Adaptive Transmit and Receive Beamforming for Interference Mitigation

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***Abstract*—We consider adaptive transmit and receive beampattern design for array radar systems. While adaptive processing is primarily employed for only receive beamforming in conventional design, we propose a fully adaptive approach involving jointly selecting the transmit correlation matrix and receive beamformer by maximizing the signal-to-interference-plus-noise ratio (SINR). The motivation of utilizing adaptive processing at the transmitter is that with imprecise knowledge of the interference (e.g., due to limited training data), only relying on adaptive receive beamforming may be inadequate for effective interference cancellation, whereas joint adaptive transmit and receive beamforming can afford a stronger ability to handle the interference. Simulations are provided to demonstrate the performance of the proposed joint beamforming approach.**

***Index Terms*—Adaptive processing, interference cancellation, receive beamforming, transmit beamforming.**

I. INTRODUCTION

OPTIMAL linear beamformers [1]–[3] employ linear weights to optimize the receive beamformer response based on the statistics of the data. Specifically, the covariance matrix of the disturbance signal (i.e., interferences and noise) is used to place nulls in the directions of interfering sources to maximize the signal-to-interference-plus-noise ratio (SINR) at the output of the beamformer. In practice, data statistics are often unknown and may change with time. To cope with the problem, adaptive algorithms are used to obtain weights that converge to the statistically optimal solution. An adaptive beamformer requires training data to estimate the unknown disturbance covariance matrix. However, the challenge is that training data are often limited in many practical scenarios, which may cause significant performance loss due to lack of sufficient training data that are needed to form a reliable covariance matrix estimate. In an effort to improve the performance under these conditions, we propose to use training data not only for adaptive reception as traditional beamformers do, but also to adaptively control the transmit beampattern for radiation.

Manuscript received November 14, 2013; accepted January 01, 2014. Date of publication January 09, 2014; date of current version January 14, 2014. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Xiaokang Yang.

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Digital Object Identifier 10.1109/LSP.2014.2298497

Transmit beampattern and waveform design for array radars has been of interest recently [4]–[10]. However, most existing studies do not consider adaptive processing for radiation. Matched-illumination design was employed for transmit beamforming in [4], [5], where the signal correlation matrix is optimized to ensure the transmission power is directed to a range of desired angles, whereas interference mitigation was not explicitly considered. Alternative designs were studied in [6]–[8] by maximizing the output SINR, thus taking into account interference mitigation. However, the limitation is that these approaches assume full prior knowledge of the disturbance covariance matrix and, hence, are non-adaptive. Meanwhile, [9] and [10] examine radar phase code design under several constraints, e.g., constraints on a similarity to known radar codes, the peak-to-average power ratio, and estimation accuracy, etc. Their designs are also non-adaptive.

We consider herein jointly adaptive transmit and receive beamforming in the presence of interferences. A closed-form expression of the transmit beamforming correlation matrix is obtained by maximizing a lower bound of the output SINR at the receiver. Our solutions of the transmit beamforming correlation matrix and the associated receive beamformer require some knowledge (i.e., locations and strengths) of the interferences, which are adaptively estimated from the training data. The advantage of employing adaptive processing at both the transmitter and receiver in the presence of interferences with uncertainties is demonstrated by numerical results.

II. PROBLEM FORMULATION

Consider a narrowband system with M_t transmit and M_r receive antennas. Let $s_m(n)$ denote the signal transmitted by the m th antenna, and θ the location of a scatterer. The baseband signal at a specific scatterer location can be described as $\mathbf{a}_t^H(\theta)\mathbf{s}(n), n = 1, \cdots, N$, where $\mathbf{s}(n) = [s_1(n) \cdots s_{M_t}(n)]^T$ and $\mathbf{a}_t(\theta)$ denotes the $M_t \times 1$ transmit steering vector containing phase shifts determined by the look angle θ. For a uniform linear array (ULA) with a half-wavelength separation between two adjacent array elements, the steering vector is given by $\mathbf{a}_t(\theta) = [1 \quad e^{j\pi \sin\theta} \quad \cdots \quad e^{j(M_t-1)\pi \sin(\theta)}]^T$.

