Robust Channel Estimation and Detection for Single-Carrier and Multicarrier Block Transmission Systems

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***Abstract*—Block transmission techniques, including single-carrier (SC) and multicarrier (MC) communication techniques, have received much research interest recently for their better ability to handle the intersymbol interference problem than continuous transmission. Numerous detection schemes for SC and MC communication systems have been proposed. While these schemes may be derived from different principles, they usually rely on some initial estimate of the communication channel and/or the covariance matrix of the received signal. However, such estimates usually contain inherent estimation errors to which most existing detection schemes are known to be sensitive. In this paper, we develop robust estimation and detection schemes that explicitly account for channel and covariance matrix estimation errors by optimizing the worst-case performance over properly selected bounded uncertainty sets. Although the prior channel and covariance matrix-estimation errors are generally not bounded, we show that it is beneficial to refine the channel and covariance matrix estimates over properly chosen bounded uncertainty sets centered on the prior channel and covariance matrix estimates. Numerical results show that an improved performance is achieved by using the proposed robust approaches over the ones that ignore the prior estimation errors.**

***Index Terms*—Multicarrier (MC) systems, robust channel estimation, single-carrier (SC) systems.**

I. Introduction

Block transmission, which is an alternative to conventional continuous transmission schemes, has been implemented in many high-speed communication standards for indoor and outdoor environments [1]. A block communication system can employ single-carrier (SC) or multicarrier (MC) modulation, along with the use of cyclic prefix (CP) or zero padding (ZP). Channel estimation for such systems often relies on either *explicit training* (e.g., [2]) or some *inherent structure* of the transmitted signal, as in the well-known class of the subspace-based methods (e.g., [3] and [4]).

Numerous detection schemes for SC and MC have been proposed. While these schemes may be derived from different principles, they usually require some prior estimate of the communication channel. There exists a rich literature on channel estimation for SC and MC communication systems (e.g., [5]–[7], and references therein). Comparative studies between SC and MC have been reported in [8]–[12]. For example, it was shown in [9] that block SC transmission has similar equalization complexity and coded performance to that of orthogonal frequency-division multiplexing (OFDM). One notable advantage of SC transmission is that it is free of the peak-to-average power-ratio problem of MC and is also less sensitive to carrier frequency shift [13]. Meanwhile, MC transmission such as OFDM is more advantageous in implementing adaptive bit or power-loading techniques based on the channel conditions at individual subcarriers.

Traditionally, channel estimation is separated from detection. When detection is optimized, it is often assumed that the channel is known. However, channel estimation methods in reality result in some estimation error that has to be dealt with. In addition, many detectors also rely on an estimate of the covariance matrix of a vector formed by samples of the received signal over a transmission block, which is, again, subject to inherent estimation error [14]. There has been constant effort in developing better channel/covariance estimation techniques with smaller estimation error and/or faster convergence rate (e.g., [15]–[17]). Unlike these works, which attempt to minimize the estimation error in the first place, our approach complements such efforts by taking care of the residual channel and covariance matrix-estimation errors through a robust design.

Over the years, there has been considerable research on robust estimation techniques based on a min–max approach [18]–[20], where the goal is to optimize the worst-case performance. Relying on these techniques, several new studies have been applied to many problems in detection and estimation. Eldar and Mehrav [21] developed a min–max approach that minimizes the worst-case difference between the mean square error (MSE) where the signal statistics are not fully known and the optimal MSE given the signal covariance. In [22], a robust precoder was designed to minimize the mean-square equalization error for the least favorable channel located in a ball centered about the channel estimate. The radius of the ball represents the channel-estimation uncertainty, and the obtained robust precoder depends on the size of this ball. A robust

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method with an adaptive threshold was proposed in [23] to estimate the DOAs of source signals in both Gaussian and non-Gaussian noise environments. In [24], a Bayesian approach was used to design robust adaptive beamforming techniques.

In this paper, we develop robust detection schemes that explicitly account for channel and covariance matrix estimation errors in SC and MC block communication systems. Our schemes build on recent developments on robust estimation for array processing and radars [25]–[28]. While the uncertainty in such applications (e.g., antenna calibration errors in array processing) is usually bounded, this is not the case in our problem, where the prior channel and covariance matrix estimation errors are often unbounded. Even so, it turns out to be beneficial to optimize the worst-case performance over a properly chosen bounded uncertainty set, as shown in this paper. By using the Chebyshev inequality [29], we show that a bounded uncertainty set can be determined by a parameter referred to as the Chebyshev bounding probability. By choosing the Chebyshev bounding probability sufficiently large (e.g., ≥ 0.9), we neglect the small probability event that the estimation error may exceed the chosen bounded set. This is because in that case, the prior estimation is so poor, and our robust scheme is not expected to help (in fact, few methods will succeed when the initial estimates are very poor). Hence, our strategy is to try to achieve robustness against small-to-medium prior estimation errors. This makes our work distinct from earlier studies.

Numerical results show that our proposed robust schemes yield significant performance improvement over the standard detectors that ignore the prior channel and covariance matrix-estimation errors.

The rest of this paper is organized as follows. In Section II, we formulate the problem of interest. We examine the estimation errors in the initial channel estimate and the sample covariance matrix in Section III. The proposed algorithms are presented in Section IV. Section V illustrates numerically the performance of the proposed schemes. Finally, we draw conclusions in Section VI.

Notation: Vectors (matrices) are denoted by boldface lower (upper) case letters; all vectors are column vectors; superscripts $(\cdot)^T$, $(\cdot)^*$, and $(\cdot)^H$ denote the transpose, conjugate, and conjugate transpose, respectively; $\mathbf{I}_N$ denotes the $N \times N$ identity matrix; $\mathbf{0}$ denotes an all-zero matrix or vector; $\mathrm{tr}\{\cdot\}$ denotes the trace; $E\{\cdot\}$ denotes the statistical expectation; $\|\cdot\|$ denotes the Frobenius norm; and finally, $\mathrm{diag}\{\cdot\}$ denotes a diagonal matrix.

II. Problem Formulation

Consider a system where a sequence of information symbols are blocked into $K \times 1$ vectors $\mathbf{s}(n) = [s(nK), \ldots, s(nK + K - 1)]^T$. To avoid multipath-induced interblock interference (IBI), a guard interval of length μ is inserted in each block, where the length μ is chosen to be greater than or equal to the channel order L. The discrete-time baseband equivalent channel, which includes the transmitter/receiver filters and the physical channel, is modeled as a finite-duration impulse response (FIR) filter $\mathbf{h} \triangleq [h(0), h(1), \ldots, h(L)]^T$. As it is widely covered in the literature (e.g., [4], [30], and [31]), we introduce, in what follows, the system models for SC with ZP (SC-ZP), MC with ZP (MC-ZP), SC with CP (SC-CP), and MC with CP (MC-CP), respectively. We note here that MC-CP is simply the famous OFDM system. In ZP block transmission, trailing zeros of length μ are introduced at the end of each block before transmission. This can be mathematically represented by multiplying $\mathbf{s}(n)$ with a $P \times K$ guard insertion matrix $\mathbf{T}_{\mathrm{zp}} = [\mathbf{I}_K^T \quad \mathbf{0}_{\mu\times K}^T]^T$, where $\mathbf{I}_K$ is a $K \times K$ identity matrix, and $P = K + \mu$. The trailing zeros will help the removal of IBI from the received signal. CP block transmission relies on stacking on top of the transmitted vector $\mathbf{s}(n)$ its last μ elements, which can be mathematically represented by multiplying $\mathbf{s}(n)$ with a $P \times K$ matrix $\mathbf{T}_{\mathrm{cp}} = [\mathbf{I}_{\mu\times K}^T \quad \mathbf{I}_K^T]^T$, where $\mathbf{I}_{\mu\times K}$ is formed from the last μ rows of the identity matrix $\mathbf{I}_K$. In what follows, we discuss the received signals for the four different block-transmission schemes.

A. SC-ZP

The nth block of the received signal in the time domain can be expressed as

$$\bar{\mathbf{y}}_{\mathrm{zp}}(n) = \mathbf{H}\mathbf{T}_{\mathrm{zp}}\mathbf{s}(n) + \bar{\mathbf{e}}_{\mathrm{zp}}(n) \tag{1}$$

where $\mathbf{H}$ is a $P \times P$ lower triangular Toeplitz matrix with the first row $[h(0), \mathbf{0}_{1\times(P-1)}]$ and the first column $[h(0), \ldots, h(L), \mathbf{0}_{1\times(P-L-1)}]^T$, and $\bar{\mathbf{e}}_{\mathrm{zp}}(n)$ denotes the $P \times 1$ channel noise vector. Because of the trailing zeros in $\mathbf{T}_{\mathrm{zp}}$, the elements of the last μ columns of $\mathbf{H}$ will be multiplied by zeros and have no impact on the received block. It follows that the matrix $\mathbf{H}$ can be regarded as a circulant matrix $\mathbf{H}_0$ that can be diagonalized with a $P \times P$ FFT matrix $\mathbf{F}_P$. That is

$$\begin{aligned}\mathbf{y}_{\mathrm{zp}}(n) = \mathbf{F}_P\bar{\mathbf{y}}_{\mathrm{zp}}(n) &= \mathbf{F}_P\mathbf{H}_0\mathbf{F}_P^H\mathbf{F}_P\mathbf{T}_{\mathrm{zp}}\mathbf{s}(n) + \mathbf{e}_{\mathrm{zp}}(n)\\ &= \mathbf{Q}(\mathbf{h})\mathbf{V}\mathbf{s}(n) + \mathbf{e}_{\mathrm{zp}}(n)\\ &\triangleq \mathbf{X}_{\mathrm{zp}}(\mathbf{h})\mathbf{s}(n) + \mathbf{e}_{\mathrm{zp}}(n)\end{aligned} \tag{2}$$

where $\mathbf{V} = \mathbf{F}_P\mathbf{T}_{\mathrm{zp}}$ has a known structure, and $\mathbf{e}_{\mathrm{zp}}(n) = \mathbf{F}_P\bar{\mathbf{e}}_{\mathrm{zp}}(n)$. The diagonal matrix $\mathbf{Q}(\mathbf{h}) = \mathbf{F}_P\mathbf{H}_0\mathbf{F}_P^H = \mathrm{diag}(\mathbf{q})$, where $\mathbf{q} = [q(1), \ldots, q(P)]^T$ and $q(p) = \sum_{l=0}^{L} h(l) \exp(-j2\pi l(p-1)/P)$. The frequency channel-response vector $\mathbf{q}$ is related to the channel-impulse response $\mathbf{h}$ by a P-point discrete Fourier transform (DFT). That is, $\mathbf{q} = \sqrt{P}\boldsymbol{\mathcal{F}}\mathbf{h}$, where $\boldsymbol{\mathcal{F}} \triangleq \mathbf{F}_{P\{:,1:L+1\}}$ stands for the first $L+1$ columns of the matrix $\mathbf{F}_P$. It follows that $\mathbf{y}_{\mathrm{zp}}(n)$ in (2) can be expressed as

$$\begin{aligned}\mathbf{y}_{\mathrm{zp}}(n) &= \mathrm{diag}(\mathbf{q})\mathbf{V}\mathbf{s}(n) + \mathbf{e}_{\mathrm{zp}}(n)\\ &= \mathrm{diag}\left(\mathbf{V}\mathbf{s}(n)\right)\mathbf{q} + \mathbf{e}_{\mathrm{zp}}(n)\\ &= \mathrm{diag}\left(\mathbf{V}\mathbf{s}(n)\right)\sqrt{P}\boldsymbol{\mathcal{F}}\mathbf{h} + \mathbf{e}_{\mathrm{zp}}(n)\\ &\triangleq \mathbf{S}_{\mathrm{zp}}(n)\mathbf{h} + \mathbf{e}_{\mathrm{zp}}(n).\end{aligned} \tag{3}$$

