleq E\{\mathbf{y}_{zp}(n)\mathbf{y}_{zp}^H(n)\}$ denotes the covariance matrix. The constraint $\mathbf{G}_{zp}^H\mathbf{H}(\mathbf{I}_N \otimes \mathbf{T}_{zp}) = \mathbf{I}_{KN}$ ensures that each filter (i.e., one column of $\mathbf{G}_{zp}$) will pass only one signal component [corresponding to one column of $\mathbf{H}(\mathbf{I}_N \otimes \mathbf{T}_{zp})$] undistorted with unit gain while completely eliminating the ISI, which modulates one symbol in $\mathbf{s}_N(n)$ caused by the other columns of $\mathbf{H}(\mathbf{I}_N \otimes \mathbf{T}_{zp})$. Meanwhile, minimizing the total output variance as in the cost function of (9) is intended to minimize the unmodeled interference in the received signal. Using the Lagrange multiplier, the solution to the aforementioned constrained quadratic minimization problem is given by (also see, e.g., [9, p. 283])

$$\mathbf{G}_{zp} = \mathbf{R}_{zp}^{-1}\mathbf{H}(\mathbf{I}_N \otimes \mathbf{T}_{zp}) \times \left[(\mathbf{I}_N \otimes \mathbf{T}_{zp})^H\mathbf{H}^H\mathbf{R}_{zp}^{-1}\mathbf{H}(\mathbf{I}_N \otimes \mathbf{T}_{zp})\right]^{-1}. \qquad (10)$$

Substituting (10) into (9), the minimized average power of the filterbank output is given by

$$V(\mathbf{h}) = \operatorname{tr}\left\{\left[(\mathbf{I}_N \otimes \mathbf{T}_{zp})^H\mathbf{H}^H\mathbf{R}_{zp}^{-1}\mathbf{H}(\mathbf{I}_N \otimes \mathbf{T}_{zp})\right]^{-1}\right\}. \qquad (11)$$

To find an estimate for $\mathbf{h}$, we want to maximize $V(\mathbf{h})$ with respect to (w.r.t.) $\mathbf{h}$ so that $\mathbf{G}_{zp}$ will maximally preserve the signal power. Because of the nonlinear nature of $V(\mathbf{h})$, this approach is computationally involved and suffers local convergence. Instead, we maximize an asymptotic lower bound of $V(\mathbf{h})$. Using the Schwartz inequality, we have

$$\begin{aligned}(KN)^2 &= \mathrm{tr}^2(\mathbf{I}_{KN})\\ &= \mathrm{tr}^2\left\{\left((\mathbf{I}_N\otimes\mathbf{T}_{zp})^H\mathbf{H}^H\mathbf{R}_{zp}^{-1}\mathbf{H}(\mathbf{I}_N\otimes\mathbf{T}_{zp})\right)^{-\frac{1}{2}}\right.\\ &\qquad\left.\times\left((\mathbf{I}_N\otimes\mathbf{T}_{zp})^H\mathbf{H}^H\mathbf{R}_{zp}^{-1}\mathbf{H}(\mathbf{I}_N\otimes\mathbf{T}_{zp})\right)^{\frac{1}{2}}\right\}\\ &\le \mathrm{tr}\left\{\left((\mathbf{I}_N\otimes\mathbf{T}_{zp})^H\mathbf{H}^H\mathbf{R}_{zp}^{-1}\mathbf{H}(\mathbf{I}_N\otimes\mathbf{T}_{zp})\right)^{-1}\right\}\\ &\qquad\times\mathrm{tr}\left\{(\mathbf{I}_N\otimes\mathbf{T}_{zp})^H\mathbf{H}^H\mathbf{R}_{zp}^{-1}\mathbf{H}(\mathbf{I}_N\otimes\mathbf{T}_{zp})\right\}\end{aligned}\tag{12}$$

where the equality is asymptotically achieved, i.e., if and only if $(\mathbf{I}_N\otimes\mathbf{T}_{zp})^H\mathbf{H}^H\mathbf{R}_{zp}^{-1}\mathbf{H}(\mathbf{I}_N\otimes\mathbf{T}_{zp})$ is a (scaled) identity. We show in Section V that this condition is satisfied (for high SNR). It follows that maximizing $V(\mathbf{h})$ w.r.t. $\mathbf{h}$ is equivalent to minimizing the following asymptotic lower bound:

$$\begin{aligned}V_1(\mathbf{h}) &= \mathrm{tr}\left\{(\mathbf{I}_N\otimes\mathbf{T}_{zp})^H\mathbf{H}^H\mathbf{R}_{zp}^{-1}\mathbf{H}(\mathbf{I}_N\otimes\mathbf{T}_{zp})\right\}\\ &= \mathrm{tr}\left\{\mathbf{H}^H\mathbf{R}_{zp}^{-1}\mathbf{H}\left[\mathbf{I}_N\otimes(\mathbf{T}_{zp}\mathbf{T}_{zp}^H)\right]\right\}\\ &= \mathrm{vec}^T(\mathbf{H}^*)\left\{\left[\mathbf{I}_N\otimes(\mathbf{T}_{zp}\mathbf{T}_{zp}^H)\right]\otimes\mathbf{R}_{zp}^{-1}\right\}\mathrm{vec}(\mathbf{H})\end{aligned}\tag{13}$$

which becomes a quadratic minimization problem. In the second equality, we used the fact that $\mathrm{tr}(\mathbf{AB}) = \mathrm{tr}(\mathbf{BA})$ for any matrices $\mathbf{A}$ and $\mathbf{B}$ with compatible size. $\mathrm{vec}(\mathbf{H})$, in the third equality, stacks the columns of matrix $\mathbf{H}$ on top of one another. Next, we explicitly express $\mathrm{vec}(\mathbf{H})$ as a linear function in $\mathbf{h}$. In particular, we can write

$$\mathrm{vec}(\mathbf{H}) = \mathbf{Sh}\tag{14}$$

where $\mathbf{S}$ has JN blocks each of size $JN\times(L+1)$ formed by elements 0 and 1 only. Explicit expression of the lth block of $\mathbf{S}$ is given by

$$\mathbf{S}_l = \begin{cases}\sum_{j=1}^{L+1}\mathbf{E}_{l+j-1,j}, & l = 1,\ldots,JN-L\\ \sum_{j=1}^{JN-l+1}\mathbf{E}_{l+j-1,j}, & l = JN-L+1,\ldots,JN\end{cases}\tag{15}$$

where $\mathbf{E}_{ij}$ is a $JN\times(L+1)$ elementary matrix with unit element at the (i,j)th location and zeros elsewhere. Using $\mathrm{vec}(\mathbf{H}) = \mathbf{Sh}$ from (14) back in $V_1(\mathbf{h})$ [see (13)], we have

$$\begin{aligned}V_1(\mathbf{h}) = {}& \mathbf{h}^H\mathbf{S}^T\left\{\left[\mathbf{I}_N\otimes(\mathbf{T}_{zp}\mathbf{T}_{zp}^H)\right]\otimes\mathbf{R}_{zp}^{-1}\right\}\\ \mathbf{Sh} \triangleq {}& \mathbf{h}^H\mathbf{\Phi}_{zp}\mathbf{h}\end{aligned}\tag{16}$$

where Φ_{zp} is an $(L+1)\times(L+1)$ matrix defined as

$$\Phi_{zp} \triangleq \mathbf{S}^T\left\{\left[\mathbf{I}_N\otimes(\mathbf{T}_{zp}\mathbf{T}_{zp}^H)\right]\otimes\mathbf{R}_{zp}^{-1}\right\}\mathbf{S}.\tag{17}$$

The solution $\hat{\mathbf{h}}_{zp}$ that minimizes $V_1(\mathbf{h})$ is given by the eigenvector of Φ_{zp} associated with the smallest eigenvalue.

B. Channel Estimation for MC-ZP

Similar to the SC case, we design a filterbank for the model in (5) as follows:

$$\begin{aligned}\bar{\mathbf{G}}_{zp} &= \arg\min_{\bar{\mathbf{G}}_{zp}\in\mathbb{C}^{JN\times KN}}\mathrm{tr}\left\{\bar{\mathbf{G}}_{zp}^H\bar{\mathbf{R}}_{zp}\bar{\mathbf{G}}_{zp}\right\}\\ &\qquad\text{subject to } \bar{\mathbf{G}}_{zp}^H\mathbf{H}\left(\mathbf{I}_N\otimes(\mathbf{T}_{zp}\mathbf{F})\right) = \mathbf{I}_{KN}\end{aligned}\tag{18}$$

where $\bar{\mathbf{R}}_{zp} \triangleq E\{\bar{\mathbf{y}}_{zp}(n)\bar{\mathbf{y}}_{zp}^H(n)\}$. The solution to the mentioned constrained problem is given by

$$\begin{aligned}\bar{\mathbf{G}}_{zp} &= \bar{\mathbf{R}}_{zp}^{-1}\mathbf{H}\left(\mathbf{I}_N\otimes(\mathbf{T}_{zp}\mathbf{F})\right)\\ &\quad\times\left[\left(\mathbf{I}_N\otimes(\mathbf{T}_{zp}\mathbf{F})\right)^H\mathbf{H}^H\bar{\mathbf{R}}_{zp}^{-1}\mathbf{H}\left(\mathbf{I}_N\otimes(\mathbf{T}_{zp}\mathbf{F})\right)\right]^{-1}.\end{aligned}\tag{19}$$