Suppose there is a target located at angle θ_0 along with K interferences located at $\boldsymbol{\theta}_i = [\theta_1 \cdots \theta_K]$, $\theta_k \neq \theta_0$ for $k = 1, \cdots, K$. Then, the received signal is given by [11]

$$\mathbf{y}(n) = \alpha_0 \mathbf{a}_r^*(\theta_0)\mathbf{a}_t^H(\theta_0)\mathbf{s}(n) + \boldsymbol{\iota}(n) + \epsilon(n)$$

where α_0 denotes the target amplitude, $\mathbf{a}_r(\theta) \in \mathbb{C}^{M_r \times 1}$ the receive steering vector similarly defined as $\mathbf{a}_t(\theta)$, $\boldsymbol{\iota}(n)$ the interferences which can be expressed as $\boldsymbol{\iota}(n) = \sum_{k=1}^{K} \alpha_k \mathbf{a}_r^*(\theta_k)\mathbf{a}_t^H(\theta_k)\mathbf{s}(n)$, and $\epsilon(n)$ the nosie with zero mean and covariance matrix $\sigma_n^2\mathbf{I}_{M_r}$. Assume the complex amplitudes α_k are uncorrelated with zero mean and variance σ_k^2. The covariance matrix of the disturbance (interferences plus noise) is

$$\mathbf{R}_d = \sum_{k=1}^{K} \sigma_k^2 \mathbf{a}_r^*(\theta_k)\mathbf{a}_t^H(\theta_k)\mathbf{R}_s\mathbf{a}_t(\theta_k)\mathbf{a}_r^T(\theta_k) + \sigma_n^2\mathbf{I}_{M_r}$$
$$\triangleq \sum_{k=1}^{K} \sigma_k^2 g(\theta_k)\mathbf{a}_r^*(\theta_k)\mathbf{a}_r^T(\theta_k) + \sigma_n^2\mathbf{I}_{M_r} \quad (1)$$

where $\mathbf{R}_s = E\{\mathbf{s}(n)\mathbf{s}^H(n)\}$ denotes the signal correlation matrix to be designed, and $g(\theta_k) \triangleq \mathbf{a}_t^H(\theta_k)\mathbf{R}_s\mathbf{a}_t(\theta_k)$ the transmit beamforming gain at direction θ_k. At the receiver side, a linear beamformer $\mathbf{w} \in \mathbb{C}^{M_r\times 1}$ is applied to $\mathbf{y}(n)$ for interference mitigation, yielding the output $z(n) = \mathbf{w}^H\mathbf{y}(n)$. The problem of interest is to jointly optimize $\mathbf{R}_s$ for transmit beamforming and $\mathbf{w}$ for receive beamforming.

III. Proposed Approach

A. Transmit and Receive Beamforming Design

Consider the receive beamformer output SINR given by

$$\rho(\mathbf{w},\mathbf{R}_s) = \sigma_0^2 \frac{\mathbf{w}^H\mathbf{a}_r^*(\theta_0)\mathbf{a}_t^H(\theta_0)\mathbf{R}_s\mathbf{a}_t(\theta_0)\mathbf{a}_r^T(\theta_0)\mathbf{w}}{\mathbf{w}^H\mathbf{R}_d\mathbf{w}}. \quad (2)$$

We take a max-SINR approach which is frequently used for radar design (e.g., [12])), by maximizing (2) jointly with respect to (w.r.t.) $\mathbf{w}$ and $\mathbf{R}_s$, subject to constraints on the transmit power and positive semi-definitiveness of $\mathbf{R}_s$:

$$\max_{\mathbf{w},\mathbf{R}_s} \rho(\mathbf{w},\mathbf{R}_s), \quad \text{s.t. } \operatorname{tr}\{\mathbf{R}_s\} \le p,\ \mathbf{R}_s \succeq 0. \quad (3)$$

To solve (3), we first solve $\mathbf{w}$ in terms of a given $\mathbf{R}_s$ as follows

$$\max_{\mathbf{w}} \quad \frac{\mathbf{w}^H\mathbf{a}_r^*(\theta_0)\mathbf{a}_t^H(\theta_0)\mathbf{R}_s\mathbf{a}_t(\theta_0)\mathbf{a}_r^T(\theta_0)\mathbf{w}}{\mathbf{w}^H\mathbf{R}_d\mathbf{w}}. \quad (4)$$