B. MC-ZP

In MC transmission, the data symbols $\mathbf{s}(n)$ are modulated with an inverse DFT (IDFT) unitary matrix $\mathbf{F}_K^H$, where $\mathbf{F}_K \in \mathbb{C}^{K\times K}$, whose (k,l)th element is given by $K^{-1/2}\exp\{j2\pi(k-1)(l-1)\}$. The received signal can be expressed as

$$\breve{\mathbf{y}}_{\mathrm{zp}}(n) = \mathbf{H}\mathbf{T}_{\mathrm{zp}}\mathbf{F}_K^H\mathbf{s}(n) + \bar{\mathbf{e}}_{\mathrm{zp}}(n). \tag{4}$$

After demodulation with the FFT matrix $\mathbf{F}_P$, and following the same steps as in the SC-ZP case, (4) can be expressed as

$$\begin{aligned}\breve{\mathbf{y}}_{\rm zp}(n) &= \breve{\mathbf{X}}_{\rm zp}(\mathbf{h})\mathbf{s}(n) + \mathbf{e}_{\rm zp}(n)\\ &= \mathbf{Q}(\mathbf{h})\mathbf{V}\mathbf{F}_K^H\mathbf{s}(n) + \mathbf{e}_{\rm zp}(n)\\ &= \mathrm{diag}\left(\mathbf{V}\mathbf{F}_K^H\mathbf{s}(n)\right)\sqrt{P}\boldsymbol{\mathcal{F}}\mathbf{h} + \mathbf{e}_{\rm zp}(n)\\ &\triangleq \breve{\mathbf{S}}_{\rm zp}(n)\mathbf{h} + \mathbf{e}_{\rm zp}(n). \end{aligned} \tag{5}$$

C. SC-CP

To eliminate the IBI, we remove the CP from the nth received block by using a truncation matrix $\boldsymbol{\Gamma} = [\mathbf{0}_{K\times\mu} \quad \mathbf{I}_K]$ and form a $K \times 1$ vector $\mathbf{y}_{\rm cp}(n)$ as follows:

$$\bar{\mathbf{y}}_{\rm cp}(n) = \boldsymbol{\Gamma}\mathbf{H}\mathbf{T}_{\rm cp}\mathbf{s}(n) + \bar{\mathbf{e}}_{\rm cp}(n) \tag{6}$$

where $\bar{\mathbf{e}}_{\rm cp}(n)$ denotes a $K \times 1$ channel noise vector. Notice that $\boldsymbol{\Gamma}\mathbf{H}\mathbf{T}_{\rm cp}$ is a $K \times K$ circulant matrix. Therefore, after demodulation with a $K \times K$ FFT matrix $\mathbf{F}_K$, the nth block of the frequency-domain data symbols can be expressed as

$$\begin{aligned}\mathbf{y}_{\rm cp}(n) = \mathbf{F}_K\bar{\mathbf{y}}_{\rm cp}(n) &= \mathbf{F}_K\boldsymbol{\Gamma}\mathbf{H}\mathbf{T}_{\rm cp}\mathbf{F}_K^H\mathbf{F}_K\mathbf{s}(n) + \mathbf{e}_{\rm cp}(n)\\ &= \mathbf{G}(\mathbf{h})\mathbf{F}_K\mathbf{s}(n) + \mathbf{e}_{\rm cp}(n)\\ &\triangleq \mathbf{X}_{\rm cp}(\mathbf{h})\mathbf{s}(n) + \mathbf{e}_{\rm cp}(n)\end{aligned} \tag{7}$$

where the diagonal matrix $\mathbf{G}(\mathbf{h}) \triangleq \mathbf{F}_K\boldsymbol{\Gamma}\mathbf{H}\mathbf{T}_{\rm cp}\mathbf{F}_K^H = \mathrm{diag}(\mathbf{g})$, and $\mathbf{g} = [g(1), g(2), \ldots, g(K)]^T$, with $g(k) = \sum_{l=0}^{L} h(l)\exp(-j2\pi l(k-1)/K)$, and $\mathbf{e}_{\rm cp}(n)$ denotes the DFT of the channel noise vector. The frequency channel-response vector $\mathbf{g}$ is related to the channel-impulse response $\mathbf{h}$ by a K-point DFT. That is, $\mathbf{g} = \sqrt{K}\bar{\boldsymbol{\mathcal{F}}}\mathbf{h}$, where $\bar{\boldsymbol{\mathcal{F}}}$ stands for the first $L+1$ columns of the DFT matrix $\mathbf{F}_K$. It follows that $\mathbf{y}_{\rm cp}(n)$ in (7) can be expressed as

$$\begin{aligned}\mathbf{y}_{\rm cp}(n) &= \mathrm{diag}(\mathbf{g})\mathbf{F}_K\mathbf{s}(n) + \mathbf{e}_{\rm cp}(n)\\ &= \mathrm{diag}\left(\mathbf{F}_K\mathbf{s}(n)\right)\mathbf{g} + \mathbf{e}_{\rm cp}(n)\\ &= \mathrm{diag}\left(\mathbf{F}_K\mathbf{s}(n)\right)\sqrt{K}\bar{\boldsymbol{\mathcal{F}}}\mathbf{h} + \mathbf{e}_{\rm cp}(n)\\ &\triangleq \mathbf{S}_{\rm cp}(n)\mathbf{h} + \mathbf{e}_{\rm cp}(n).\end{aligned} \tag{8}$$

D. MC-CP

Similar to MC-ZP, the transmitted signal $\mathbf{s}(n)$ needs to be converted by the IDFT prior to transmission. The received signal can be expressed as

$$\breve{\bar{\mathbf{y}}}_{\rm cp}(n) = \boldsymbol{\Gamma}\mathbf{H}\mathbf{T}_{\rm cp}\mathbf{F}_K^H\mathbf{s}_N(n) + \bar{\mathbf{e}}_{\rm cp}(n). \tag{9}$$

After demodulation with the matrix $\mathbf{F}_K$, and following the same steps as in the SC-CP case, (9) can be expressed as

$$\begin{aligned}\breve{\mathbf{y}}_{\rm cp}(n) &= \breve{\mathbf{X}}_{\rm cp}(\mathbf{h})\mathbf{s}(n) + \mathbf{e}_{\rm cp}(n)\\ &= \mathbf{G}(\mathbf{h})\mathbf{s}(n) + \mathbf{e}_{\rm cp}(n)\\ &= \mathrm{diag}\left(\mathbf{s}(n)\right)\sqrt{K}\bar{\boldsymbol{\mathcal{F}}}\mathbf{h} + \mathbf{e}_{\rm cp}(n)\\ &\triangleq \breve{\mathbf{S}}_{\rm cp}(n)\mathbf{h} + \mathbf{e}_{\rm cp}(n).\end{aligned} \tag{10}$$

The problem of interest is to find robust channel estimates $\tilde{\mathbf{h}}$ for each of the SC-ZP, MC-ZP, SC-CP, and MC-CP communication systems, and then use these estimates to develop robust detectors for all of the above systems.

III. Characterization of Estimation Error

To facilitate our discussion, we first analyze the estimation error of a standard least squares (LS)-based channel estimator and then of the sample covariance matrix for both SC-ZP and SC-CP. The channel and covariance matrix-estimation errors for the other two MC schemes can be similarly analyzed like their SC counterparts. These analyses can help us determine the boundaries of the uncertainty sets in our robust receiver design in Section IV.

A. Channel Estimation

We assume that unit energy constellations are used, i.e., $|s(k)|^2 = 1$, for $k = 0, \ldots, K-1$. Furthermore, we assume that $\mathbf{e}_{\rm zp}$ and $\mathbf{e}_{\rm cp}$ are zero-mean white Gaussian noise with $E\{\mathbf{e}_{\rm zp}\mathbf{e}_{\rm zp}^H\} = \sigma_{\rm zp}^2\mathbf{I}_P$ and $E\{\mathbf{e}_{\rm cp}\mathbf{e}_{\rm cp}^H\} = \sigma_{\rm cp}^2\mathbf{I}_K$ for ZP and CP, respectively.