Substituting (19) into (18), the minimized average power of the filterbank output is given by

$$\mathrm{tr}\left\{\left[\left(\mathbf{I}_N\otimes(\mathbf{T}_{zp}\mathbf{F})\right)^H\mathbf{H}^H\bar{\mathbf{R}}_{zp}^{-1}\mathbf{H}\left(\mathbf{I}_N\otimes(\mathbf{T}_{zp}\mathbf{F})\right)\right]^{-1}\right\}.\tag{20}$$

We notice that (20) is similar to (11). As was done in the SC-ZP case, we use the Schwartz inequality and the fact that $\mathbf{FF}^H = \mathbf{I}_K$, and following similar steps as in (13)–(16), the channel estimate $\hat{\bar{\mathbf{h}}}_{zp}$ can then be obtained as the eigenvector of $\bar{\Phi}_{zp}$ associated with the smallest eigenvalue, where

$$\bar{\Phi}_{zp} \triangleq \mathbf{S}^T\left\{\left[\mathbf{I}_N\otimes(\mathbf{T}_{zp}\mathbf{T}_{zp}^H)\right]\otimes\bar{\mathbf{R}}_{zp}^{-1}\right\}\mathbf{S}.\tag{21}$$

As shown in (17) and (21), both SC-ZP and MC-ZP estimators take the same form, and therefore, either one can be applied to the other with minor changes, i.e., mainly replacing $\bar{\mathbf{R}}_{zp}$ with $\mathbf{R}_{zp}$ and *vice versa*.

C. Channel Estimation for SC-CP

We design an equalizing filterbank according to the following minimum variance criterion:

$$\begin{aligned}\mathbf{G}_{cp} &= \arg\min_{\mathbf{G}_{cp}\in\mathbb{C}^{(JN-L)\times KN}}\mathrm{tr}\left\{\mathbf{G}_{\mathrm{cp}}^{\mathrm{H}}\mathbf{R}_{\mathrm{cp}}\mathbf{G}_{\mathrm{cp}}\right\}\\ &\qquad\text{subject to } \mathbf{G}_{cp}^H\bar{\mathbf{H}}(\mathbf{I}_N\otimes\mathbf{T}_{cp}) = \mathbf{I}_{KN}\end{aligned}\tag{22}$$

where $\mathbf{R}_{cp} \triangleq E\{\mathbf{y}_{cp}(n)\mathbf{y}_{cp}^H(n)\}$ denotes the covariance matrix, and $\bar{\mathbf{H}} = \Gamma\mathbf{H}$. The solution to the constrained quadratic minimization problem is given by

$$\mathbf{G}_{cp} = \mathbf{R}_{cp}^{-1}\bar{\mathbf{H}}(\mathbf{I}_N \otimes \mathbf{T}_{cp}) \times \left[(\mathbf{I}_N \otimes \mathbf{T}_{cp})^H \bar{\mathbf{H}}^H \mathbf{R}_{cp}^{-1}\bar{\mathbf{H}}(\mathbf{I}_N \otimes \mathbf{T}_{cp})\right]^{-1}. \quad (23)$$

Substituting (23) into (22), and following the same ideas as in Section III-A, we minimize the following asymptotic lower bound:

$$\begin{aligned} V_2(\mathbf{h}) &= \mathrm{tr}\Big\{\left[(\mathbf{I}_N \otimes \mathbf{T}_{cp})^H \bar{\mathbf{H}}^H \mathbf{R}_{cp}^{-1}\bar{\mathbf{H}}(\mathbf{I}_N \otimes \mathbf{T}_{cp})\right]\Big\} \\ &= \mathrm{tr}\Big\{\bar{\mathbf{H}}^H \mathbf{R}_{cp}^{-1}\bar{\mathbf{H}}\left[\mathbf{I}_N \otimes (\mathbf{T}_{cp}\mathbf{T}_{cp}^H)\right]\Big\} \\ &= \mathrm{vec}^T(\bar{\mathbf{H}}^*)\Big\{\left[\mathbf{I}_N \otimes (\mathbf{T}_{cp}\mathbf{T}_{cp}^H)\right] \otimes \mathbf{R}_{cp}^{-1}\Big\}\mathrm{vec}(\bar{\mathbf{H}}) \\ &= \mathbf{h}^H \bar{\mathbf{S}}^T \Big\{\left[\mathbf{I}_N \otimes (\mathbf{T}_{cp}\mathbf{T}_{cp}^H)\right] \otimes \mathbf{R}_{cp}^{-1}\Big\}\bar{\mathbf{S}}\mathbf{h} \end{aligned} \quad (24)$$

where $\bar{\mathbf{S}}$ has JN blocks each of size $(JN-L)\times(L+1)$ formed by elements 0 and 1 only. Explicit expression of the lth block of $\bar{\mathbf{S}}$ is given by

$$\bar{\mathbf{S}}_l = \begin{cases} \sum_{j=1}^{l} \bar{\mathbf{E}}_{l-j+1,L+2-j}, & l = 1,\ldots,L \\ \sum_{j=1}^{L+1} \bar{\mathbf{E}}_{l-j+1,L+2-j}, & l = L+1,\ldots,JN-L \\ \sum_{j=1}^{JN-l+1} \bar{\mathbf{E}}_{j+l-L-1,j}, & l = JN-L+1,\ldots,JN \end{cases} \quad (25)$$

where the elementary matrix $\bar{\mathbf{E}}_{ij}$ is a $(JN-L)\times(L+1)$ matrix with unit element at the (i,j)th location and zeros elsewhere. The solution $\hat{\mathbf{h}}_{cp}$ that minimizes $V_2(\mathbf{h})$ is given by the eigenvector of Φ_{cp} associated with the smallest eigenvalue, where

$$\Phi_{cp} \triangleq \bar{\mathbf{S}}^T \Big\{\left[\mathbf{I}_N \otimes (\mathbf{T}_{cp}\mathbf{T}_{cp}^H)\right] \otimes \mathbf{R}_{cp}^{-1}\Big\}\bar{\mathbf{S}}. \quad (26)$$

D. *Channel Estimation for MC-CP*

Following the same minimization criterion as in the SC-CP case, we have

$$\bar{\mathbf{G}}_{cp} = \arg \min_{\bar{\mathbf{G}}_{cp}\in\mathbb{C}^{(JN-L)\times KN}} \mathrm{tr}\{\bar{\mathbf{G}}_{cp}^H \bar{\mathbf{R}}_{cp}\bar{\mathbf{G}}_{cp}\} \quad \text{subject to } \bar{\mathbf{G}}_{cp}^H \bar{\mathbf{H}}(\mathbf{I}_N \otimes \mathbf{T}_{cp}) = \mathbf{I}_{KN} \quad (27)$$

where $\bar{\mathbf{R}}_{cp} \triangleq E\{\bar{\mathbf{y}}_{cp}(n)\bar{\mathbf{y}}_{cp}^H(n)\}$. As was done in (23) and (24), the channel estimate $\hat{\bar{\mathbf{h}}}_{cp}$ is obtained by the eigenvector of $\bar{\Phi}_{cp}$ associated with the smallest eigenvalue, where

$$\bar{\Phi}_{cp} \triangleq \bar{\mathbf{S}}^T \Big\{\left[\mathbf{I}_N \otimes (\mathbf{T}_{cp}\mathbf{T}_{cp}^H)\right] \otimes \bar{\mathbf{R}}_{cp}^{-1}\Big\}\bar{\mathbf{S}}. \quad (28)$$

Again, we notice here that both estimators SC-CP and MC-CP are the same, except for different covariance matrices.

Remark: Note that for the implementation of all schemes, the covariance matrix $\mathbf{R}_{zp}$, $\bar{\mathbf{R}}_{zp}$, $\mathbf{R}_{cp}$, or $\bar{\mathbf{R}}_{cp}$ of the received signal $\mathbf{y}_{zp}(n)$, $\bar{\mathbf{y}}_{zp}(n)$, $\mathbf{y}_{cp}(n)$, or $\bar{\mathbf{y}}_{zp}(n)$, respectively, has to be replaced by some covariance matrix estimate, e.g., the sample covariance matrix $\hat{\mathbf{R}} = P^{-1}\sum_{n=0}^{P-1}\mathbf{y}(n)\mathbf{y}^H(n)$, or some adaptive estimate of the true covariance matrix, where $\mathbf{y}(n)$ denotes the received signal corresponding to the four different transmissions. It can be shown (e.g., using similar techniques as in [10]) that the channel estimate $\hat{\mathbf{h}}$ (which denotes any of the four channel estimates) converges to the true channel $\mathbf{h}$ (up to a scalar factor) as the interference and noise vanish. For finite SNR and in the presence of interference, we evaluate the accuracy of $\hat{\mathbf{h}}$ via simulations in Section VII. Finally, like all other blind schemes, the channel estimate $\hat{\mathbf{h}}$ has a scalar ambiguity, which can be resolved either by differential coding or by transmitting a *few* pilot symbols.