The solution is given by (see Appendix A for a proof)

$$\mathbf{w} = \tau\mathbf{R}_d^{-1}\mathbf{a}_r^*(\theta_0) \quad (5)$$

where $\tau \neq 0$ is an arbitrary constant, and the associated maximum value of (4), denoted by λ_1, is

$$\lambda_1(\mathbf{R}_s) = \mathbf{a}_t^H(\theta_0)\mathbf{R}_s\mathbf{a}_t(\theta_0)\mathbf{a}_r^T(\theta_0)\mathbf{R}_d^{-1}\mathbf{a}_r^*(\theta_0). \quad (6)$$

The remaining step is to find $\mathbf{R}_s$ to maximize the output SINR:

$$\max_{\mathbf{R}_s} \lambda_1(\mathbf{R}_s), \quad \text{s.t. } \operatorname{tr}\{\mathbf{R}_s\} \le p,\ \mathbf{R}_s \succeq \mathbf{0}. \quad (7)$$

A main difficulty of (7) is that the objective function involves a non-diagonal matrix inverse. To circumvent this difficulty, we propose to maximize a lower bound of the objective function (7) (see Appendix B for derivation of the lower bound)

$$\lambda_1 \ge \frac{M_r^2 \operatorname{tr}\{\mathbf{R}_s\mathbf{B}_t(\theta_0)\}}{\operatorname{tr}\{\mathbf{R}_s\mathbf{C}\} + M_r\sigma_n^2} \quad (8)$$

where $\mathbf{B}_t(\theta_0) \triangleq \mathbf{a}_t(\theta_0)\mathbf{a}_t^H(\theta_0)$, $\mathbf{B}_r(\theta_0) \triangleq \mathbf{a}_r^*(\theta_0)\mathbf{a}_r^T(\theta_0)$, and

$$\mathbf{C} \triangleq \sum_{k=1}^{K} \sigma_k^2 \mathbf{a}_t(\theta_k)\mathbf{a}_r^T(\theta_k)\mathbf{B}_r(\theta_0)\mathbf{a}_r^*(\theta_k)\mathbf{a}_t^H(\theta_k). \quad (9)$$

Thus, we have the following new optimization problem:

$$\max_{\mathbf{R}_s} \frac{\operatorname{tr}\{\mathbf{R}_s\mathbf{B}_t(\theta_0)\}}{\operatorname{tr}\{\mathbf{R}_s\mathbf{C}\} + M_r\sigma_n^2}, \quad \text{s.t. } \operatorname{tr}\{\mathbf{R}_s\} \le p,\ \mathbf{R}_s \succeq \mathbf{0}. \quad (10)$$

This is a constrained fractional semidefinite programming (SDP) problem whose solution can be obtained by solving its equivalent SDP via the so-called Charnes-Cooper transformation [13]. Specifically, since the denominator of the fractional SDP is strictly positive [see (21)], we can define $\mathbf{Q} \triangleq q\mathbf{R}_s$, where $q > 0$ is a scaling parameter which makes $\operatorname{tr}\{q\mathbf{R}_s\mathbf{C}\} + qM_r\sigma_n^2 = 1$. Hence, multiplying by q the numerator and the denominator of the objective function in (10), we obtain the equivalent SDP problem as

$$\begin{aligned} \max_{\mathbf{Q},q} \quad & \operatorname{tr}\{\mathbf{Q}\mathbf{B}_t(\theta_0)\} \\ \text{s.t.} \quad & \operatorname{tr}\{\mathbf{Q}\mathbf{C}\} = 1 - qM_r\sigma_n^2 \\ & \operatorname{tr}\{\mathbf{Q}\} = qp \\ & \mathbf{Q} \succeq \mathbf{0}, \quad q > 0. \end{aligned} \quad (11)$$

The optimal solution $(\mathbf{Q}_\star, q_\star)$ of (11) can be found by using standard convex optimization software. In turn, the solution of (10) can be obtained as $\mathbf{R}_{s\star} = \frac{1}{q_\star}\mathbf{Q}_\star$.