1) ZP Case: To simplify notations in our presentation, we will drop the index n and rewrite (3) as

$$\mathbf{y}_{\rm zp} = \mathbf{S}_{\rm zp}\mathbf{h} + \mathbf{e}_{\rm zp}. \tag{11}$$

Following the LS criterion, the initial channel estimate using one block of training symbols is given by

$$\hat{\mathbf{h}}_{\rm zp} = \left(\mathbf{S}_{\rm zp}^H\mathbf{S}_{\rm zp}\right)^{-1}\mathbf{S}_{\rm zp}^H\mathbf{y}_{\rm zp}. \tag{12}$$

The channel-estimation error associated with the channel estimate $\hat{\mathbf{h}}_{\rm zp}$ is

$$\Delta\mathbf{h}_{\rm zp} = \hat{\mathbf{h}}_{\rm zp} - \mathbf{h} = \left(\mathbf{S}_{\rm zp}^H\mathbf{S}_{\rm zp}\right)^{-1}\mathbf{S}_{\rm zp}^H\mathbf{e}_{\rm zp}. \tag{13}$$

It follows that $\Delta\mathbf{h}_{\rm zp}$ is zero-mean Gaussian with covariance matrix, i.e.,

$$\begin{aligned}\mathrm{cov}\{\Delta\mathbf{h}_{\rm zp}\} &= \sigma_{\rm zp}^2\left(\mathbf{S}_{\rm zp}^H\mathbf{S}_{\rm zp}\right)^{-1}\\ &= \sigma_{\rm zp}^2\left(P\boldsymbol{\mathcal{F}}^H\mathrm{diag}(\mathbf{s}^H\mathbf{V}^H)\mathrm{diag}(\mathbf{V}\mathbf{s})\boldsymbol{\mathcal{F}}\right)^{-1}\\ &\simeq \sigma_{\rm zp}^2\left(P\boldsymbol{\mathcal{F}}^H\mathrm{diag}\left(\mathrm{diag}(\mathbf{V}\mathbf{V}^H)\right)\boldsymbol{\mathcal{F}}\right)^{-1}\\ &= \frac{\sigma_{\rm zp}^2}{K}\mathbf{I}_{L+1}.\end{aligned} \tag{14}$$

The final equality in (14) was due to

$$\begin{aligned}\mathbf{V}\mathbf{V}^H &= \mathbf{F}_P\mathbf{T}_{\rm zp}\mathbf{T}_{\rm zp}^H\mathbf{F}_P^H\\ &= \mathbf{F}_P\begin{bmatrix}\mathbf{I}_K & \mathbf{0}_{K\times\mu}\\ \mathbf{0}_{\mu\times K} & \mathbf{0}_\mu\end{bmatrix}\mathbf{F}_P^H\\ &= \begin{bmatrix}\frac{K}{P} & \star & \star & \cdots & \star\\ \star & \frac{K}{P} & \star & \cdots & \star\\ \vdots & & \vdots & & \vdots\\ \star & \star & \cdots & \star & \frac{K}{P}\end{bmatrix}_{P\times P}\end{aligned} \tag{15}$$

where $\star$ stands for elements that are irrelevant simply because we are interested in the diagonal only, i.e.,

$$\operatorname{diag}\left(\operatorname{diag}(\mathbf{V}\mathbf{V}^H)\right) = \frac{K}{P}\mathbf{I}_P. \tag{16}$$

Replacing $\operatorname{diag}(\operatorname{diag}(\mathbf{V}\mathbf{V}^H))$ in (14) with $(K/P)\mathbf{I}_P$ leads to

$$\sigma_{\rm zp}^2\left(P\boldsymbol{\mathcal{F}}^H\frac{K}{P}\mathbf{I}_P\boldsymbol{\mathcal{F}}\right)^{-1} = \sigma_{\rm zp}^2(K\boldsymbol{\mathcal{F}}^H\boldsymbol{\mathcal{F}})^{-1} = \frac{\sigma_{\rm zp}^2}{K}\mathbf{I}_{L+1}. \tag{17}$$

Let $\beta_{\rm zp} \triangleq \|\Delta\mathbf{h}_{\rm zp}\|^2$. Clearly, $\beta_{\rm zp}$ has a central $\mathcal{X}^2$ distribution with $2(L+1)$ degrees of freedom, whose mean $\mu_{\beta_{\rm zp}}$ and variance $\sigma^2_{\beta_{\rm zp}}$ are given by

$$\begin{aligned}\mu_{\beta_{\rm zp}} &= E\{\beta_{\rm zp}\} = \sigma_{\rm zp}^2(L+1)/K\\ \sigma^2_{\beta_{\rm zp}} &= \operatorname{var}\{\beta_{\rm zp}\} = \sigma_{\rm zp}^4(L+1)/K^2.\end{aligned} \tag{18}$$

We note that the mean and variance for the MC-ZP channel estimation error is exactly the same as the ones we have in (18).

2) CP Case: Again, we drop the index n for simplicity and rewrite (8) as

$$\mathbf{y}_{\rm cp} = \mathbf{S}_{\rm cp}\mathbf{h} + \mathbf{e}_{\rm cp}. \tag{19}$$

Similar to the ZP case, we use the LS criterion along with one block of training symbols to obtain an initial channel estimate $\hat{\mathbf{h}}_{\rm cp} = (\mathbf{S}_{\rm cp}^H\mathbf{S}_{\rm cp})^{-1}\mathbf{S}_{\rm cp}^H\mathbf{y}_{\rm cp}$. The channel-estimation error associated with the channel estimate $\hat{\mathbf{h}}_{\rm cp}$ is $\Delta\mathbf{h}_{\rm cp} = \hat{\mathbf{h}}_{\rm cp} - \mathbf{h} = (\mathbf{S}_{\rm cp}^H\mathbf{S}_{\rm cp})^{-1}\mathbf{S}_{\rm cp}^H\mathbf{e}_{\rm cp}$. It follows that $\Delta\mathbf{h}_{\rm cp}$ is zero-mean Gaussian with covariance matrix, i.e.,

$$\begin{aligned}\operatorname{cov}\{\Delta\mathbf{h}_{\rm cp}\} &= \sigma_{\rm cp}^2\left(\mathbf{S}_{\rm cp}^H\mathbf{S}_{\rm cp}\right)^{-1}\\ &= \sigma_{\rm cp}^2\left(K\bar{\boldsymbol{\mathcal{F}}}^H\operatorname{diag}\left(\mathbf{s}^H\mathbf{F}_K^H\right)\operatorname{diag}(\mathbf{F}_K\mathbf{s})\bar{\boldsymbol{\mathcal{F}}}\right)^{-1}\\ &= \frac{\sigma_{\rm cp}^2}{K}\mathbf{I}_{L+1}.\end{aligned} \tag{20}$$

Let $\beta_{\rm cp} \triangleq \|\Delta\mathbf{h}_{\rm cp}\|^2$. Then, $\beta_{\rm cp}$ has a central $\mathcal{X}^2$ distribution with $2(L+1)$ degrees of freedom, whose mean $\mu_{\beta_{\rm cp}}$ and variance $\sigma^2_{\beta_{\rm cp}}$ are given by

$$\begin{aligned}\mu_{\beta_{\rm cp}} &= E\{\beta_{\rm cp}\} = \sigma_{\rm cp}^2(L+1)/K\\ \sigma^2_{\beta_{\rm cp}} &= \operatorname{var}\{\beta_{\rm cp}\} = \sigma_{\rm cp}^4(L+1)/K^2.\end{aligned} \tag{21}$$

Similar to the ZP case, the mean and variance for the MC-CP channel estimation error is identical to the results that we have in (21).

B. Covariance Matrix Estimation

The covariance matrix $\mathbf{R}_{\rm zp}$, $\breve{\mathbf{R}}_{\rm zp}$, $\mathbf{R}_{\rm cp}$, or $\breve{\mathbf{R}}_{\rm cp}$ of the received signal $\mathbf{y}_{\rm zp}(n)$, $\breve{\mathbf{y}}_{\rm zp}(n)$, $\mathbf{y}_{\rm cp}(n)$, or $\breve{\mathbf{y}}_{\rm zp}(n)$, respectively, is not available in reality and has to be replaced by some covariance matrix estimate, e.g., the sample covariance matrix $\hat{\mathbf{R}} = M^{-1}\sum_{n=1}^{M}\mathbf{y}(n)\mathbf{y}^H(n)$ obtained by using M blocks of received signals, where $\mathbf{y}(n)$ denotes the received signal corresponding to any of the four different transmissions. As mentioned earlier, we analyze the covariance matrix-estimation error for the SC-ZP and SC-CP, while the other two schemes follow the same analysis, which leads to identical results.

1) ZP Case: Let $\gamma_{\rm zp} \triangleq \|\hat{\mathbf{R}}_{\rm zp} - \mathbf{R}_{\rm zp}\|^2$ be the estimation error, and let $\hat{R}_{\rm zp}(i,j)$ and $R_{\rm zp}(i,j)$ be the (i,j)th element of $\hat{\mathbf{R}}_{\rm zp}$ and $\mathbf{R}_{\rm zp}$, respectively. Then

$$\gamma_{\rm zp} = \sum_{i=1}^{P}\sum_{j=1}^{P}\left[\hat{R}_{\rm zp}(i,j) - R_{\rm zp}(i,j)\right]\left[\hat{R}_{\rm zp}(i,j) - R_{\rm zp}(i,j)\right]^* \tag{22}$$

is a sum of P^2 random variables. As shown in the Appendix, the mean and variance of $\gamma_{\rm zp}$ are given by

$$\begin{aligned}\mu_{\gamma_{\rm zp}} &= E\{\gamma_{\rm zp}\} = \frac{1}{M}\operatorname{tr}^2\{\mathbf{R}_{\rm zp}\}\\ \sigma^2_{\gamma_{\rm zp}} &= \operatorname{var}\{\gamma_{\rm zp}\} = \frac{2}{M^2}\|\mathbf{R}_{\rm zp}\|^4.\end{aligned} \tag{23}$$

2) CP Case: Similar to ZP, we define $\gamma_{\rm cp} \triangleq \|\hat{\mathbf{R}}_{\rm cp} - \mathbf{R}_{\rm cp}\|^2$ to be the estimation error and $\hat{R}_{\rm cp}(i,j)$ and $R_{\rm cp}(i,j)$ the (i,j)th element of $\hat{\mathbf{R}}_{\rm cp}$ and $\mathbf{R}_{\rm cp}$, respectively. Then

$$\gamma_{\rm cp} = \sum_{i=1}^{K}\sum_{j=1}^{K}\left[\hat{R}_{\rm cp}(i,j) - R_{\rm cp}(i,j)\right]\left[\hat{R}_{\rm cp}(i,j) - R_{\rm cp}(i,j)\right]^* \tag{24}$$

is a sum of K^2 random variables, whose mean and variance are given by

$$\begin{aligned}\mu_{\gamma_{\rm cp}} &= E\{\gamma_{\rm cp}\} = \frac{1}{M}\operatorname{tr}^2\{\mathbf{R}_{\rm cp}\}\\ \sigma^2_{\gamma_{\rm cp}} &= \operatorname{var}\{\gamma_{\rm cp}\} = \frac{2}{M^2}\|\mathbf{R}_{\rm cp}\|^4.\end{aligned} \tag{25}$$

IV. Proposed Robust Schemes

In the following, we will develop robust estimation and detection schemes by taking into account the estimation error in $\hat{\mathbf{h}}$ and $\hat{\mathbf{R}}$, where $\hat{\mathbf{h}}$ and $\hat{\mathbf{R}}$ represent the channel and covariance matrix estimates, respectively, for the four different block transmissions. Our schemes optimize the worst-case performance over two bounded sets of the above estimation errors. In the following, we first discuss how to bound the channel-estimation error and covariance matrix-estimation error. Then, we present the robust channel estimation and detection schemes.