IV. Implementation

Using either the ZP or CP technique, we have seen that the SC and MC estimators are the same, except that they use different covariance matrices. Therefore, we present here efficient implementation for SC-CP and SC-ZP, where the other two MC schemes can be identically implemented as their counterparts with CP and ZP. Before proceeding, we remark that efficient implementation is needed for the calculation of Φ_{cp} and Φ_{zp} used in the two estimators, which involves multiplications of large-size matrices. In particular, $\mathbf{T}_{cp}\mathbf{T}_{cp}^H$ is $J\times J$, $\mathbf{I}_N \otimes (\mathbf{T}_{cp}\mathbf{T}_{cp}^H)$ is $JN\times JN$, $\mathbf{S}$ is $(JN-L)JN\times(L+1)$, and, as such, $[\mathbf{I}_N \otimes (\mathbf{T}_{cp}\mathbf{T}_{cp}^H)]\otimes\mathbf{R}_{cp}^{-1}$ is a $(JN-L)JN \times (JN-L)JN$ matrix (e.g., 14336 × 14336 for modest values of $N=2$, $J=64$, and $\mu=16$). Similarly, $[\mathbf{I}_N \otimes (\mathbf{T}_{zp}\mathbf{T}_{zp}^H)]\otimes\mathbf{R}_{zp}^{-1}$ is a $(JN)^2\times(JN)^2$ matrix (e.g., 16384 × 16384 for $N=2$, $J=64$, and $\mu=16$). Hence, brute-force computation is impractical/inefficient, except for small values of J and μ. In what follows, we exploit the sparse/special structure of the matrices involved for efficient implementation.

A. *SC-CP*

Note that

$$\mathbf{T}_{cp}\mathbf{T}_{cp}^H = \begin{bmatrix} \mathbf{I}_\mu & & \mathbf{I}_\mu \\ & \mathbf{I}_{K-\mu} & \\ \mathbf{I}_\mu & & \mathbf{I}_\mu \end{bmatrix}. \quad (29)$$

Then

$$(\mathbf{T}_{cp}\mathbf{T}_{cp}^H)\otimes\mathbf{R}_{cp}^{-1} = \left[\begin{array}{c|c|c} \mathbf{I}_\mu\otimes\mathbf{R}_{cp}^{-1} & & \mathbf{I}_\mu\otimes\mathbf{R}_{cp}^{-1} \\ \hline & \mathbf{I}_{K-\mu}\otimes\mathbf{R}_{cp}^{-1} & \\ \hline \mathbf{I}_\mu\otimes\mathbf{R}_{cp}^{-1} & & \mathbf{I}_\mu\otimes\mathbf{R}_{cp}^{-1} \end{array}\right]. \quad (30)$$

Let

$$\bar{\mathbf{S}} = \begin{bmatrix} \bar{\mathbf{S}}_1 \\ \vdots \\ \bar{\mathbf{S}}_N \end{bmatrix}_{(JN-L)JN\times(L+1)}$$

$$\bar{\mathbf{S}}_N = \begin{bmatrix} \mathbf{S}_{n,1} \\ \vdots \\ \mathbf{S}_{n,J} \end{bmatrix}_{(JN-L)J\times(L+1)} \quad (31)$$

where $\mathbf{S}_{n,j}$ is $(JN-L)\times(L+1)$, and $n=1,\ldots,N$. It follows that Φ_{cp} can be expressed as [cf. (14)]

$$\begin{aligned}\Phi_{cp} &= \sum_{n=1}^{N} \bar{\mathbf{S}}_N^T \left[\left(\mathbf{T}_{cp}\mathbf{T}_{cp}^H\right)\otimes \mathbf{R}_{cp}^{-1}\right]\bar{\mathbf{S}}_n \\ &= \sum_{n=1}^{N}\left[\sum_{j=1}^{J}\mathbf{S}_{n,j}^T\mathbf{R}_{cp}^{-1}\mathbf{S}_{n,j}\right. \\ &\qquad \left. + \sum_{j=1}^{\mu}\left(\mathbf{S}_{n,j}^T\mathbf{R}_{cp}^{-1}\mathbf{S}_{n,K+j}+\mathbf{S}_{n,K+j}^T\mathbf{R}_{cp}^{-1}\mathbf{S}_{n,j}\right)\right]. \quad (32)\end{aligned}$$

To compute $\mathbf{S}_{n,j}$ for all n and j, we can reformulate $\mathbf{S}_{n,j}$ as

$$\begin{aligned}\mathbf{S}_{n,j} &= \left[\mathbf{0}_{(JN-L)\times L}\quad \mathbf{I}_{JN-L}\quad \mathbf{0}_{(JN-L)\times L}\right] \\ &\quad\times\begin{bmatrix}\mathbf{0}_{[(n-1)J+j-1]\times(L+1)}\\ \mathbf{I}_{L+1}\\ \mathbf{0}_{[JN-(n-1)J-j]\times(L+1)}\end{bmatrix} \\ &\triangleq \left[\mathbf{0}_{(JN-L)\times L}\quad \mathbf{I}_{JN-L}\quad \mathbf{0}_{(JN-L)\times L}\right]\tilde{\mathbf{S}}_{n,j}. \quad (33)\end{aligned}$$

Following this reformulation, one can see that the first item in the second equality of (32), i.e., $\mathbf{S}_{n,j}^T\mathbf{R}_{cp}^{-1}\mathbf{S}_{n,j}$, can be simplified as

$$\begin{aligned}&\mathbf{S}_{n,j}^T\mathbf{R}_{cp}^{-1}\mathbf{S}_{n,j} \\ &= \tilde{\mathbf{S}}_{n,j}^T\begin{bmatrix}\mathbf{0}_{L\times(JN-L)}\\ \mathbf{I}_{JN-L}\\ \mathbf{0}_{L\times(JN-L)}\end{bmatrix}\mathbf{R}_{cp}^{-1} \\ &\quad\times\left[\mathbf{0}_{(JN-L)\times L}\quad \mathbf{I}_{JN-L}\quad \mathbf{0}_{(JN-L)\times L}\right]\tilde{\mathbf{S}}_{n,j} \\ &= \tilde{\mathbf{S}}_{n,j}^T\begin{bmatrix}\mathbf{0}_{L\times L} & \mathbf{0}_{L\times(JN-L)} & \mathbf{0}_{L\times L}\\ \mathbf{0}_{(JN-L)\times L} & \mathbf{R}_{cp}^{-1} & \mathbf{0}_{(JN-L)\times L}\\ \mathbf{0}_{L\times L} & \mathbf{0}_{L\times(JN-L)} & \mathbf{0}_{L\times L}\end{bmatrix}\tilde{\mathbf{S}}_{n,j} \\ &\triangleq \tilde{\mathbf{S}}_{n,j}^T\boldsymbol{\mathcal{R}}\tilde{\mathbf{S}}_{n,j}. \quad (34)\end{aligned}$$

Note that $\boldsymbol{\mathcal{R}}$ is independent of n and j. Additionally, $\tilde{\mathbf{S}}_{n,j}$ is essentially performing a matrix truncation operation. As such, we have

$$\begin{aligned}\mathbf{S}_{n,j}^T\mathbf{R}_{cp}^{-1}\mathbf{S}_{n,j} = \tilde{\mathbf{S}}_{n,j}^T\boldsymbol{\mathcal{R}}\tilde{\mathbf{S}}_{n,j} &= \boldsymbol{\mathcal{R}}(i_1:i_1+L, i_1:i_1+L) \\ &\text{for } n=1,\ldots,N;\quad j=1,\ldots,J \quad (35)\end{aligned}$$

where $i_1=(n-1)J+j$. Similarly, for the second term in (32), we have

$$\begin{aligned}\mathbf{S}_{n,j}^T\mathbf{R}_{cp}^{-1}\mathbf{S}_{n,K+j} &= \tilde{\mathbf{S}}_{n,j}^T\boldsymbol{\mathcal{R}}\tilde{\mathbf{S}}_{n,K+j} \\ &= \boldsymbol{\mathcal{R}}(i_1:i_1+L, i_1+K:i_1+K+L) \\ \mathbf{S}_{n,K+j}^T\mathbf{R}_{cp}^{-1}\mathbf{S}_{n,j} &= (\mathbf{S}_{n,j}^T\mathbf{R}_{cp}^{-1}\mathbf{S}_{n,K+j})^T \\ &\text{for } n=1,\ldots,N;\quad j=1,\ldots,\mu. \quad (36)\end{aligned}$$

We summarize the implementation steps of Φ_{cp} along with the computational complexity in terms of the number of flops needed in each step as follows.

Step 1) Set $\Phi_{cp}=\mathbf{0}_{(L+1)\times(L+1)}$. $\Rightarrow$ Total $O(0)$ flops.

Step 2) Form $\boldsymbol{\mathcal{R}}$ using (34). $\Rightarrow$ Total $O(0)$ flops.

Step 3) Partition $\boldsymbol{\mathcal{R}}$ and sum them together using (32), (35), and (36). Note that in the following loops, $i_1=(n-1)J+j$:

```
for n = 1 : N
  for j = 1 : J
    Φcp = Φcp + R(i1 : i1 + L, i1 : i1 + L);
  end   ⇒ Total O(JN(L + 1)^2) flops
  forj = 1 : μ
    Φcp = Φcp + R(i1 : i1 + L, i1 + K : i1 + K + L)
      +R^T(i1 : i1 + L, i1 + K : i1 + K + L);
  end   ⇒ Total O(μJ(L + 1)^2) flops
end
```

Therefore, the total number of flops involved in the aforementioned implementation is $O(J(N+\mu)(L+1)^2)$. In contrast, with direct implementation, the number of flops needed to calculate Φ_{cp} is $O((L+1)((JN-L)JN)^2)$, that is, approximately $O(J^3N^4/(N+\mu)(L+1))$-fold saving in complexity by the efficient approach when $JN\gg L$ (i.e., $JN-L\approx JN$).