In fact, a closed-form solution to (10) can be derived as shown next. Let the eigenvalue decomposition (EVD) of $\mathbf{R}_s$ be $\mathbf{R}_s = \mathbf{U}\mathbf{\Gamma}\mathbf{U}^H$, where $\mathbf{\Gamma}$ contains the eigenvalues $\{\gamma_m\}_{m=1}^{M_t}$ on its diagonal. The optimization problem can be rewritten as

$$\max_{\mathbf{\Gamma},\mathbf{U}} \frac{\operatorname{tr}\{\mathbf{\Gamma}\mathbf{U}^H\mathbf{B}_t(\theta_0)\mathbf{U}\}}{\operatorname{tr}\{\mathbf{\Gamma}\mathbf{U}^H\mathbf{C}\mathbf{U}\} + M_r\sigma_n^2}, \quad \text{s.t. } \sum_{m=1}^{M_t}\gamma_m \le p,\ \gamma_m \ge 0. \quad (12)$$

Let $\eta_\star$ denote the optimum value of the objective function. Then, for any eigen-pair $\mathbf{\Gamma}$ and $\mathbf{U}$

$$\frac{\operatorname{tr}\{\mathbf{\Gamma}\mathbf{U}^H\mathbf{B}_t(\theta_0)\mathbf{U}\}}{\operatorname{tr}\{\mathbf{\Gamma}\mathbf{U}^H\mathbf{C}\mathbf{U}\} + M_r\sigma_n^2} \le \eta_\star.$$

The problem is to construct $\mathbf{U}_\star$ and $\mathbf{\Gamma}_\star$ which satisfy the constraints in (12) as well as the relation

$$\operatorname{tr}\{\mathbf{\Gamma}_\star\mathbf{U}_\star^H(\mathbf{B}_t(\theta_0) - \eta_\star\mathbf{C})\mathbf{U}_\star\} = M_r\sigma_n^2\eta_\star. \quad (13)$$

Let $\mathbf{C}_0 \triangleq \mathbf{C} + \frac{M_r\sigma_n^2}{p}\mathbf{I}_{M_t}$. It is shown in Appendix C that $\eta_\star = \mathbf{a}_t^H(\theta_0)\mathbf{C}_0^{-1}\mathbf{a}_t(\theta_0)$ and $\mathbf{R}_{s\star} = p\mathbf{v}\mathbf{v}^H$ with

$$\mathbf{v} = \mathbf{C}_0^{-1}\mathbf{a}_t(\theta_0)\left[\mathbf{a}_t^H(\theta_0)\mathbf{C}_0^{-2}\mathbf{a}_t(\theta_0)\right]^{-\frac{1}{2}}. \quad (14)$$

The optimal $\mathbf{R}_{s\star}$ is a rank-one matrix, which is consistent with numerical results obtained by the SDP approach.

B. Adaptive Estimation

Our method requires to know interference locations θ_k and strengths σ_k^2. We discuss here how to adaptively estimated these parameters from training signals. Specifically, training signals are obtained by sending a selected waveform $\mathbf{s}_0(n), n = 1,\ldots,N$, to probe the environment when the target is absent (prior to target sensing). For simplicity, we use orthogonal waveforms. Let $\mathbf{Y}_0 = [\mathbf{y}_0(1) \cdots \mathbf{y}_0(N)] \in \mathbb{C}^{M_r \times N}$ contains the corresponding received signal which can be expressed as

$$\mathbf{Y}_0 = \sum_{k=1}^{K} \alpha_k \mathbf{a}_r^*(\theta_k)\mathbf{a}_t^H(\theta_k)\mathbf{S}_0 + \mathbf{E}_0 \tag{15}$$

where $\mathbf{S}_0 = [\mathbf{s}_0(1) \cdots \mathbf{s}_0(N)] \in \mathbb{C}^{M_t \times N}$ and $\mathbf{E}_0 = [\epsilon(1) \cdots \epsilon(N)] \in \mathbb{C}^{M_r \times N}$. Given the training data $\mathbf{Y}_0$, a multitude of methods (see [14]) can be used to obtain the interference location estimates $\hat{\theta}_k$ (we use the MUSIC algorithm in Section IV). From (15),

$$\mathbf{Y}_0\mathbf{S}_0^{\dagger} = \sum_{k=1}^{K} \alpha_k \mathbf{a}_r^*(\theta_k)\mathbf{a}_t^H(\theta_k) + \mathbf{E}_0\mathbf{S}_0^{\dagger} \tag{16}$$