A. Bounding Channel-Estimation Error

Herein, we use the Chebyshev inequality to determine the size of the uncertainty set. One reason for using the Chebyshev inequality rather than an exact expression is that the former involves only the first- and second-order statistics of the estimation error and, as such, is much easier to compute. Exact expressions may require computationally intensive numerical evaluation and sometimes may be unavailable (e.g., the distribution of the covariance matrix-estimation error γ is unknown). Not only is our Chebyshev bounding approach easy to use, but it also leads to robust detection and estimation performance, as shown by numerical simulation in Section V.

1) ZP Case: We consider here the ZP case, which is identical for both SC-ZP and MC-ZP. Since $\beta_{\rm zp} \triangleq \|\Delta\mathbf{h}_{\rm zp}\|^2$ has a central $\mathcal{X}^2$ distribution and is therefore unbounded, we use the Chebyshev inequality, where the unbounded channel estimation error is bounded in the following probability:

$$P_{\beta_{\rm zp}}\left(\left|\beta_{\rm zp} - \mu_{\beta_{\rm zp}}\right| > \delta_{\beta_{\rm zp}}\right) \leq \frac{\sigma^2_{\beta_{\rm zp}}}{\delta^2_{\beta_{\rm zp}}} \tag{26}$$

for any positive number $\delta_{\beta_{\rm zp}}$. For large $\delta_{\beta_{\rm zp}}$, we ignore the unmodeled channel estimation error (which is a small probability event) and consider a bounded set

$$P_{\beta_{\rm zp}}(\beta_{\rm zp} \leq \mu_{\beta_{\rm zp}} + \delta_{\beta_{\rm zp}}) \geq 1 - \frac{\sigma^2_{\beta_{\rm zp}}}{\delta^2_{\beta_{\rm zp}}}. \tag{27}$$

Let

$$\epsilon_{\rm zp} \triangleq \mu_{\beta_{\rm zp}} + \delta_{\beta_{\rm zp}} \tag{28}$$

denote a boundary of $\beta_{\rm zp}$. Then

$$P_{\beta_{\rm zp}}\left(\|\Delta\mathbf{h}_{\rm zp}\|^2 \leq \epsilon_{\rm zp}\right) \geq 1 - \frac{\sigma^2_{\beta_{\rm zp}}}{\delta^2_{\beta_{\rm zp}}} \tag{29}$$

where $P_{\beta_{\rm zp}}(\|\Delta\mathbf{h}_{\rm zp}\|^2 \leq \epsilon_{\rm zp})$ is a *Chebyshev bounding probability*. Although we can choose a sufficiently large $\epsilon_{\rm zp}$ to make the Chebyshev bounding probability approach 1, it is not worth seeking robustness when the channel estimation error is very large. Hence, our strategy is to improve detection robustness against small to moderate channel-estimation errors. For a given probability $P_{\beta_{\rm zp}}$, the boundary $\epsilon_{\rm zp}$ can be determined by setting the two sides of (29) equal to each other and replacing $\delta_{\beta_{\rm zp}}$ from (28), which gives

$$\epsilon_{\rm zp} = \mu_{\beta_{\rm zp}} + \sqrt{\frac{\sigma^2_{\beta_{\rm zp}}}{1 - P_{\beta_{\rm zp}}}} = \frac{\sigma^2_{\rm zp}}{K}\left(L + 1 + \sqrt{\frac{L+1}{1 - P_{\beta_{\rm zp}}}}\right). \tag{30}$$

2) CP Case: Following the same steps as in the ZP case, the boundary $\epsilon_{\rm cp}$ for both SC-CP and MC-CP is given by

$$\epsilon_{\rm cp} = \mu_{\beta_{\rm cp}} + \sqrt{\frac{\sigma^2_{\beta_{\rm cp}}}{1 - P_{\beta_{\rm cp}}}} = \frac{\sigma^2_{\rm cp}}{K}\left(L + 1 + \sqrt{\frac{L+1}{1 - P_{\beta_{\rm cp}}}}\right). \tag{31}$$

B. Bounding Covariance Matrix-Estimation Error

1) ZP Case: Similar to the channel-estimation error, we use the Chebyshev inequality to bound the covariance matrix-estimation error $\gamma_{\rm zp} \triangleq \|\hat{\mathbf{R}}_{\rm zp} - \mathbf{R}_{\rm zp}\|^2$ for the SC-ZP. That is

$$P_{\gamma_{\rm zp}}\left(\left|\gamma_{\rm zp} - \mu_{\gamma_{\rm zp}}\right| > \delta_{\gamma_{\rm zp}}\right) \leq \frac{\sigma^2_{\gamma_{\rm zp}}}{\delta^2_{\gamma_{\rm zp}}} \tag{32}$$

for any positive number $\delta_{\gamma_{\rm zp}}$. Again, for large $\delta_{\gamma_{\rm zp}}$, we ignore the unmodeled covariance matrix-estimation error and consider a bounded set

$$P_{\gamma_{\rm zp}}\left(\gamma_{\rm zp} \leq \mu_{\gamma_{\rm zp}} + \delta_{\gamma_{\rm zp}}\right) \geq 1 - \frac{\sigma^2_{\gamma_{\rm zp}}}{\delta^2_{\gamma_{\rm zp}}}. \tag{33}$$

For convenience, we consider the estimation error $\sqrt{\gamma}_{\rm zp}$ instead of $\gamma_{\rm zp}$. Let $\eta_{\rm zp} \triangleq \sqrt{\mu_{\gamma_{\rm zp}} + \delta_{\gamma_{\rm zp}}}$ denote the boundary for $\gamma_{\rm zp}$. Then

$$P_{\gamma_{\rm zp}}\left(\|\hat{\mathbf{R}}_{\rm zp} - \mathbf{R}_{\rm zp}\|^2 \leq \eta_{\rm zp}\right) \geq 1 - \frac{\sigma^2_{\gamma_{\rm zp}}}{\delta^2_{\gamma_{\rm zp}}}. \tag{34}$$

For a given Chebyshev bounding probability $P_{\gamma_{\rm zp}}$, we determine $\eta_{\rm zp}$ by setting the two sides of (34) equal to each other. Then, we have

$$\begin{aligned}\eta_{\rm zp} &= \sqrt{\mu_{\gamma_{\rm zp}} + \sqrt{\frac{\sigma^2_{\gamma_{\rm zp}}}{1 - P_{\gamma_{\rm zp}}}}} \\ &= \frac{1}{\sqrt{M}}\sqrt{\operatorname{tr}^2\{\mathbf{R}_{\rm zp}\} + \frac{\sqrt{2}\|\mathbf{R}_{\rm zp}\|^2}{\sqrt{1 - P_{\gamma_{\rm zp}}}}}.\end{aligned} \tag{35}$$

For the MC-ZP case, we replace $\mathbf{R}_{\rm zp}$ with $\breve{\mathbf{R}}_{\rm zp}$ in (35).

2) CP Case: The boundary for the covariance matrix-estimation error $\eta_{\rm cp}$ for the SC-CP can be derived exactly as in the ZP case, and it is given by

$$\begin{aligned}\eta_{\rm cp} &= \sqrt{\mu_{\gamma_{\rm cp}} + \sqrt{\frac{\sigma^2_{\gamma_{\rm cp}}}{1 - P_{\gamma_{\rm cp}}}}} \\ &= \frac{1}{\sqrt{M}}\sqrt{\operatorname{tr}^2\{\mathbf{R}_{\rm cp}\} + \frac{\sqrt{2}\|\mathbf{R}_{\rm cp}\|^2}{\sqrt{1 - P_{\gamma_{\rm cp}}}}}.\end{aligned} \tag{36}$$

For the MC-CP case, we replace $\mathbf{R}_{\rm cp}$ with $\breve{\mathbf{R}}_{\rm cp}$ in (36).

C. Robust Channel Estimation and Detection

1) Channel Estimation and Detection for SC-ZP: From (2) and following the minimum variance (MV) criterion (e.g., [32] and [33]), we can determine a linear detector (expressed as a $P \times P$ matrix) as follows:

$$\begin{aligned}\mathbf{W}_{\text{zp-MV}} &= \arg\min_{\mathbf{W}_{\rm zp}\in\mathbb{C}^{P\times P}} \operatorname{tr}\left\{\mathbf{W}^H_{\rm zp}\mathbf{R}_{\rm zp}\mathbf{W}_{\rm zp}\right\} \\ &\text{subject to} \qquad \mathbf{W}^H_{\rm zp}\mathbf{X}_{\rm zp}(\mathbf{h}) = \mathbf{I}_P\end{aligned} \tag{37}$$

where $\mathbf{R}_{\rm zp}$ denotes the covariance matrix, and the constraint $\mathbf{W}^H_{\rm zp}\mathbf{X}_{\rm zp}(\mathbf{h}) = \mathbf{I}_P$ ensures that each column of $\mathbf{W}_{\rm zp}$ will pass only one signal component [corresponding to one column of $\mathbf{X}_{\rm zp}(\mathbf{h})$] undistorted with unit gain, while completely eliminating the intersymbol interference. Meanwhile, minimizing the total output variance as in the cost function of (37) is intended to minimize the unmodeled interference that may be present in the received signal. Using the Lagrange multiplier, the solution to the above constrained quadratic minimization problem is given by (see also [34, p. 283])

$$\mathbf{W}_{\text{zp-MV}} = \mathbf{R}^{-1}_{\rm zp}\mathbf{X}_{\rm zp}(\mathbf{h})\left(\mathbf{X}^H_{\rm zp}(\mathbf{h})\mathbf{R}^{-1}_{\rm zp}\mathbf{X}_{\rm zp}(\mathbf{h})\right)^{-1}. \tag{38}$$

Substituting (38) into (37), the minimized average output power of $\mathbf{W}_{\text{zp-MV}}$ is given by

$$\mathcal{V}_{\rm zp1}(\mathbf{h}) = \operatorname{tr}\left\{\left[\mathbf{X}^H_{\rm zp}(\mathbf{h})\mathbf{R}^{-1}_{\rm zp}\mathbf{X}_{\rm zp}(\mathbf{h})\right]^{-1}\right\}. \tag{39}$$