We note here that for the proposed schemes, the tricky part is the calculation of the Φ matrix. The remaining steps (e.g., computing the covariance matrix and finding the eigenvector of Φ, etc.) are rather standard and, therefore, are not discussed in detail.

B. SC-ZP

Due to the presence of trailing zeros introduced by $\mathbf{T}_{zp}$, the implementation of the SC-ZP estimator takes a simpler form than the one for the CP case. Observe that

$$\mathbf{T}_{zp}\mathbf{T}_{zp}^H = \left[\begin{array}{c|c}\mathbf{I}_K & \mathbf{0}_{K\times\mu}\\ \hline \mathbf{0}_{\mu\times K} & \mathbf{0}_{\mu\times\mu}\end{array}\right] \qquad \mathbf{S} = \begin{bmatrix}\mathbf{S}_1\\ \vdots\\ \mathbf{S}_{JN}\end{bmatrix} \quad (37)$$

where $\mathbf{S}_i$ is a $(JN)\times(L+1)$ block matrix. Then

$$(\mathbf{T}_{zp}\mathbf{T}_{zp}^H)\otimes\mathbf{R}_{zp}^{-1} = \left[\begin{array}{c|c}\mathbf{I}_K\otimes\mathbf{R}_{zp}^{-1} & \mathbf{0}_{JNK\times JN\mu}\\ \hline \mathbf{0}_{JN\mu\times JNK} & \mathbf{0}_{JN\mu\times JN\mu}\end{array}\right]. \quad (38)$$

It follows that

$$\begin{aligned}\Phi_{zp} &= \sum_{n=1}^{N}\left[\mathbf{S}_{(n-1)(K+\mu)+1}^H\quad\cdots\quad\mathbf{S}_{nK+(n-1)\mu}^H\right] \\ &\quad\times\left[\mathbf{I}_K\otimes\mathbf{R}_{zp}^{-1}\right]\begin{bmatrix}\mathbf{S}_{(n-1)(K+\mu)+1}\\ \vdots\\ \mathbf{S}_{nK+(n-1)\mu}\end{bmatrix} \\ &= \sum_{n=1}^{N}\sum_{k=1}^{K}\mathbf{S}_j^H\mathbf{R}_{zp}^{-1}\mathbf{S}_j \quad (39)\end{aligned}$$

where $j = J(n-1)+k$. The total number of flops involved in the ZP scheme implementation is $O(NK(L+1)^2)$. Similar to the CP case, there is approximately a $O(J^2N^3K/L+1)$-fold saving in complexity compared to direct implementation.

V. Asymptotic Analysis

We discuss here the asymptotic structure for the SC-ZP and SC-CP cases, mainly, the structure of $(\mathbf{I}_N \otimes \mathbf{T}_{zp})^H \mathbf{H}^H \mathbf{R}_{zp}^{-1} \mathbf{H}(\mathbf{I}_N \otimes \mathbf{T}_{zp})$ and $(\mathbf{I}_N \otimes \mathbf{T}_{cp})^H \bar{\mathbf{H}}^H \mathbf{R}_{cp}^{-1} \bar{\mathbf{H}}(\mathbf{I}_N \otimes \mathbf{T}_{cp})$ from (11) and (24) simultaneously. The MC cases are identical to the SC ones as we explain at the end of this section.

Proposition 1: Let

$$\boldsymbol{\mathcal{H}}_{zp} = \mathbf{H}(\mathbf{I}_N \otimes \mathbf{T}_{zp}) \quad \mathcal{H}_{cp} = \bar{\mathbf{H}}(\mathbf{I}_N \otimes \mathbf{T}_{cp}) \tag{40}$$

and suppose the following.

1) The channel noise is white with

$$\begin{aligned} E\{\mathbf{e}_{zp}(n)\mathbf{e}_{zp}^H(n)\} &= \sigma^2 \mathbf{I}_{JN} \\ E\{\mathbf{e}_{cp}(n)\mathbf{e}_{cp}^H(n)\} &= \sigma^2 \mathbf{I}_{JN-L} \end{aligned} \tag{41}$$

where σ^2 denotes the noise variance.

2) The interference covariance matrix defined as

$$\begin{aligned} \mathbf{W}_{zp} &\stackrel{\triangle}{=} E\left\{\mathbf{w}_{zp}(n)\mathbf{w}_{zp}^H(n)\right\} \\ \mathbf{W}_{zp} &\stackrel{\triangle}{=} E\left\{\mathbf{w}_{cp}(n)\mathbf{w}_{cp}^H(n)\right\} \end{aligned} \tag{42}$$

has a low-rank decomposition

$$\mathbf{W}_{zp} = \mathbf{\Pi}_{zp}\mathbf{\Pi}_{zp}^H, \quad \mathbf{W}_{cp} = \mathbf{\Pi}_{cp}\mathbf{\Pi}_{cp}^H \tag{43}$$

where $\mathbf{\Pi}_{zp}$ is a $JN \times M_{zp}$ matrix with $M_{zp} < JN - KN = \mu N$, and $\mathbf{\Pi}_{cp}$ is a $(JN-L) \times M_{cp}$ matrix with $M_{cp} < JN - L - KN = \mu N - L$.

3) $\tilde{\boldsymbol{\mathcal{H}}}_{zp} \stackrel{\triangle}{=} [\boldsymbol{\mathcal{H}}_{zp}, \mathbf{\Pi}_{zp}]$ and $\tilde{\boldsymbol{\mathcal{H}}}_{cp} \stackrel{\triangle}{=} [\boldsymbol{\mathcal{H}}_{cp}, \mathbf{\Pi}_{cp}]$ has full column rank.

Then

$$\lim_{\sigma^2 \to 0} \boldsymbol{\mathcal{H}}_{zp}^H \mathbf{R}_{zp}^{-1}(\sigma^2)\boldsymbol{\mathcal{H}}_{zp} = \mathbf{I}_{KN} \tag{44}$$

$$\lim_{\sigma^2 \to 0} \boldsymbol{\mathcal{H}}_{cp}^H \mathbf{R}_{cp}^{-1}(\sigma^2)\boldsymbol{\mathcal{H}}_{cp} = \mathbf{I}_{KN}. \tag{45}$$

In the following, we present the proof for the ZP system. The CP case exactly follows the same steps.

Proof: Assume that the information symbols $\mathbf{s}_N(n)$ are independent with unit average energy. Then, the covariance matrix of $\mathbf{y}_{zp}(n)$ [see (2)] can be written as

$$\begin{aligned} \mathbf{R}_{zp}(\sigma^2) &= \boldsymbol{\mathcal{H}}_{zp}\boldsymbol{\mathcal{H}}_{zp}^H + \mathbf{W}_{zp} + \sigma^2 \mathbf{I}_{JN} \\ &= \boldsymbol{\mathcal{H}}_{zp}\boldsymbol{\mathcal{H}}_{zp}^H + \mathbf{\Pi}_{zp}\mathbf{\Pi}_{zp}^H + \sigma^2 \mathbf{I}_{JN} \\ &= \tilde{\boldsymbol{\mathcal{H}}}_{zp}\tilde{\boldsymbol{\mathcal{H}}}_{zp}^H + \sigma^2 \mathbf{I}_{JN}. \end{aligned} \tag{46}$$

As it is clear from (3), the matrix $\boldsymbol{\mathcal{H}}_{zp}$ has full column rank unless all channel coefficients are zeros, which is not the case. Therefore, the eigenvalue decomposition (EVD) of $\mathbf{R}_{zp}$ can be expressed as

$$\mathbf{R}_{zp}(\sigma^2) = [\mathbf{u}_s,\ \mathbf{u}_N]\begin{bmatrix} \mathbf{\Lambda}_s + \sigma^2 \mathbf{I}_{KN+M_{zp}} & \\ & \sigma^2 \mathbf{I}_{JN-KN-M_{zp}} \end{bmatrix}\begin{bmatrix} \mathbf{U}_s^H \\ \mathbf{U}_N^H \end{bmatrix}. \tag{47}$$

Note that the EVD of $\tilde{\boldsymbol{\mathcal{H}}}_{zp}\tilde{\boldsymbol{\mathcal{H}}}_{zp}^H$ can be written as

$$\tilde{\boldsymbol{\mathcal{H}}}_{zp}\tilde{\boldsymbol{\mathcal{H}}}_{zp}^H = \mathbf{U}_s\mathbf{\Lambda}_s\mathbf{U}_s^H. \tag{48}$$

Left multiplying both sides of the aforementioned equation by $\mathbf{U}_s^H$ and right multiplying by $\mathbf{U}_s$ leads to

$$\mathbf{U}_s^H\tilde{\boldsymbol{\mathcal{H}}}_{zp}\tilde{\boldsymbol{\mathcal{H}}}_{zp}^H\mathbf{U}_s = \mathbf{U}_s^H\mathbf{U}_s\Lambda_s\mathbf{U}_s^H\mathbf{U}_s = \mathbf{\Lambda}_s. \tag{49}$$