where $\mathbf{S}_0^{\dagger}$ denotes the pseudo inverse of $\mathbf{S}_0$, or, equivalently,

$$\mathrm{vec}(\mathbf{Y}_0\mathbf{S}_0^{\dagger}) = \sum_{k=1}^{K} \alpha_k [\mathbf{a}_t^*(\theta_k) \otimes \mathbf{a}_r^*(\theta_k)] + \mathrm{vec}(\mathbf{E}_0\mathbf{S}_0^{\dagger})$$

where $\mathrm{vec}(\cdot)$ stacks the columns of a matrix and $\otimes$ denotes the Kronecker product. Then, the least-squares estimate of $\boldsymbol{\alpha}$ is given by

$$\hat{\boldsymbol{\alpha}} = (\mathbf{A}_{t,r}^H \mathbf{A}_{t,r})^{-1}\mathbf{A}_{t,r}^H \mathrm{vec}(\mathbf{Y}_0\mathbf{S}_0^{\dagger})$$

where $\mathbf{A}_{t,r} \triangleq [\mathbf{a}_t^*(\hat{\theta}_1) \otimes \mathbf{a}_r^*(\hat{\theta}_1), \cdots, \mathbf{a}_t^*(\hat{\theta}_K) \otimes \mathbf{a}_r^*(\hat{\theta}_K)]$. In addition, the variance of the amplitude of the interference can be simply estimated as $\hat{\sigma}_k^2 = \hat{\alpha}_k^2$.

Finally, we use the estimates $\hat{\theta}_k$ and $\hat{\sigma}_k^2$ in (14). The resulting signal correlation matrix can be written as $\hat{\mathbf{R}}_{s\star} = p\hat{\mathbf{v}}\hat{\mathbf{v}}^H$, and its associate receive beamformer is

$$\hat{\mathbf{w}} = \tau \hat{\mathbf{R}}_d^{-1}\mathbf{a}_r^*(\theta_0) \tag{17}$$

where $\hat{\mathbf{R}}_d$ is computed as in (1). In Section IV, beampattern is used to compare different adaptive beamforming schemes. We consider the joint transmit-receive beampattern given by

$$\hat{P}(\theta) = \left[\mathbf{a}_t^H(\theta)\hat{\mathbf{R}}_{s\star}\mathbf{a}_t(\theta)\right]\left|\hat{\mathbf{w}}^H\mathbf{a}_r^*(\theta)\right|^2 \tag{18}$$

which includes the contribution from transmit beamforming $\mathbf{a}_t^H(\theta)\hat{\mathbf{R}}_{s\star}\mathbf{a}_t(\theta)$ and receive beamforming $|\hat{\mathbf{w}}^H\mathbf{a}_r^*(\theta)|^2$. We also set the non-zero scalar τ in (17) as $\tau \triangleq [\mathbf{a}_r^T(\theta_0)\hat{\mathbf{R}}_d^{-1}\mathbf{a}_r^*(\theta_0)]^{-1}[\mathbf{a}_t^H(\theta_0)\hat{\mathbf{R}}_{s\star}\mathbf{a}_t(\theta_0)]^{-\frac{1}{2}}$ to normalize the beampattern (18) such that the gain at the target direction is one (to facilitate comparison). Note the normalization does not change the shape of the beampattern.

IV. Numerical Results

We present numerical results to demonstrate the merits of the proposed beamforming scheme. We compare it with the phased-array (PA) scheme that points to the target location θ_0 at transmission. The transmit correlation matrix of the PA is $\mathbf{R}_{s,\mathrm{PA}} = \frac{p}{\|\mathbf{a}_t(\theta_0)\|^2}\mathbf{a}_t(\theta_0)\mathbf{a}_t^H(\theta_0)$[5], whereas the receive beamforming vector is similarly given by (5), except that its disturbance covariance matrix $\mathbf{R}_d$ depends on $\mathbf{R}_{s,\mathrm{PA}}$. Moreover, for adaptive interference cancellation, the PA scheme also requires knowledge of the interferences in order to compute $\mathbf{R}_d$. Here, $\mathbf{R}_d$ for the PA system is estimated in a similar approach as described in Section III-B.