Since the MV detector is sensitive to signal mismatch due to errors in some initial channel and covariance matrix estimates $\hat{\mathbf{h}}_{\rm zp}$ and $\hat{\mathbf{R}}_{\rm zp}$, we consider robust channel estimation by maximizing the output $\mathcal{V}_{\rm zp1}(\mathbf{h})$ so that $\mathbf{W}_{\rm zp\text{-}MV}$ will maximally preserve the signal power. Because of the nonlinear nature of $\mathcal{V}_{\rm zp1}(\mathbf{h})$, this approach is computationally involved and suffers local convergence. Instead, we maximize an asymptotic lower bound of $\mathcal{V}_{\rm zp1}(\mathbf{h})$. Using the Schwartz inequality, we have

$$\begin{aligned} K^2 &= \mathrm{tr}^2(\mathbf{I}_K) \\ &= \mathrm{tr}^2\left\{\left(\mathbf{X}_{\rm zp}^H(\mathbf{h})\mathbf{R}_{\rm zp}^{-1}\mathbf{X}_{\rm zp}(\mathbf{h})\right)^{-\frac{1}{2}}\left(\mathbf{X}_{\rm zp}^H(\mathbf{h})\mathbf{R}_{\rm zp}^{-1}\mathbf{X}_{\rm zp}(\mathbf{h})\right)^{\frac{1}{2}}\right\} \\ &\le \mathrm{tr}\left\{\left(\mathbf{X}_{\rm zp}^H(\mathbf{h})\mathbf{R}_{\rm zp}^{-1}\mathbf{X}_{\rm zp}(\mathbf{h})\right)^{-1}\right\}\mathrm{tr}\left\{\mathbf{X}_{\rm zp}^H(\mathbf{h})\mathbf{R}_{\rm zp}^{-1}\mathbf{X}_{\rm zp}(\mathbf{h})\right\} \end{aligned} \tag{40}$$

where the equality is achieved asymptotically [33], i.e., if and only if $\mathbf{X}_{\rm zp}^H(\mathbf{h})\mathbf{R}_{\rm zp}^{-1}\mathbf{X}_{\rm zp}(\mathbf{h})$ is a (scaled) identity. It follows that maximizing $\mathcal{V}_{\rm zp1}(\mathbf{h})$ is equivalent to minimizing $\mathrm{tr}\{\mathbf{X}_{\rm zp}^H(\mathbf{h})\mathbf{R}_{\rm zp}^{-1}\mathbf{X}_{\rm zp}(\mathbf{h})\}$. The robust channel estimation can now be obtained using the following constrained optimization:

$$\begin{aligned} &\min_{\mathbf{h},\mathbf{R}_{\rm zp}} \mathrm{tr}\left\{\mathbf{X}_{\rm zp}^H(\mathbf{h})\mathbf{R}_{\rm zp}^{-1}\mathbf{X}_{\rm zp}(\mathbf{h})\right\} \\ &\text{s.t.}\quad \|\hat{\mathbf{h}}_{\rm zp}-\mathbf{h}\|^2 \le \epsilon_{\rm zp} \text{ and } \|\hat{\mathbf{R}}_{\rm zp}-\mathbf{R}_{\rm zp}\| \le \eta_{\rm zp}. \end{aligned} \tag{41}$$

Here, we seek to optimize the cost function over two spherical constrained sets centered on the initial estimates with radius determined by their statistical uncertainty. Our updated estimates optimize the best worst-case performance over the two uncertainty sets. To proceed, we first simplify the cost function in (41) as follows:

$$\begin{aligned} \mathrm{tr}\left\{\mathbf{X}_{\rm zp}^H(\mathbf{h})\mathbf{R}_{\rm zp}^{-1}\mathbf{X}_{\rm zp}(\mathbf{h})\right\} &= \mathrm{tr}\left\{\mathbf{V}^H\mathbf{Q}^H(\mathbf{h})\mathbf{R}_{\rm zp}^{-1}\mathbf{Q}(\mathbf{h})\mathbf{V}\right\} \\ &= \sum_{k=1}^{K}\mathbf{x}_k^H\mathbf{R}_{\rm zp}^{-1}\mathbf{x}_k \end{aligned} \tag{42}$$

where $\mathbf{X}_{\rm zp}(\mathbf{h}) = \mathbf{Q}(\mathbf{h})\mathbf{V} \triangleq [\mathbf{x}_1,\ldots,\mathbf{x}_k]$. Due to the nonlinearity introduced by the matrix inverse $\mathbf{R}_{\rm zp}^{-1}$, we will replace the cost function by using the following upperbound:

$$\min_{\|\hat{\mathbf{R}}_{\rm zp}-\mathbf{R}_{\rm zp}\|\le\eta_{\rm zp}} \sum_{k=1}^{K}\mathbf{x}_k^H\mathbf{R}_{\rm zp}^{-1}\mathbf{x}_k \le \sum_{k=1}^{K}\mathbf{x}_k^H\left(\hat{\mathbf{R}}_{\rm zp}+\eta_{\rm zp}\mathbf{I}\right)^{-1}\mathbf{x}_k. \tag{43}$$

To see this, let $\mathbf{R}_{\rm zp} = \hat{\mathbf{R}}_{\rm zp} + \mathbf{\Delta}_{\rm zp}$, where the covariance error is a Hermitian matrix with $\|\mathbf{\Delta}_{\rm zp}\| \le \eta_{\rm zp}$. Let $\lambda_p, p = 1,\ldots,P$ denote the eigenvalues of $\mathbf{\Delta}_{\rm zp}$. The norm constraint implies that

$$\sum_{p=1}^{P}|\lambda_p|^2 \le \eta_{\rm zp}^2, \quad |\lambda_{\max}| \le \eta_{\rm zp} \tag{44}$$

where $\lambda_{\max} = \max_p|\lambda_p|$. Hence, $\eta_{\rm zp}\mathbf{I} \ge \mathbf{\Delta}_{\rm zp}$, or equivalently, $\hat{\mathbf{R}}_{\rm zp} + \eta_{\rm zp}\mathbf{I} \ge \hat{\mathbf{R}}_{\rm zp} + \Delta_{\rm zp} = \mathbf{R}_{\rm zp}$. It follows that $\mathbf{R}_{\rm zp}^{-1} \ge (\hat{\mathbf{R}}_{\rm zp}+\eta_{\rm zp}\mathbf{I})^{-1}$, which means that $\mathbf{R}_{\rm zp}^{-1} - (\hat{\mathbf{R}}_{\rm zp}+\eta_{\rm zp}\mathbf{I})^{-1}$ is positive semidefinite, and (43) follows.

We now consider minimizing the above upperbound with respect to the uncertainty set due to the channel-estimation error. Note that the $P\times 1$ vectors $\mathbf{x}_k$ introduced earlier is a function of the channel

$$\mathbf{x}_k = \mathbf{Q}(\mathbf{h})\mathbf{v}_k = \sqrt{P}\,\mathrm{diag}(\boldsymbol{\mathcal{F}}\mathbf{h})\mathbf{v}_k = \sqrt{P}\,\mathrm{diag}(\mathbf{v}_k)\boldsymbol{\mathcal{F}}\mathbf{h} \tag{45}$$

where $\mathbf{v}_k$ is the kth column of the $P\times K$ matrix $\mathbf{V}$. By substituting (45) into the right-hand side of (43), we get the following optimization problem seeking an updated channel estimate:

$$\tilde{\mathbf{h}}_{\rm zp} = \arg\min_{\|\hat{\mathbf{h}}_{\rm zp}-\mathbf{h}\|^2\le\epsilon_{\rm zp}} \mathbf{h}^H\mathbf{\Phi}_{\rm zp}\mathbf{h} \tag{46}$$

where $\mathbf{\Phi}_{\rm zp}=P\sum_{k=1}^{K}\boldsymbol{\mathcal{F}}^H\mathrm{diag}(\mathbf{v}_k)^H(\hat{\mathbf{R}}_{\rm zp}+\eta_{\rm zp}\mathbf{I})^{-1}\mathrm{diag}(\mathbf{v}_k)\boldsymbol{\mathcal{F}}$. Since the solution of (46) will evidently occur on the boundary of the uncertainty set (i.e., the worst case) [35], we have

$$\begin{aligned} \tilde{\mathbf{h}}_{\rm zp} &= \arg\min_{\mathbf{h}} \mathbf{h}^H\mathbf{\Phi}_{\rm zp}\mathbf{h} \\ &\text{subject to}\quad \|\hat{\mathbf{h}}_{\rm zp}-\mathbf{h}\|^2 = \epsilon_{\rm zp} \end{aligned} \tag{47}$$

where the inequality constraint has been replaced by a quadratic equality constraint. The problem in (47) can be solved by using the Lagrange multiplier in a manner similar to [25]. Specifically, let

$$\mathcal{V}_{\rm zp2}(\mathbf{h},\xi_{\rm zp}) = \mathbf{h}^H\mathbf{\Phi}_{\rm zp}\mathbf{h} + \xi_{\rm zp}\left[(\hat{\mathbf{h}}_{\rm zp}-\mathbf{h})^H(\hat{\mathbf{h}}_{\rm zp}-\mathbf{h}) - \epsilon_{\rm zp}\right]. \tag{48}$$

Then, the new channel estimate is obtained by taking the partial derivative and setting it to zero, i.e.,

$$\frac{\partial\mathcal{V}_{\rm zp2}(\mathbf{h},\xi_{\rm zp})}{\partial\mathbf{h}} = \mathbf{\Phi}_{\rm zp}\tilde{\mathbf{h}}_{\rm zp} + \xi_{\rm zp}(\tilde{\mathbf{h}}_{\rm zp}-\hat{\mathbf{h}}_{\rm zp}) = 0 \tag{49}$$

which yields

$$\tilde{\mathbf{h}}_{\rm zp} = \xi_{\rm zp}(\mathbf{\Phi}_{\rm zp}+\xi_{\rm zp}\mathbf{I}_P)^{-1}\hat{\mathbf{h}}_{\rm zp} = \hat{\mathbf{h}}_{\rm zp} - \left(\mathbf{I}_{(L+1)}+\xi_{\rm zp}\mathbf{\Phi}_{\rm zp}^{-1}\right)^{-1}\hat{\mathbf{h}}_{\rm zp}. \tag{50}$$