It is easy to show that $\mathbf{U}_s^H\tilde{\boldsymbol{\mathcal{H}}}_{zp}$ is a square and full-rank matrix. Taking the inverse of both sides of the aforementioned equation and noticing that $\mathbf{U}_s^H\tilde{\boldsymbol{\mathcal{H}}}_{zp}$ is a square and full-rank matrix, we have

$$\tilde{\boldsymbol{\mathcal{H}}}_{zp}^H\mathbf{U}_s\Lambda_s^{-1}\mathbf{u}_s^H\tilde{\boldsymbol{\mathcal{H}}}_{zp} = \mathbf{I}_{KN+M_{zp}}. \tag{50}$$

A Taylor expansion of $\mathbf{R}_{zp}^{-1}(\sigma^2)$ at high SNR is [10]

$$\mathbf{R}_{zp}^{-1}(\sigma^2) = \sigma^{-2}\mathbf{U}_N\mathbf{U}_N^H + \mathbf{U}_s\Lambda_s^{-1}\mathbf{U}_s^H + O(\sigma^2). \tag{51}$$

Using the above Taylor expansion, along with (50) and the observation $\tilde{\boldsymbol{\mathcal{H}}}_{zp}^H\mathbf{U}_n = \mathbf{0}$, we have

$$\tilde{\boldsymbol{\mathcal{H}}}_{zp}^H\mathbf{R}_{zp}^{-1}\tilde{\boldsymbol{\mathcal{H}}}_{zp} = \mathbf{I}_{KN+M_{zp}} + O(\sigma^2) \stackrel{\sigma^2 \to 0}{\longrightarrow} \mathbf{I}_{KN+M_{zp}}. \tag{52}$$

Equation (44) directly follows from (52). ■

We note here that the asymptotic structure for the MC-ZP and MC-CP systems is identical to the SC-ZP and SC-CP cases, respectively, since

$$\begin{aligned} &\mathbf{H}\left(\mathbf{I}_N \otimes (\mathbf{T}_{zp}\mathbf{F})\right)\left(\mathbf{I}_N \otimes (\mathbf{T}_{zp}\mathbf{F})\right)^H\mathbf{H}^H \\ &\quad = \mathbf{H}\left(\mathbf{I}_N \otimes \left(\mathbf{T}_{zp}\mathbf{F}\mathbf{F}^H\mathbf{T}_{zp}^H\right)\right)\mathbf{H}^H = \boldsymbol{\mathcal{H}}_{zp}\boldsymbol{\mathcal{H}}_{zp}^H \\ &\bar{\mathbf{H}}\left(\mathbf{I}_N \otimes (\mathbf{T}_{cp}\mathbf{F})\right)\left(\mathbf{I}_N \otimes (\mathbf{T}_{cp}\mathbf{F})\right)^H\bar{\mathbf{H}}^H \\ &\quad = \bar{\mathbf{H}}\left(\mathbf{I}_N \otimes \left(\mathbf{T}_{cp}\mathbf{F}\mathbf{F}^H\mathbf{T}_{cp}^H\right)\right)\bar{\mathbf{H}}^H = \boldsymbol{\mathcal{H}}_{cp}\boldsymbol{\mathcal{H}}_{cp}^H \end{aligned} \tag{53}$$

where we used the fact that $\mathbf{F}\mathbf{F}^H = \mathbf{I}_K$.

Remark: The second assumption made in Proposition 1 implies that the interference occupies only a portion of the overall bandwidth, i.e., $M_{zp} < \mu N$ and $M_{cp} < \mu N - L$ out of the KN subcarriers for MC-ZP and MC-CP,[1] respectively. Recall that μ is the length of the introduced redundancy. So, the redundancy μ that was initially introduced for IBI removal is now helping with narrowband interference cancellation. For the proposed schemes to handle more interfering tones, we can either increase the length of μ or increase the number of symbols N that are simultaneously processed. If the bounds on

[1]For SC systems, the condition translates to a constraint on the number of degrees of freedom that can be exploited for interference suppression.

the number of interfering tones shown above are not satisfied, the performance will significantly degrade, as we show in our simulation in Section VII. In addition, the channel matrices $\boldsymbol{\mathcal{H}}_{zp}$ and $\boldsymbol{\mathcal{H}}_{cp}$ need to be tall. As a result, we must have $N > 1$ for CP (see Section II-C), but this restriction is not needed for ZP.

We summarize the interference suppression ability of the proposed schemes in the following corollary.

Corollary 1: For a block of N symbols simultaneously processed in a system with a channel impulse response of order L and redundancy of length μ, the maximum number of interfering tones that can effectively be suppressed using ZP and CP techniques is equal to $N\mu - 1$ and $N\mu - L - 1$, respectively.

We briefly comment on the channel identifiability issue as follows. As in [11], at high SNR and in the absence of interference, channel identifiability is guaranteed if $\boldsymbol{\mathcal{H}}_{zp}$ or $\boldsymbol{\mathcal{H}}_{cp}$ has full rank. To see this, let us consider the ZP case. The cost function (13) used for channel estimation is asymptotically (high SNR) equivalent to [see (51)]

$$\mathrm{tr}\left\{\boldsymbol{\mathcal{H}}_{zp}^H \mathbf{U}_N \mathbf{U}_N^H \boldsymbol{\mathcal{H}}_{zp}\right\}. \tag{54}$$

Since $\mathrm{span}(\boldsymbol{\mathcal{H}}_{zp}) = \mathrm{span}(\mathbf{U}_s)$, we have $\mathbf{U}_n^H \boldsymbol{\mathcal{H}}_{zp} = \mathbf{0}$, which indicates that the *true* channel matrix $\boldsymbol{\mathcal{H}}_{zp}$ is a minimizer of the aforementioned asymptotic cost function. Suppose that there is another channel matrix $\boldsymbol{\mathcal{H}}'_{zp}$ that is also a minimizer, i.e., $\mathbf{U}_n^H \boldsymbol{\mathcal{H}}'_{zp} = \mathbf{0}$. Since $\mathbf{U}_n$ spans the orthogonal complement space to the range of $\boldsymbol{\mathcal{H}}_{zp}$, the range of $\boldsymbol{\mathcal{H}}'_{zp}$ is contained in the range of $\boldsymbol{\mathcal{H}}_{zp}$. Since both matrices have full column rank and have the same linear convolutional matrix structure as shown in (3) and (4), we must have $\mathbf{h} = c\mathbf{h}'$ for some scalar c, which implies identifiability of the channel to within a scalar ambiguity (see [12, p. 68]). While a similar argument carries over to the CP case, we note the following difference: $\boldsymbol{\mathcal{H}}_{zp}$ always has full column rank [see (3) and (4)] for any nontrivial $\mathbf{h}$, but this does not hold for $\boldsymbol{\mathcal{H}}_{cp}$. In other words, channel identifiability is always guaranteed for ZP but not for CP; for the latter, we need a full-rank $\boldsymbol{\mathcal{H}}_{cp}$ to ensure identifiability. This is a well-known result [2].

VI. CRB

In this section, we derive the *unconditional* CRB that is averaged over the unknown information symbols. The CRB provides a suitable lower bound for all unbiased blind estimators, which do not assume knowledge of the information symbols and channel coefficients. Conditional CRBs (i.e., CRBs that are conditioned on the information symbols) for various blind-channel identification problems have been investigated in the literature; see, e.g., [13]–[15] and references therein.

We will next derive CRBs for SC-ZP and SC-CP models in (2) and (6), respectively. The CRBs for the MC case are identical to those for the SC case [see (53)]. We assume here that the information symbols are independent identically distributed and drawn from some unit-energy constellation, i.e., $E\{\mathbf{s}_N(n)\mathbf{s}_N^H(n)\} = \mathbf{I}_{KN}$. Additionally, we assume that the noise is white and normally distributed random vector with zero mean and covariance matrix $\sigma^2\mathbf{I}$, where the identity matrix $\mathbf{I}$ is $JN \times JN$ for the ZP case and $(JN-L)\times(JN-L)$ for the CP case. The unmodeled interference is also assumed to have a Gaussian distribution. The Gaussian assumption is known to yield an upper bound for all CRB based on all other distributions [9].