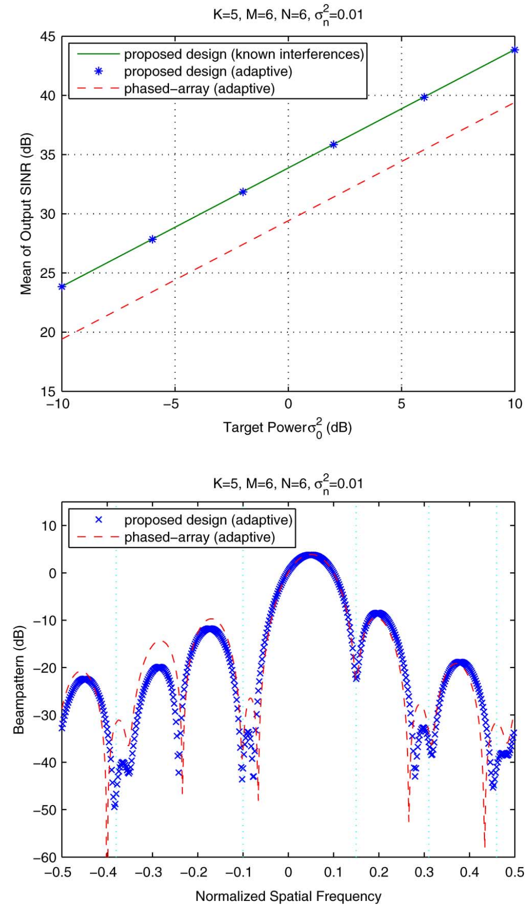

Fig. 1. Mean output SNR and joint transmit and receive beampattern.

Consider a system where the transmitter and receiver share a ULA of $M_t = M_r = M = 6$ elements with half-wavelength inter-element separation. The total transmit power is set to $p = M$, a target is located at 0.1 and five interferences are at $(-0.38, -0.1, 0.15, 0.31, 0.46)$ in normalized spatial frequency $f \triangleq (\sin\theta)/2$. The overall power for the interferences is 1 and the noise variance is $\sigma_n^2 = 0.01$. The target power is either $\sigma_0^2 = 1$ or varied over a range of values as specified. We consider a training-limited scenario where the number of training data used for adaptive estimation is $N = M = 6$.

Fig. 1 depicts the mean of the output SINR for the proposed and the PA schemes based on adaptive estimation. The output SINR for the proposed scheme with known interferences is also shown as a benchmark. We note that the proposed scheme has a similar SINR with known or adaptive estimated interferences. Moreover, it outperforms the PA scheme by 4.3 dB. The joint beampatterns of the two adaptive approaches are also shown in Fig. 1. It can be seen that the proposed adaptive design is able to suppress all the five interferences, while the PA scheme cannot effectively mitigate the interferences at $(-0.38, 0.46)$. Therefore, our proposed adaptive approach has a stronger ability to handle the interferences in the training-limited situation.

V. Conclusions

We have proposed a jointly adaptive transmit and receive beamforming for array radars. The transmit and receive beampattern is obtained by jointly designing the transmit beamforming correlation matrix and receive beamforming vector in terms of maximizing the output SINR. Numerical results show that by applying adaptive processing for both radiation and receiving in a training-limited situation, we can achieve a better beampattern, a stronger ability to handle interference, and a higher output SINR.

Appendix A
Proof of (5) and (6)

Define $\mathbf{w}_1 \triangleq \mathbf{R}_d^{1/2}\mathbf{w}$. The problem (4) becomes

$$\max_{\mathbf{w}_1} \frac{\mathbf{w}_1^H \mathbf{R}_d^{-\frac{1}{2}} \mathbf{a}_r^*(\theta_0)\mathbf{a}_t^H(\theta_0)\mathbf{R}_s\mathbf{a}_t(\theta_0)\mathbf{a}_r^T(\theta_0)\mathbf{R}_d^{-\frac{1}{2}}\mathbf{w}_1}{\|\mathbf{w}_1\|^2} \tag{19}$$

or equivalently,

$$\max_{\mathbf{w}_1} \quad \mathbf{w}_1^H \mathbf{R}_d^{-\frac{1}{2}} \mathbf{a}_r^*(\theta_0)\mathbf{a}_t^H(\theta_0)\mathbf{R}_s\mathbf{a}_t(\theta_0)\mathbf{a}_r^T(\theta_0)\mathbf{R}_d^{-\frac{1}{2}}\mathbf{w}_1$$
$$\text{s.t.} \quad \|\mathbf{w}_1\| = 1.$$