The Lagrange multiplier $\xi_{\rm zp}$ can be determined by setting

$$\mathcal{V}_{\rm zp3}(\xi_{\rm zp}) \triangleq \|\tilde{\mathbf{h}}_{\rm zp}-\hat{\mathbf{h}}_{\rm zp}\|^2 = \left\|\left(\mathbf{I}_{(L+1)}+\xi_{\rm zp}\mathbf{\Phi}_{\rm zp}^{-1}\right)^{-1}\hat{\mathbf{h}}_{\rm zp}\right\|^2 = \epsilon_{\rm zp}. \tag{51}$$

Let the eigenvalue decomposition of $\mathbf{\Phi}_{\rm zp}^{-1}$ be $\mathbf{\Phi}_{\rm zp}^{-1} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$, $\boldsymbol{\alpha} \triangleq \mathbf{U}^H\hat{\mathbf{h}}_{\rm zp} = [\alpha_1,\ldots,\alpha_{L+1}]^T$, and $\mathbf{\Lambda} = \mathrm{diag}[\lambda_1,\ldots,\lambda_{L+1}]$ with $\lambda_1 \ge \cdots \ge \lambda_{L+1}$. Then, $\mathcal{V}_{\rm zp3}(\xi_{\rm zp})$ can be rewritten as

$$\mathcal{V}_{\rm zp3}(\xi_{\rm zp}) = \sum_{l=1}^{L+1}\frac{|\alpha_l|^2}{[1+\xi_{\rm zp}\lambda_l]^2} = \epsilon_{\rm zp}. \tag{52}$$

Since $\mathcal{V}_{\rm zp3}(\xi_{\rm zp})$ is monotonically decreasing, we can determine a unique solution lying in the upper and lower bounds given by (cf. [25])

$$\begin{aligned} \frac{\|\hat{\mathbf{h}}_{\rm zp}\|-\sqrt{\epsilon_{\rm zp}}}{\lambda_1\sqrt{\epsilon_{\rm zp}}} &\le \xi_{\rm zp} \\ &\le \min\left\{\left[\epsilon_{\rm zp}^{-1}\sum_{l=1}^{L+1}\frac{|\alpha_l|^2}{\lambda_l^2}\right]^{1/2}, \frac{\|\hat{\mathbf{h}}_{\rm zp}\|-\sqrt{\epsilon_{\rm zp}}}{\lambda_{L+1}\sqrt{\epsilon_{\rm zp}}}\right\}. \end{aligned} \tag{53}$$

Once $\xi_{\rm zp}$ is obtained, our robust channel estimate is given by

$$\tilde{\mathbf{h}}_{\rm zp} = \left(\xi_{\rm zp}^{-1}\mathbf{\Phi}_{\rm zp} + \mathbf{I}_{L+1}\right)^{-1}\hat{\mathbf{h}}_{\rm zp}. \tag{54}$$

With the knowledge of $\tilde{\mathbf{h}}_{\rm zp}$, $\mathbf{q}$ can be updated as $\tilde{\mathbf{q}} = \sqrt{P}\boldsymbol{\mathcal{F}}\tilde{\mathbf{h}}_{\rm zp}$, and $\tilde{\mathbf{X}}_{\rm zp}(\mathbf{h}) = \mathrm{diag}(\tilde{\mathbf{q}})\mathbf{V}$. The robust MV detector is given by

$$\begin{aligned}\tilde{\mathbf{W}}_{\text{zp-RMV}} = &(\hat{\mathbf{R}}_{\rm zp} + \eta_{\rm zp}\mathbf{I}_P)^{-1}\tilde{\mathbf{X}}_{\rm zp}(\mathbf{h})\\ &\times\left(\tilde{\mathbf{X}}_{\rm zp}^H(\mathbf{h})(\hat{\mathbf{R}}_{\rm zp} + \eta_{\rm zp}\mathbf{I}_P)^{-1}\tilde{\mathbf{X}}_{\rm zp}(\mathbf{h})\right)^{-1}.\end{aligned} \tag{55}$$

We briefly discuss the complexity of the proposed scheme versus that of the conventional MV detector. Note that the conventional MV detector uses similar matrix inversions. The additional Lagrangian optimization requires a 1-D search. With the bounds derived for the 1-D search, the associated complexity is small compared with the matrix operation. Overall, we found that the additional complexity of our robust schemes over the conventional methods is modest.

2) Channel Estimation and Detection for MC-ZP: As in the SC case, we use the MV criterion to determine a linear detector

$$\begin{aligned}\breve{\mathbf{W}}_{\text{zp-MV}} = \arg &\min_{\breve{\mathbf{W}}_{\rm zp}\in\mathbb{C}^{P\times P}} \mathrm{tr}\left\{\breve{\mathbf{W}}_{\rm zp}^H\breve{\mathbf{R}}_{\rm zp}\breve{\mathbf{W}}_{\rm zp}\right\}\\ \text{subject to} \quad &\breve{\mathbf{W}}_{\rm zp}^H\breve{\mathbf{X}}_{\rm zp}(\mathbf{h}) = \mathbf{I}_P.\end{aligned} \tag{56}$$

The solution to the constrained problem above is given by

$$\breve{\mathbf{W}}_{\text{zp-MV}} = \breve{\mathbf{R}}_{\rm zp}^{-1}\breve{\mathbf{X}}_{\rm zp}(\mathbf{h})\left(\breve{\mathbf{X}}_{\rm zp}^H(\mathbf{h})\breve{\mathbf{R}}_{\rm zp}^{-1}\breve{\mathbf{X}}_{\rm zp}(\mathbf{h})\right)^{-1}. \tag{57}$$

Following the same steps as was done in the SC case, the robust channel estimate for the MC-ZP is given by

$$\tilde{\breve{\mathbf{h}}}_{\rm zp} = \left(\breve{\xi}_{\rm zp}^{-1}\breve{\mathbf{\Phi}}_{\rm zp} + \mathbf{I}_{L+1}\right)^{-1}\hat{\breve{\mathbf{h}}}_{\rm zp} \tag{58}$$

where $\breve{\mathbf{\Phi}}_{\rm zp} \triangleq P\sum_{k=1}^K \boldsymbol{\mathcal{F}}^H\mathrm{diag}(\mathbf{v}_k)^H(\hat{\breve{\mathbf{R}}}_{\rm zp} + \breve{\eta}_{\rm zp}\mathbf{I})^{-1}\mathrm{diag}(\mathbf{v}_k)\boldsymbol{\mathcal{F}}$, $\breve{\xi}_{\rm zp}$ is calculated as in (53), and $\breve{\eta}_{\rm zp}$ is calculated from (35) after replacing $\mathbf{R}_{\rm zp}$ with $\breve{\mathbf{R}}_{\rm zp}$.

With the knowledge of $\tilde{\breve{\mathbf{h}}}_{\rm zp}$, we update $\tilde{\breve{\mathbf{q}}} = \sqrt{P}\boldsymbol{\mathcal{F}}\tilde{\breve{\mathbf{h}}}_{\rm zp}$, and $\tilde{\breve{\mathbf{X}}}_{\rm zp}(\mathbf{h}) = \mathrm{diag}(\tilde{\breve{\mathbf{q}}})\mathbf{V}\mathbf{F}_K^H$. It follows that the robust MV detector is given by

$$\begin{aligned}\tilde{\breve{\mathbf{W}}}_{\text{zp-RMV}} = &(\hat{\breve{\mathbf{R}}}_{\rm zp} + \breve{\eta}_{\rm zp}\mathbf{I}_P)^{-1}\tilde{\breve{\mathbf{X}}}_{\rm zp}(\mathbf{h})\\ &\times\left(\tilde{\breve{\mathbf{X}}}_{\rm zp}^H(\mathbf{h})(\hat{\breve{\mathbf{R}}}_{\rm zp} + \breve{\eta}_{\rm zp}\mathbf{I}_P)^{-1}\tilde{\breve{\mathbf{X}}}_{\rm zp}(\mathbf{h})\right)^{-1}.\end{aligned} \tag{59}$$

3) Channel Estimation and Detection for SC-CP: The following equations are obtained in a similar fashion as we did for the SC-ZP case. The robust channel estimate is given by

$$\tilde{\mathbf{h}}_{\rm cp} = \left(\xi_{\rm cp}^{-1}\mathbf{\Phi}_{\rm cp} + \mathbf{I}_{L+1}\right)^{-1}\hat{\mathbf{h}}_{\rm cp} \tag{60}$$

where $\mathbf{\Phi}_{\rm cp} \triangleq K\sum_{k=1}^K \mathbf{I}^H(:,k)\otimes\bar{\boldsymbol{\mathcal{F}}}_k^H(\hat{\mathbf{R}}_{\rm cp} + \eta_{\rm cp}\mathbf{I})^{-1}\bar{\boldsymbol{\mathcal{F}}}_k\otimes \mathbf{I}(:,k)$, in which $\bar{\boldsymbol{\mathcal{F}}}_k$ stands for the kth row of the matrix $\bar{\boldsymbol{\mathcal{F}}}$, the Matlab notation $\mathbf{I}(:,k)$ stands for the kth column of the identity matrix $\mathbf{I}_K$, and $\xi_{\rm cp}$ is calculated as in (53). With the knowledge of $\tilde{\mathbf{h}}_{\rm cp}$, we update $\tilde{\mathbf{g}} = \sqrt{K}\bar{\boldsymbol{\mathcal{F}}}\tilde{\mathbf{h}}_{\rm cp}$, and $\tilde{\mathbf{X}}_{\rm cp}(\mathbf{h}) = \mathrm{diag}(\tilde{\mathbf{g}})\mathbf{F}_K$. It follows that the robust MV detector is given by

$$\begin{aligned}\tilde{\mathbf{W}}_{\text{cp-RMV}} = &(\hat{\mathbf{R}}_{\rm cp} + \eta_{\rm cp}\mathbf{I}_K)^{-1}\tilde{\mathbf{X}}_{\rm cp}(\mathbf{h})\\ &\times\left(\tilde{\mathbf{X}}_{\rm cp}^H(\mathbf{h})(\hat{\mathbf{R}}_{\rm cp} + \eta_{\rm cp}\mathbf{I}_K)^{-1}\tilde{\mathbf{X}}_{\rm cp}(\mathbf{h})\right)^{-1}.\end{aligned} \tag{61}$$

4) Channel Estimation and Detection for MC-CP: Following similar steps as in the SC-ZP case, the robust channel estimate is given by

$$\tilde{\breve{\mathbf{h}}}_{\rm cp} = \left(\breve{\xi}_{\rm cp}^{-1}\breve{\mathbf{\Phi}}_{\rm cp} + \mathbf{I}_{L+1}\right)^{-1}\hat{\breve{\mathbf{h}}}_{\rm cp} \tag{62}$$

where $\breve{\mathbf{\Phi}}_{\rm cp} \triangleq K\sum_{k=1}^K \mathbf{I}^H(:,k)\otimes\bar{\boldsymbol{\mathcal{F}}}_k^H(\hat{\breve{\mathbf{R}}}_{\rm cp} + \breve{\eta}_{\rm cp}\mathbf{I})^{-1}\bar{\boldsymbol{\mathcal{F}}}_k\otimes \mathbf{I}(:,k)$, $\breve{\xi}_{\rm cp}$ is calculated as in (53), and $\breve{\eta}_{\rm cp}$ is calculated from (36) after replacing $\mathbf{R}_{\rm cp}$ with $\breve{\mathbf{R}}_{\rm cp}$.