A. CRB for SC-ZP

Let the observation time consist of a total of P blocks, where each block consists of N symbols, each of size J. We collect all samples within this observation time into an $PJN \times 1$ vector $\mathbf{y}$ defined as

$$\mathbf{y} \triangleq [\mathbf{y}_{zp}^T(P), \mathbf{y}_{zp}^T(P-1), \ldots, \mathbf{y}_{zp}^T(1)]^T. \tag{55}$$

With the aforementioned assumptions, it is clear that $\mathbf{y}$ follows a Gaussian distribution with zero-mean and covariance matrix

$$\begin{aligned}\mathbf{R}_P &= \mathbf{I}_P \otimes \left[\mathbf{H}\left(\mathbf{I}_N \otimes \left(\mathbf{T}_{zp}\mathbf{T}_{zp}^H\right)\right)\mathbf{H}^H + \mathbf{R}_w + \sigma^2\mathbf{I}_{JN}\right] \\ &\triangleq \mathbf{I}_P \otimes \mathbf{R}_{zp}\end{aligned} \tag{56}$$

where

$$\begin{aligned}\mathbf{R}_w &= E\left\{\mathbf{w}_{zp}(n)\mathbf{w}_{zp}^H(n)\right\} \\ &= \begin{bmatrix} r(0) & r^*(1) & \cdots & r^*(JN-1) \\ r(1) & r(0) & \cdots & r^*(JN-2) \\ \vdots & \vdots & \ddots & \vdots \\ r(JN-1) & r(JN-2) & \cdots & r(0) \end{bmatrix}\end{aligned} \tag{57}$$

and $r(k)$ denotes the autocorrelation of the interference samples that are assumed stationary. Let $m = 2JN + 2L + 1$ be the total number of unknown parameters, and define the $m \times 1$ vector of unknown parameters as

$$\theta = [\,\mathbf{h}_r^T \quad \mathbf{h}_i^T \quad \mathbf{r}_r^T \quad \mathbf{r}_i^T\,]^T \tag{58}$$

where $\mathbf{h}_r = [h_r(0), \ldots, h_r(L)]$, $\mathbf{h}_i = [h_i(0), \ldots, h_i(L)]$, $\mathbf{r}_r = [r(0), r_r(1), \ldots, r_r(JN-1)]$, $\mathbf{r}_i = [r_i(1), \ldots, r_i(JN-1)]$, and the subscripts r and i denote the real and imaginary parts, respectively. By the Slepian–Bangs formula, the $m \times m$ Fisher information matrix (FIM) is given, element wise, by (see, e.g., [9])

$$\begin{aligned}[\mathbf{J}(\theta)]_{s,t} &= \mathrm{tr}\left[\mathbf{R}_P^{-1}\frac{\partial \mathbf{R}_P}{\partial[\theta]_s}\mathbf{R}_P^{-1}\frac{\partial \mathbf{R}_P}{\partial[\theta]_t}\right] \\ &= P\,\mathrm{tr}\left[\mathbf{R}_{zp}^{-1}\frac{\partial \mathbf{R}_{zp}}{\partial[\theta]_s}\mathbf{R}_{zp}^{-1}\frac{\partial \mathbf{R}_{zp}}{\partial[\theta]_t}\right]\end{aligned} \tag{59}$$

where $[\mathbf{J}(\theta)]_{s,t}$ denotes the (s,t)th element of the FIM matrix, $[\theta]_s$ is the sth element of θ, and the second equality is due to the block diagonal structure of the covariance matrix (56). We next calculate the partial differentiation w.r.t. each unknown parameter. First, consider the partial differentiation w.r.t. the channel real and imaginary part parameters contained in $\mathbf{h}_r$ and $\mathbf{h}_i$, respectively, as

$$\frac{\partial \mathbf{R}_{zp}}{\partial h_r(l)} = \mathbf{X}_l \left(\mathbf{I}_N \otimes \left(\mathbf{T}_{zp}\mathbf{T}_{zp}^H\right)\right)\mathbf{H}^H + \mathbf{H}\left(\mathbf{I}_N \otimes \left(\mathbf{T}_{zp}\mathbf{T}_{zp}^H\right)\right)\mathbf{X}_l^H \quad (60)$$

$$\frac{\partial \mathbf{R}_{zp}}{\partial h_i(l)} = j\left[\mathbf{X}_l \left(\mathbf{I}_N \otimes \left(\mathbf{T}_{zp}\mathbf{T}_{zp}^H\right)\right)\mathbf{H}^H - \mathbf{H}\left(\mathbf{I}_N \otimes \left(\mathbf{T}_{zp}\mathbf{T}_{zp}^H\right)\right)\mathbf{X}_l^H\right] \quad (61)$$

where

$$\mathbf{X}_l \triangleq \begin{bmatrix} \mathbf{0}_{l\times(JN-l)} & \mathbf{0}_{l\times l} \\ \mathbf{I}_{JN-l} & \mathbf{0}_{(JN-l)\times l} \end{bmatrix}, \quad l = 0,\dots,L. \quad (62)$$

Next, we calculate the partial differentiation w.r.t. the interference real and imaginary part parameters contained in $\mathbf{r}_r$ and $\mathbf{r}_i$, respectively, as

$$\frac{\partial \mathbf{R}_{zp}}{\partial [\mathbf{r}_r]_z} = \frac{\partial \mathbf{R}_w}{\partial [\mathbf{r}_r]_z} = \begin{cases} \mathbf{I}_{JN}, & z = 1 \\ \mathbf{Q}_z + \mathbf{Q}_z^T, & z = 2,\dots,JN \end{cases} \quad (63)$$

$$\frac{\partial \mathbf{R}_{zp}}{\partial [\mathbf{r}_i]_z} = \frac{\partial \mathbf{R}_w}{\partial [\mathbf{r}_i]_z} = j(\mathbf{Q}_z - \mathbf{Q}_z^T), \quad z = 2,\dots,JN \quad (64)$$

where

$$\mathbf{Q}_z \triangleq \begin{bmatrix} \mathbf{0}_{(z-1)\times(JN-z+1)} & \mathbf{0}_{(z-1)\times(z-1)} \\ \mathbf{I}_{JN-z+1} & \mathbf{0}_{(JN-z+1)\times(z-1)} \end{bmatrix}. \quad (65)$$

Using (60) and (61), (63) and (64), which are subsequently substituted into (59), the FIM can be computed entry by entry.

The FIR $\mathbf{J}(\theta)$, however, is singular due to the scalar ambiguity inherent in all blind-channel identification problems [13]–[15]. To eliminate the ambiguity, various constraints can be enforced to regularize the estimation problem. In what follows, we present the CRB for parameters that satisfy the constraints

$$\mathbf{f}(\theta) = \begin{bmatrix} h_r(0) \\ h_i(0) \end{bmatrix} - \begin{bmatrix} a_r \\ a_i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad (66)$$

Then, the $2 \times m$ gradient matrix for the constraints is given by

$$\mathbf{F}(\theta) = \frac{\partial \mathbf{F}(\theta)}{\partial \theta^T} = \begin{bmatrix} 1 & \mathbf{0}_{1\times(m-1)} & \\ \mathbf{0}_{1\times(L+1)} & 1 & \mathbf{0}_{1\times(m-L-2)} \end{bmatrix}. \quad (67)$$

The gradient matrix $\mathbf{F}(\theta)$ has full row rank; therefore, there exists a matrix $\mathbf{U} \in \mathbb{C}^{m\times(m-2)}$ whose columns form basis for the null space of $\mathbf{F}(\theta)$, that is

$$\mathbf{F}(\theta)\mathbf{U} = \mathbf{0}. \quad (68)$$

It follows that the constrained CRB is given by [14], [16]

$$\mathrm{CRB}\,(\theta; \mathbf{f}(\theta) = \mathbf{0}) = \mathbf{U}\left(\mathbf{U}^T\mathbf{J}(\theta)\mathbf{U}\right)^{-1}\mathbf{U}^T. \quad (69)$$

B. CRB for SC-CP

Similar to the ZP case, we collect all samples within a P-block observation time and form a $P(JN-L)\times 1$ vector $\bar{\mathbf{y}}$ defined as

$$\bar{\mathbf{y}} \triangleq \left[\mathbf{y}_{cp}^T(P), \mathbf{y}_{cp}^T(P-1), \dots, \mathbf{y}_{cp}^T(1)\right]^T. \quad (70)$$

It is clear that $\bar{\mathbf{y}}$ follows a Gaussian distribution with zero mean and covariance matrix

$$\bar{\mathbf{R}}_P = \mathbf{I}_P \otimes \left[\bar{\mathbf{H}}\left(\mathbf{I}_N \otimes \left(\mathbf{T}_{cp}\mathbf{T}_{cp}^H\right)\right)\bar{\mathbf{H}}^H + \bar{\mathbf{R}}_w + \sigma^2\mathbf{I}_{(JN-L)}\right] = \mathbf{I}_P \otimes \mathbf{R}_{cp} \quad (71)$$

where $\bar{\mathbf{R}}_w = \mathbf{R}_w(1:JN-L, 1:JN-L)$, and $\bar{\mathbf{H}} = \Gamma\mathbf{H}$.

Let $\bar{m} = 2JN + 1$ be the total number of unknown parameters, and define the $\bar{m}\times 1$ vector of unknown parameters as

$$\bar{\theta} = [\,\mathbf{h}_r^T \quad \mathbf{h}_i^T \quad \bar{\mathbf{r}}_r^T \quad \bar{\mathbf{r}}_i^T\,]^T \quad (72)$$

where $\bar{\mathbf{r}}_r = [r(0), r_r(1), \dots, r_r(JN-L-1)]$, and $\bar{\mathbf{r}}_i = [r_i(1), \dots, r_i(JN-L-1)]$. The $\bar{m}\times\bar{m}$ FIM is given by

$$\left[\bar{\mathbf{J}}(\theta)\right]_{s,t} = P\,\mathrm{tr}\left[\mathbf{R}_{cp}^{-1}\frac{\partial \mathbf{R}_{cp}}{\partial[\bar{\theta}]_s}\mathbf{R}_{cp}^{-1}\frac{\partial \mathbf{R}_{cp}}{\partial[\bar{\theta}]_t}\right]. \quad (73)$$