The maximum of the objective function is the largest eigenvalue λ_1 of $\mathbf{R}_d^{-\frac{1}{2}} \mathbf{a}_r^*(\theta_0)\mathbf{a}_t^H(\theta_0)\mathbf{R}_s\mathbf{a}_t(\theta_0)\mathbf{a}_r^T(\theta_0)\mathbf{R}_d^{-\frac{1}{2}}$, and the solution of $\mathbf{w}_1$ is the associated principal eigenvector. Since $\mathbf{a}_r^*(\theta_0)\mathbf{a}_t^H(\theta_0)$ is a rank one matrix, there is only one non-zero eigenvalue of $\mathbf{R}_d^{-\frac{1}{2}} \mathbf{a}_r^*(\theta_0)\mathbf{a}_t^H(\theta_0)\mathbf{R}_s\mathbf{a}_t(\theta_0)\mathbf{a}_r^T(\theta_0)\mathbf{R}_d^{-\frac{1}{2}}$, which is $\lambda_1 = \mathbf{a}_t^H(\theta_0)\mathbf{R}_s\mathbf{a}_t(\theta_0)\mathbf{a}_r^T(\theta_0)\mathbf{R}_d^{-1}\mathbf{a}_r^*(\theta_0) \geq 0$. The associated eigenvector $\mathbf{w}_1$ is

$$\mathbf{w}_1 = \mathbf{R}_d^{-\frac{1}{2}}\mathbf{a}_r^*(\theta_0)\left[\mathbf{a}_r^T(\theta_0)\mathbf{R}_d^{-1}\mathbf{a}_r^*(\theta_0)\right]^{-\frac{1}{2}}.$$

In turn, we can write

$$\mathbf{w} = \mathbf{R}_d^{-\frac{1}{2}}\mathbf{w}_1 = \tau\mathbf{R}_d^{-1}\mathbf{a}_r^*(\theta_0)$$

where τ can be any non-zero constant since scaling does not change the value of (19).

Appendix B
Proof of (8)

Let $\mathbf{E} = \mathbf{R}_s^{\frac{1}{2}}\mathbf{a}_t(\theta_0)\mathbf{a}_r^T(\theta_0)\mathbf{R}_d^{-\frac{1}{2}}$ and $\mathbf{F} = \mathbf{R}_s^{\frac{1}{2}}\mathbf{a}_t(\theta_0)\mathbf{a}_r^T(\theta_0)\mathbf{R}_d^{\frac{1}{2}}$. Then we can write (6) as $\lambda_1 = \mathrm{tr}(\mathbf{E}\mathbf{E}^H)$. By the Cauchy-Schwarz inequality, a lower bound is given as

$$\lambda_1 \geq \frac{\left[\mathrm{tr}(\mathbf{E}\mathbf{F}^H)\right]^2}{\mathrm{tr}(\mathbf{F}\mathbf{F}^H)} = \frac{M_r^2\mathbf{a}_t^H(\theta_0)\mathbf{R}_s\mathbf{a}_t(\theta_0)}{\mathbf{a}_r^T(\theta_0)\mathbf{R}_d\mathbf{a}_r^*(\theta_0)}. \tag{20}$$

The lower bound is tight if $\mathbf{E} = \alpha\mathbf{F}$, where α is a non-zero constant. The condition is met if the interference is (approximately) spectrally white, or $\mathbf{R}_d \propto \mathbf{I}$. Following from (1) and (9), we can write the denominator of (20)

$$\mathbf{a}_r^T(\theta_0)\mathbf{R}_d\mathbf{a}_r^*(\theta_0) = \mathrm{tr}(\mathbf{R}_s\mathbf{C}) + M_r\sigma_n^2 \tag{21}$$

and the numerator as

$$M_r^2\mathbf{a}_t^H(\theta_0)\mathbf{R}_s\mathbf{a}_t(\theta_0) = M_r^2\ \mathrm{tr}[\mathbf{R}_s\mathbf{B}_t(\theta_0)].$$