Now that $\tilde{\breve{\mathbf{h}}}_{\rm cp}$ is known, we update $\tilde{\breve{\mathbf{g}}} = \sqrt{K}\bar{\boldsymbol{\mathcal{F}}}\tilde{\breve{\mathbf{h}}}_{\rm cp}$, and $\tilde{\breve{\mathbf{X}}}_{\rm cp}(\mathbf{h}) = \mathrm{diag}(\tilde{\breve{\mathbf{g}}})$. It follows that the robust MV detector is given by

$$\begin{aligned}\tilde{\breve{\mathbf{W}}}_{\text{cp-RMV}} = &(\hat{\breve{\mathbf{R}}}_{\rm cp} + \breve{\eta}_{\rm cp}\mathbf{I}_K)^{-1}\tilde{\breve{\mathbf{X}}}_{\rm cp}(\mathbf{h})\\ &\times\left(\tilde{\breve{\mathbf{X}}}_{\rm cp}^H(\mathbf{h})(\hat{\breve{\mathbf{R}}}_{\rm cp} + \breve{\eta}_{\rm cp}\mathbf{I}_K)^{-1}\tilde{\breve{\mathbf{X}}}_{\rm cp}(\mathbf{h})\right)^{-1}.\end{aligned} \tag{63}$$

V. Numerical Results

The systems under study utilize a binary phase-shift keying modulation, with number of symbols per block $K = 48$, $\mu = 16$, block size $P = 64$, and a four-tap $(L = 3)$ FIR Rayleigh-fading channel. We compare here the proposed robust MV detectors with the standard MV detectors that ignore the estimation errors in the prior estimate of the channel. For all examples, results are obtained over 200 realizations.

We first examine the impact of the size of the uncertainty set pertaining to the initial channel estimate when SNR $= 10$ dB. Fig. 1 depicts the receiver output signal-to-interference-plus-noise ratio (SINR) as a function of the normalized channel uncertainty $\epsilon/E\{\|\mathbf{h}\|^2\}$ for (left) SC with ZP and CP and (right) MC with ZP and CP when the variance σ^2 of the initial channel-estimation error is fixed. Since the conventional detectors ignore the prior estimation error, they are independent of ϵ. While the robust detectors require a choice of ϵ, they are insensitive to the choice. Compared with the standard detectors, the robust detectors show a notable improvement in SINR for both ZP and CP. It is seen that the performance of SC and MC are identical for the proposed as well as for the standard schemes.

To investigate the impact of the size η of the uncertainty set related to the initial sample covariance matrix estimate, we use the same set of simulation parameters as in the previous example and vary the size of the uncertainty set

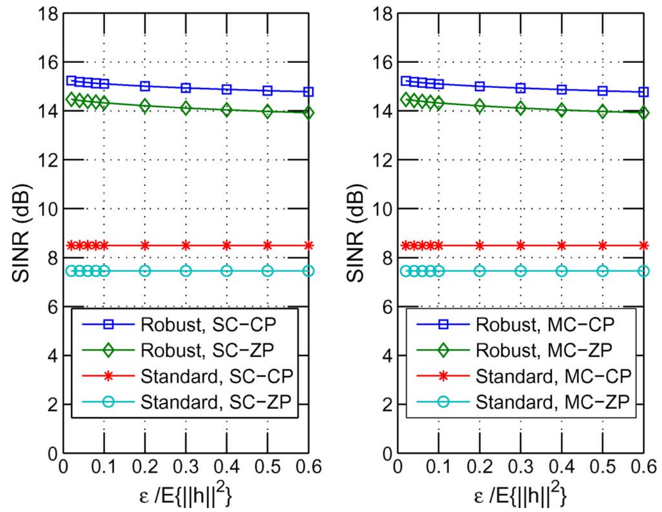

Fig. 1. Receiver output SINR performance versus the normalized channel uncertainty $\epsilon/E\{\|\mathbf{h}\|^2\}$ when $\mathrm{SNR}=10$ dB. (Left) SC with ZP and CP. (Right) MC with ZP and CP.

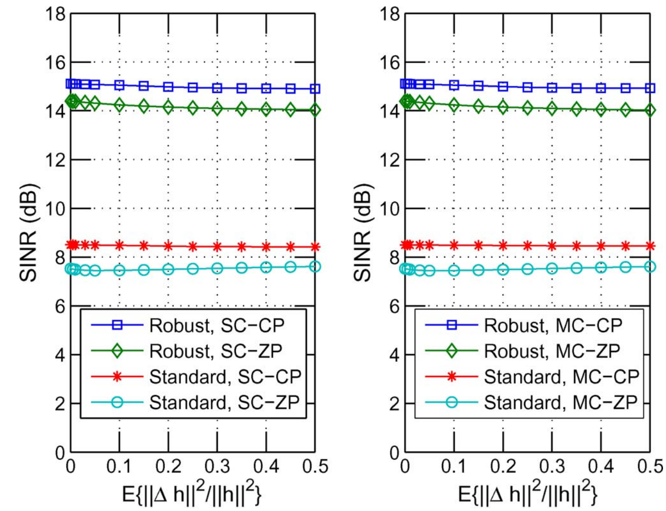

Fig. 3. Receiver output SINR performance versus initial channel estimation error $E\{\|\Delta\mathbf{h}\|/\|\mathbf{h}\|^2\}$ when $\mathrm{SNR}=10$ dB and $\epsilon/E\{\|\mathbf{h}\|^2\}=0.1$. (Left) SC with ZP and CP. (Right) MC with ZP and CP.

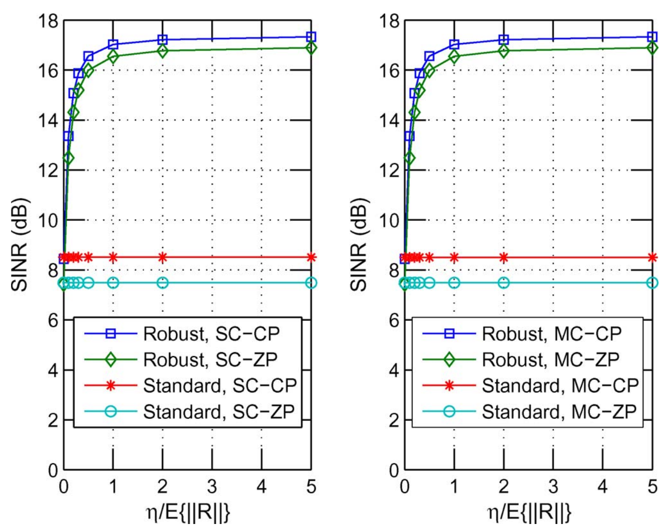

Fig. 2. Receiver output SINR performance versus the normalized covariance matrix uncertainty $\eta/E\{\|\mathbf{R}_y\|\}$ when $\mathrm{SNR}=10$ dB and $\epsilon/E\{\|\mathbf{h}\|^2\}=0.1$. (Left) SC with ZP and CP. (Right) MC with ZP and CP.

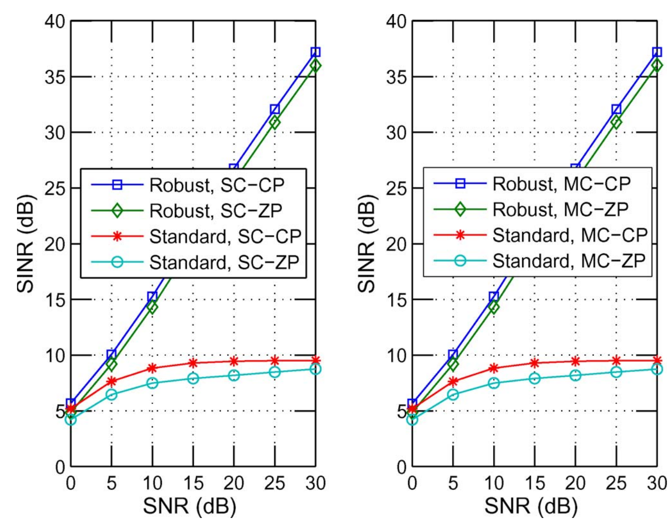

Fig. 4. Receiver output SINR versus the input SNR when $\epsilon/E\{\|\mathbf{h}\|^2\}=0.1$. (Left) SC with ZP and CP. (Right) MC with ZP and CP.

$\eta/E\{\|\mathbf{R}_y\|\}$. The results are shown in Fig. 2 for (left) SC with ZP and CP and (right) MC with ZP and CP. As in the previous example, the standard schemes are independent of η since they ignore the prior estimation error. Meanwhile, the performance of the proposed robust approach seems to be insensitive to the choice of bound. Compared with the nonrobust schemes, the proposed robust schemes obtain significant improvement.

To show how the proposed robust detectors compare to the standard detectors when initial estimates are sufficiently accurate, we consider a case where the amount of the prior estimation error varies but the assumed size of the uncertainty set is fixed. In particular, Fig. 3 depicts the performance as a function of the channel estimation error $E\{\|\Delta\mathbf{h}\|^2/\|\mathbf{h}\|^2\}$ for (left) SC with ZP and CP and (right) MC with ZP and CP when $\epsilon/E\{\|\mathbf{h}\|^2\}=0.1$ and $\mathrm{SNR}=10$ dB. Even with a small channel-estimation error, the robust detectors outperform the standard ones, leading to a large gap between them. Meanwhile, the performance of SC and MC are identical for the proposed as well as for the standard schemes.