The partial differentiations w.r.t. each parameter are given by

$$\frac{\partial \mathbf{R}_{cp}}{\partial h_r(l)} = \bar{\mathbf{X}}_l\left(\mathbf{I}_N \otimes \left(\mathbf{T}_{cp}\mathbf{T}_{cp}^H\right)\right)\bar{\mathbf{H}}^H + \bar{\mathbf{H}}\left(\mathbf{I}_N \otimes \left(\mathbf{T}_{cp}\mathbf{T}_{cp}^H\right)\right)\bar{\mathbf{X}}_l^H \quad (74)$$

$$\frac{\partial \mathbf{R}_{cp}}{\partial h_i(l)} = j\left[\bar{\mathbf{X}}_l\left(\mathbf{I}_N \otimes \left(\mathbf{T}_{cp}\mathbf{T}_{cp}^H\right)\right)\bar{\mathbf{H}}^H - \bar{\mathbf{H}}\left(\mathbf{I}_N \otimes \left(\mathbf{T}_{cp}\mathbf{T}_{cp}^H\right)\right)\bar{\mathbf{X}}_l^H\right] \quad (75)$$

$$\frac{\partial \mathbf{R}_{cp}}{\partial [\bar{\mathbf{r}}_r]_z} = \frac{\partial \bar{\mathbf{R}}_w}{\partial [\bar{\mathbf{r}}_r]_z} = \begin{cases} \mathbf{I}_{JN-L}, & z = 1 \\ \bar{\mathbf{Q}}_z + \bar{\mathbf{Q}}_z^T, & z = 2,\dots,JN-L \end{cases} \quad (76)$$

$$\frac{\partial \mathbf{R}_{cp}}{\partial [\bar{\mathbf{r}}_i]_z} = \frac{\partial \bar{\mathbf{R}}_w}{\partial [\bar{\mathbf{r}}_i]_z} = j(\bar{\mathbf{Q}}_z - \bar{\mathbf{Q}}_z^T), \quad z = 2,\dots,JN-L \quad (77)$$

where

$$\bar{\mathbf{X}}_l \triangleq \left[\mathbf{0}_{(JN-L)\times(L-l)} \quad \mathbf{I}_{JN-L} \quad \mathbf{0}_{(JN-L)\times l}\right], \quad l = 0,\dots,L \quad (78)$$

$$\bar{\mathbf{Q}}_z \triangleq \begin{bmatrix} \mathbf{0}_{(z-1)\times(JN-L-z+1)} & \mathbf{0}_{(z-1)\times(z-1)} \\ \mathbf{I}_{JN-L-z+1} & \mathbf{0}_{(JN-L-z+1)\times(z-1)} \end{bmatrix}. \quad (79)$$

Using (74)–(77), which are subsequently substituted into (73), the FIM can be computed entry by entry. To eliminate the scalar ambiguity, we can use the same constraint and follow similar steps as in the ZP case.

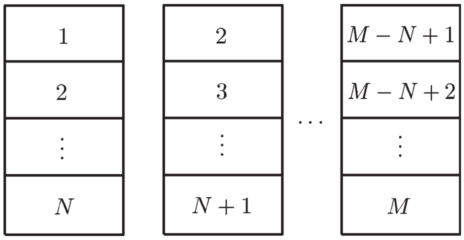


Fig. 1. Overlapping block processing of M received symbols.

VII. Numerical Results

In what follows, we present the simulation results by processing a block of N symbols that are arranged in an overlapping fashion. For a total number of symbols equal to M, this will result in $M - N + 1$ blocks, as illustrated by Fig. 1. This approach increases the number of blocks by using the same available received signal in an efficient way. We compare here the proposed methods with the subspace blind-channel estimators [5], [6] for CP and ZP, respectively. While [5] and [6] only cover the MC cases, straightforward extensions can be made to the SC cases. The systems under study utilize a binary phase shift keying constellation with $K = 48$ and a total of $M = 200$ symbols for channel estimation. The channel is a four-tap ($L = 3$) FIR channel. The narrowband frequencies are randomly generated and then fixed for all examples. The narrowband interference is then generated as a number of interfering tones at these different frequencies. The signal-to-interference ratio (SIR) is defined as the average signal energy divided by the power of the total interfering tones. As a performance measure, we consider here the normalized root-mean-squared error (RMSE) defined as $1/\|\mathbf{h}\|\sqrt{1/D(L+1)\sum_{i=1}^{D}\|\hat{\mathbf{h}}_i - \mathbf{h}\|^2}$, which is averaged over $D = 500$ Monte Carlo runs.

A. Performance versus SNR and SIR

In this example, we show the performance of the proposed and subspace methods as well as the CRB for SIR equal to 20 and -20 dB, respectively, as the SNR varies from 15 to 40 dB. In addition, a training-based method is included for comparison, which applies least-squares fitting using (2), (5), (6), and (8), respectively, with a set of training signals $\{\mathbf{s}_N(n)\}$ for each of the four block transmission schemes. The training-based method is a standard channel-estimation technique. It works well in signal-plus-white-noise environments but significantly suffers in the presence of unknown interference.

The simulation parameters are $N = 2$ and $\mu = 16$, and 19 narrowband interfering tones are added. Fig. 2(a) and (b) shows the performance for SC-ZP and MC-ZP systems, respectively. It is seen that at both interference levels (i.e., SIR $= 20$ dB and -20 dB), the performances of SC and MC are nearly identical for the proposed as well as for training-based and subspace schemes. When the interference is relatively weak (i.e., SIR $= 20$ dB), the subspace estimators significantly degrade and exhibit irreducible error. At the same interference level, the performance of the training-based schemes is also significantly affected since interference is not taken care of. The flat behavior of the training based (RMSE is not decreasing with increasing SNR) is due to the presence of interference, which is the primary limiting factor of the training-based techniques at the high-SNR region. As for the proposed schemes, even in the presence of strong interference (i.e., SIR $= -20$ dB), they can effectively suppress the interference and approach the CRB as the SNR is increased.

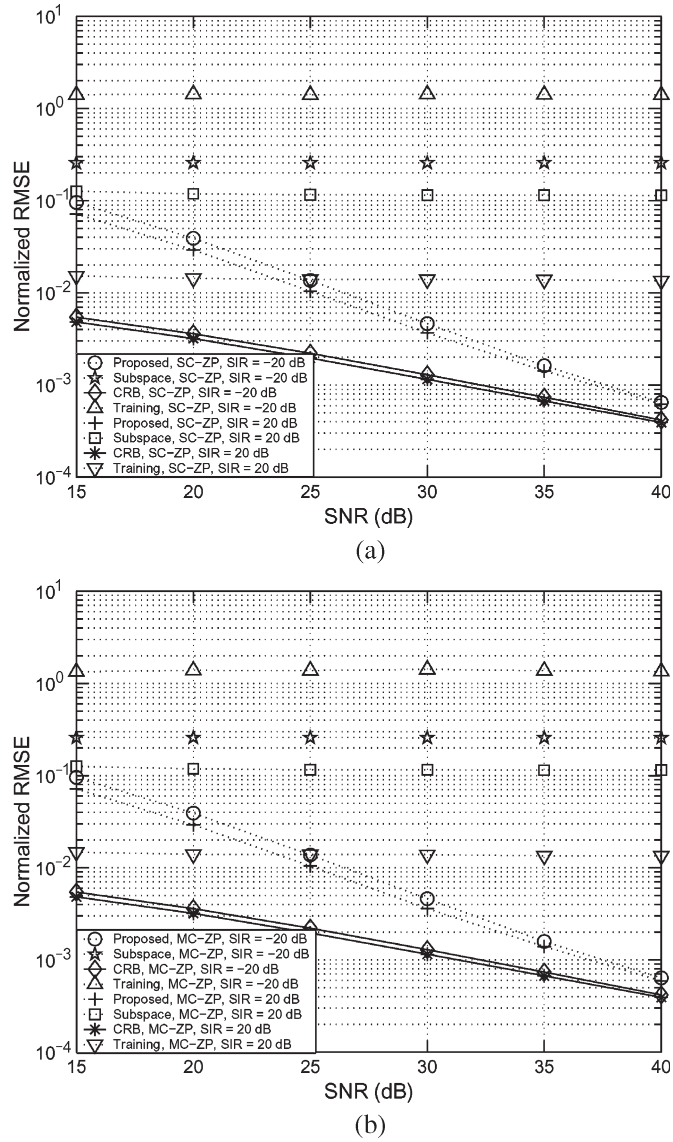
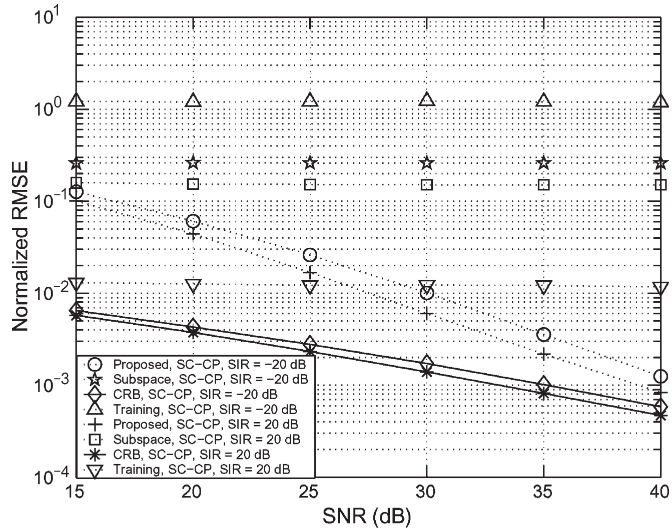


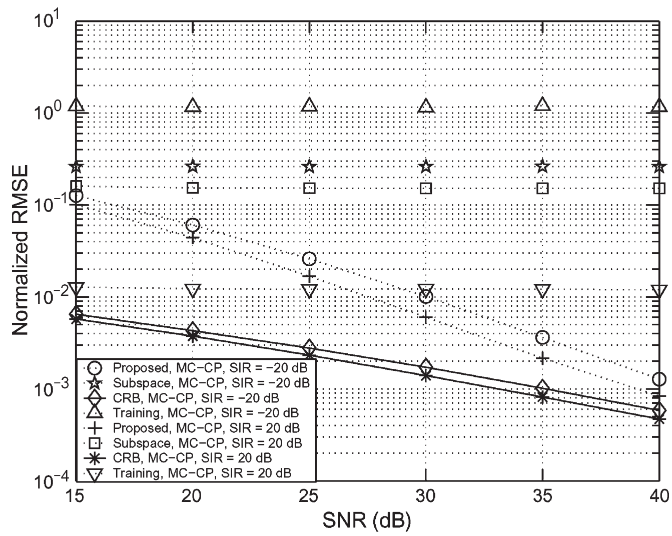
Fig. 2. Normalized RMSE of the CRB, proposed, training-based, and subspace blind-channel estimates versus SNR and SIR for $K = 48$, $L = 3$, $N = 2$, $\mu = 16$, and 19 interfering tones. (a) SC-ZP. (b) MC-ZP.