Next, we compare the output SINR performance of the proposed robust and the standard nonrobust detectors as a function of the input SNR for (left) SC with ZP and CP and (right) MC with ZP and CP when $\epsilon/E\{\|\mathbf{h}\|^2\}=0.1$. In Fig. 4, it is seen that the robust detectors outperform the standard nonrobust detectors, particularly in the higher SNR region.

In the last example, we examine the average bit error rate performance of all detectors versus the input SNR, where ϵ is chosen the same as in the previous examples. As shown in Fig. 5, when the input SNR varies from -2 to 10 dB, the performance gap between the robust detectors and the nonrobust detectors increase. Moreover, the conventional detectors have an irreducible error floor due to poor initial channel estimates.

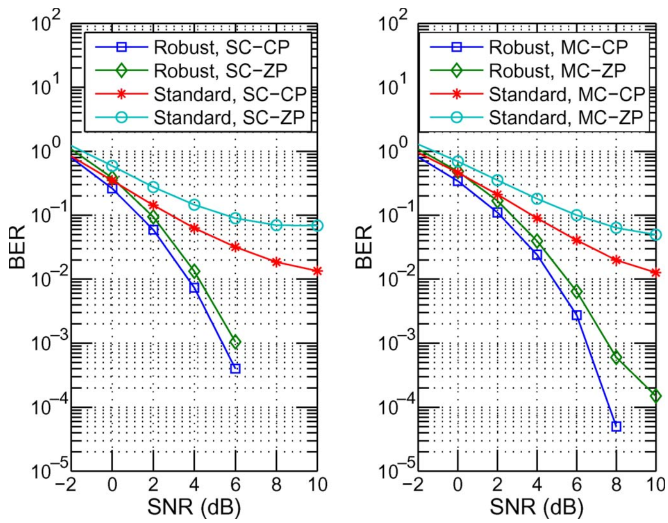

Fig. 5. Average BER versus the input SNR when $\epsilon/E\{\|\mathbf{h}\|^2\} = 0.1$. (Left) SC with ZP and CP. (Right) MC with ZP and CP.

VI. Conclusion

We have presented a unified approach to robust channel estimation and detection for block transmission using ZP and CP for both SC and MC systems. The proposed robust detectors were obtained by optimizing the worst-case performance over two bounded uncertainty sets pertaining to the prior estimation error in the initial channel estimate and the sample covariance matrix, respectively. We have shown that, although the prior estimation error is generally unbounded, it is beneficial to optimize the worst-case performance over the properly chosen bounded uncertainty set, which is determined by a bounding probability. Numerical results show that the proposed robust schemes yield improved performance over the ones that ignore the prior channel estimation and sample covariance matrix-estimation errors. We have seen that the performance of the SC and MC schemes are nearly identical for the ZP, as well as for the CP case.

Appendix
Derivations of $E\{\gamma_{\text{zp}}\}$ and $\text{var}\{\gamma_{\text{zp}}\}$

The sample covariance matrix $\hat{\mathbf{R}}_{\text{zp}}$ is an unbiased estimate of $\mathbf{R}_{\text{zp}}$, i.e., $E\{\hat{\mathbf{R}}_{\text{zp}}\} = \mathbf{R}_{\text{zp}}$. Let $y_i(n)$ be the ith element of $\mathbf{y}_{\text{zp}}(n)$. The unbiasedness can easily be checked by

$$E\left\{\hat{R}_{\text{zp}}(i,j)\right\} = \frac{1}{M}\sum_{n=1}^{M} E\left\{y_i(n)y_j^*(n)\right\} = R_{\text{zp}}(i,j). \quad (64)$$

To simplify notations, the subscript zp will be dropped in the following derivations. Using (64), we can rewrite the mean of γ as follows:

$$\begin{aligned} E\{\gamma\} &= \sum_{i=1}^{P}\sum_{j=1}^{P} E\left\{\left[\hat{R}(i,j)-R(i,j)\right]\left[\hat{R}(i,j)-R(i,j)\right]^*\right\} \\ &= \sum_{i=1}^{P}\sum_{j=1}^{P} E\left\{\hat{R}(i,j)\hat{R}(j,i)\right\} - R(i,j)R(j,i). \end{aligned} \quad (65)$$

Note that in (65), we have utilized the fact that both $\hat{\mathbf{R}}$ and $\mathbf{R}$ are Hermitian matrices, and therefore, $\hat{R}^*(i,j) = \hat{R}(j,i)$, and $R^*(i,j) = R(j,i)$. Furthermore

$$\begin{aligned} &E\left\{\hat{R}(i,j)\hat{R}(j,i)\right\} \\ &= \frac{1}{M^2}\sum_{n_1=1}^{M}\sum_{n_2=1}^{M} E\left\{y_i(n_1)y_j^*(n_1)y_j(n_2)y_i^*(n_2)\right\} \\ &= \frac{1}{M^2}\sum_{n_1=1}^{M}\sum_{n_2\neq n_1}^{M} E\left\{y_i(n_1)y_j^*(n_1)\right\}E\left\{y_j(n_2)y_i^*(n_2)\right\} \\ &\quad + \frac{1}{M^2}\sum_{n_1=1}^{M} E\left\{y_i(n_1)y_j^*(n_1)y_j(n_1)y_i^*(n_1)\right\} \\ &= \frac{M^2-M}{M^2}R(i,j)R(j,i) \\ &\quad + \frac{1}{M}\left[R(i,j)R(j,i) + R(i,i)R(j,j)\right] \\ &= R(i,j)R(j,i) + \frac{1}{M}R(i,i)R(j,j) \end{aligned} \quad (66)$$

where we have used the standard result on the fourth-order moment of Gaussian random variables in the second equality [34]. Substituting (66) in (65), we have

$$E\{\gamma\} = \frac{1}{M}\sum_{i=1}^{P}\sum_{j=1}^{P} R(i,i)R(j,j) = \frac{1}{M}\text{tr}^2\{\mathbf{R}\}. \quad (67)$$

Next, we compute $E\{\gamma^2\}$. Let $v(i,j) \triangleq \hat{R}(i,j) - R(i,j)$. Then, using the standard result on the fourth-order moment of Gaussian random variables, we have

$$\begin{aligned} E\{\gamma^2\} &= E\left\{\sum_{i=1}^{P}\sum_{j=1}^{P} v(i,j)v^*(i,j)\sum_{p=1}^{P}\sum_{q=1}^{P} v(p,q)v^*(p,q)\right\} \\ &= \sum_{i=1}^{P}\sum_{j=1}^{P}\sum_{p=1}^{P} \\ &\quad \times \sum_{q=1}^{P}\left[E\left\{v(i,j)v^*(i,j)\right\}E\left\{v(p,q)v^*(p,q)\right\}\right. \\ &\quad + E\left\{v(i,j)v(p,q)\right\}E\left\{v^*(i,j)v^*(p,q)\right\} \\ &\quad \left. + E\left\{v(i,j)v^*(p,q)\right\}E\left\{v^*(i,j)v(p,q)\right\}\right] \end{aligned} \quad (68)$$

where we have utilized the fact that $E\{v(i,j)\} = E\{\hat{R}(i,j)\} - R(i,j) = 0$. We first consider the item with the most general case in (68), i.e.,

$$\begin{aligned} &E\left\{v(i,j)v^*(p,q)\right\} \\ &= E\left\{\left[\hat{R}(i,j) - R(i,j)\right]\left[\hat{R}(p,q) - R(p,q)\right]^*\right\} \\ &= E\left\{\hat{R}(i,j)\hat{R}^*(p,q)\right\} + R(i,j)R^*(p,q) \\ &\quad - R(i,j)E\left\{\hat{R}^*(p,q)\right\} - E\left\{\hat{R}(i,j)\right\}R^*(p,q) \\ &= E\left\{\hat{R}(i,j)\hat{R}(q,p)\right\} - R(i,j)R(q,p). \end{aligned} \quad (69)$$

Here, $E\{\hat{R}(i,j)\hat{R}(q,p)\}$ can be rewritten as

$$\begin{aligned}
&E\left\{\hat{R}(i,j)\hat{R}(q,p)\right\}\\
&= \frac{1}{M^2}\sum_{n_1=1}^{M}\sum_{n_2=1}^{M} E\left\{y_i(n_1)y_j^*(n_1)y_q(n_2)y_p^*(n_2)\right\}\\
&= \frac{1}{M^2}\sum_{n_1=1}^{M}\sum_{n_2\neq n_1}^{M} E\left\{y_i(n_1)y_j^*(n_1)\right\} E\left\{y_q(n_2)y_p^*(n_2)\right\}\\
&\quad + \frac{1}{M^2}\sum_{n_1=1}^{M} E\left\{y_i(n_1)y_j^*(n_1)y_q(n_1)y_p^*(n_1)\right\}\\
&= \frac{M^2-M}{M^2}R(i,j)R^*(p,q)\\
&\quad + \frac{1}{M}\left[R(i,j)R^*(p,q) + R(i,p)R(q,j)\right]\\
&= R(i,j)R(q,p) + \frac{1}{M}R(i,p)R(q,j). \qquad (70)
\end{aligned}$$

Substituting (70) into (69), we have

$$E\left\{v(i,j)v^*(p,q)\right\} = \frac{1}{M}R(i,p)R(q,j). \qquad (71)$$

Based on (71), (68) can be simplified as

$$\begin{aligned}
E\{\gamma^2\} &= \frac{1}{M^2}\sum_{i=1}^{P}\sum_{j=1}^{P}\sum_{p=1}^{P}\sum_{q=1}^{P}\Big[R(i,i)R(j,j)R(p,p)R(q,q)\\
&\qquad + |R(i,q)R(p,j)|^2\\
&\qquad + |R(i,p)R(q,j)|^2\Big]\\
&= \frac{1}{M^2}\left[\mathrm{tr}^4\{\mathbf{R}\} + 2\|\mathbf{R}\|^4\right]. \qquad (72)
\end{aligned}$$

Using (67) and (72), we have

$$\mathrm{var}\{\gamma\} = E\{\gamma^2\} - E^2\{\gamma\} = \frac{2}{M^2}\|\mathbf{R}\|^4. \qquad (73)$$

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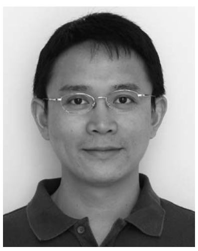

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