Fig. 3(a) and (b) shows the performance versus the SNR and SIR for SC and MC systems, respectively, when CP is used. Similar to the ZP case, for all methods, the performances of SC and MC are nearly identical at both interference levels. Additionally, similar performance degradation happens to both training-based and subspace schemes when SIR $= 20$ dB. Meanwhile, the proposed estimators continue to perform well, even at low SIR, and approach the CRB.

We note that in the absence of interference, the training-based method is better than the proposed for both ZP and CP cases; the result is not shown since otherwise, the figures will be too crowded and will not be readable. Additionally, although

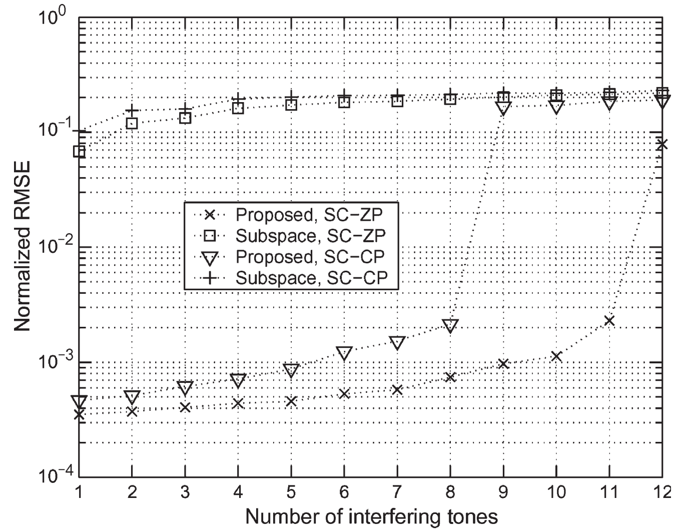


(a)

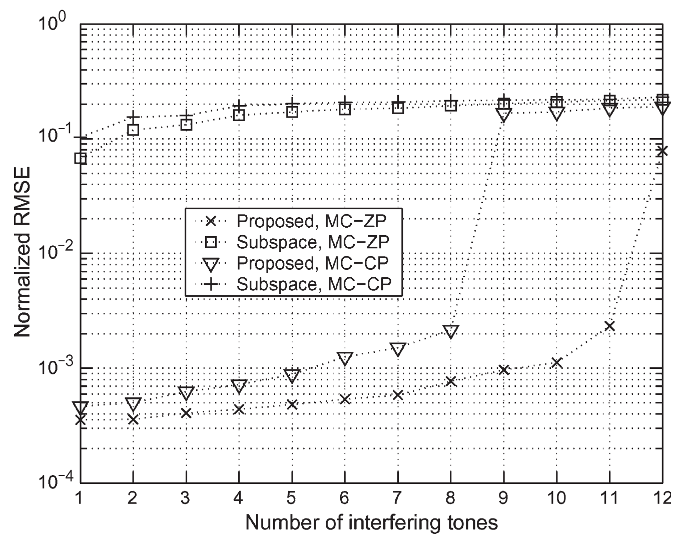


(b)

Fig. 3. Normalized RMSE of the CRB, proposed, training-based, and subspace blind-channel estimates versus SNR and SIR for $K = 48$, $L = 3$, $N = 2$, $\mu = 16$, and 19 interfering tones. (a) SC-CP. (b) MC-CP.



(a)



(b)

Fig. 4. Normalized RMSE of the proposed and subspace blind-channel estimates versus number of interfering tones for $K = 48$, $L = 3$, $\mu = 6$, $N = 2$, SIR $= -10$ dB, and SNR $= 40$ dB. (a) SC. (b) MC.

the complexity of the training-based method (about $O(JN(L+1)^2)$) is lower than the proposed method, the training-based method reduces the spectral efficiency, and training symbols have to periodically be retransmitted, leading to throughput reduction.

B. Performance versus Number of Interfering Tones

Fig. 4(a) and (b) shows the performance versus the number of narrowband interfering tones with ZP and CP for the (top) SC and (bottom) MC systems when SIR $= -10$ dB, SNR $= 40$ dB, $N = 2$, and $\mu = 6$. It is seen that the performances of SC and MC are nearly identical for the proposed as well as for the subspace schemes. As we remarked in Section V, Fig. 4 shows the maximum number of interfering tones that can be handled by each one of the proposed methods. For the ZP schemes, the number of interfering tones needs to be less than μN and less than $\mu N - L$ for the CP schemes, which means that we can suppress L more interfering tones by employing ZP over CP. On the other hand, subspace schemes are considerably degrading due to their sensitivity to unknown narrowband interference.

C. Performance versus SNR and N

Finally, we consider the effect of N, i.e., the number of symbols simultaneously processed, as the SNR varies from 0 to 40 dB. Fig. 5 depicts the RMSE of the proposed channel estimators with (top) ZP and (bottom) CP for SIR $= -10$ dB, $\mu = L = 3$, and number of interfering tones equal to 2. Similar to what was seen in the previous examples, the performances of SC and MC schemes are nearly identical. For the (top) ZP case, $N = 2$ gives a better performance than $N = 1$. Similarly, improved performance is obtained when using $N = 3$ compared to $N = 2$ for (bottom) the CP case. Because a larger N leads to more degrees of freedom. As mentioned earlier in Section V, we must have $N > 1$ for CP, but this restriction is not needed for ZP.

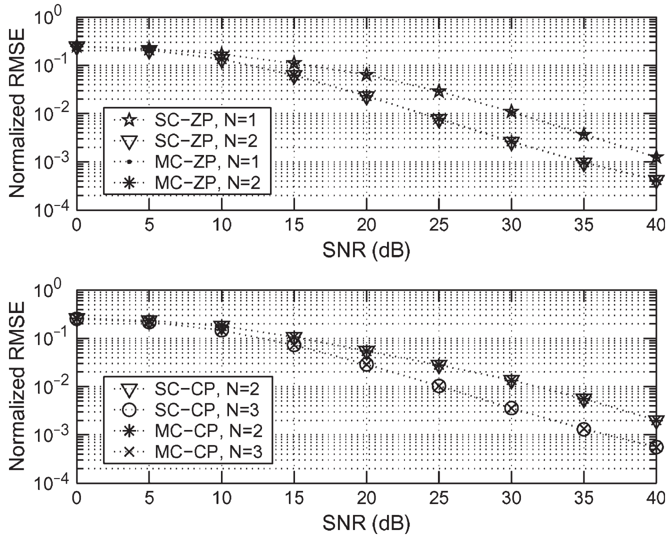


Fig. 5. Normalized RMSE of the proposed blind-channel estimates versus SNR and N for $K = 48$, $L = 3$, $\mu = L$, SIR $= -10$ dB, and two interfering tones. (Top) SC and MC with ZP. (Bottom) SC and MC with CP.

VIII. Conclusion

We have presented a unified approach to blind-channel estimation and interference suppression for block transmission using ZP and CP for both SC and MC systems. A generalized multichannel minimum-variance principle was invoked to design an equalizing filterbank that preserves desired signal components and suppresses the overall interference. Our unified approach leads to estimators having similar forms, which gives flexibility for application to all cases with minor changes. Through an asymptotic analysis of the subspace of the received signal, we determined an upper bound for the number of interfering tones that can be handled by the proposed schemes. To assess the performance of the proposed schemes, an unconditional CRB was derived for each scheme. The proposed schemes favorably compare with the subspace blind-channel estimators in the presence of unknown narrowband interference and approach the CRB as the SNR increases. We have seen that the performances of the SC and MC schemes are nearly identical for ZP as well as for the CP case. We have also observed that in the presence of interference, ZP schemes give better performance and can handle more interfering tones than the CP schemes.

In this paper, we only considered batch implementations of the proposed channel estimators, which are appropriate when the channel remains (nearly) stationary over the observation time. Batch implementations are not recommended for time-varying channels, in which case, adaptive implementations can provide some tracking ability (providing channels are not changing too fast) and, therefore, are more suitable. Adaptive implementations of the proposed channel estimators, which have not been pursued here due to space limitations, can be obtained by following similar steps as in [17]. Their performance in time-varying fading channels will be reported elsewhere.

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