Appendix C
Solution to (12)

A solution to the problem is obtained by construction. Let the rank of $\mathbf{R}_{s\star}$ be M with $M \leq M_t$, and $\mathbf{u}_m$ be the eigenvector of $\mathbf{R}_{s\star}$ corresponding to the eigenvalue γ_m. Then, (13) can be written as:

$$\sum_{m=1}^{M}\left[\gamma_m\mathbf{u}_m^H\left(\mathbf{B}_t(\theta_0) - \eta_\star\mathbf{C}\right)\mathbf{u}_m - b_m M_r\sigma_n^2\eta_\star\right] = 0$$

where $\{b_m\}_{m=1}^M$ are a set of coefficients which satisfy $\sum_{m=1}^M b_m = 1$. By selecting $b_m \triangleq \gamma_m\mathbf{u}_m^H(\mathbf{B}_t(\theta_0) - \eta_\star\mathbf{C})\mathbf{u}_m/M_r\sigma_n^2\eta_\star$, we have

$$\gamma_m\mathbf{u}_m^H\left(\mathbf{B}_t(\theta_0) - \eta_\star\mathbf{C}\right)\mathbf{u}_m - b_m M_r\sigma_n^2\eta_\star = 0$$

or equivalently,

$$\mathbf{u}_m^H\left[\mathbf{B}_t(\theta_0) - \eta_\star\left(\mathbf{C} + \frac{b_m M_r\sigma_n^2}{\gamma_m}\mathbf{I}_{M_t}\right)\right]\mathbf{u}_m = 0$$

for $m = 1, \ldots, M$. Hence, $\eta_\star$ is a non-zero generalized eigenvalue of $\{\mathbf{B}_t(\theta_0), \mathbf{C}_m\}$ with $\mathbf{C}_m \triangleq \mathbf{C} + \frac{b_m M_r\sigma_n^2}{\gamma_m}\mathbf{I}_{M_t}$, and $\mathbf{u}_m$ is the generalized eigenvector corresponding to $\eta_\star$. That is,

$$\mathbf{B}_t(\theta_0)\mathbf{u}_m = \eta_\star\mathbf{C}_m\mathbf{u}_m. \tag{22}$$

Since $\mathbf{B}_t(\theta_0) = \mathbf{a}_t(\theta_0)\mathbf{a}_t^H(\theta_0)$ is rank-one, it is easy to show

$$\eta_\star = \mathbf{a}_t^H(\theta_0)\mathbf{C}_m^{-1}\mathbf{a}_t(\theta_0).$$

$\eta_\star$ obtained for different m should be identical, i.e.,

$$\mathbf{a}_t^H(\theta_0)\mathbf{C}_j^{-1}\mathbf{a}_t(\theta_0) - \mathbf{a}_t^H(\theta_0)\mathbf{C}_l^{-1}\mathbf{a}_t(\theta_0) = 0, \quad \forall j, l \in \{1, 2, \ldots, M\}. \tag{23}$$

Denote the EVD of $\mathbf{C}$ as $\mathbf{C} = \mathbf{U}_c\mathbf{D}\mathbf{U}_c^H$, where the unitary matrix $\mathbf{U}_c$ contains the eigenvectors while the diagonal matrix $\mathbf{D}$ contains the eigenvalues. Let $\mu_j = \frac{b_j M_r\sigma_n^2}{\gamma_j}, j = 1, \ldots, M$. Then, (23) can be expressed as

$$\mathbf{a}_t^H(\theta_0)\mathbf{U}_c\left[(\mathbf{D} + \mu_j\mathbf{I})^{-1} - (\mathbf{D} + \mu_l\mathbf{I})^{-1}\right]\mathbf{U}_c^H\mathbf{a}_t(\theta_0) = 0$$

which implies $\frac{b_1}{\gamma_1} = \frac{b_2}{\gamma_2} = \cdots = \frac{b_M}{\gamma_M}$. In addition, from $\sum_{m=1}^M \gamma_m = p$ and $\sum_{m=1}^M b_m = 1$, we conclude $\frac{b_m}{\gamma_m} = \frac{1}{p}$. As such,

$$\mathbf{C}_m = \mathbf{C} + \frac{M_r\sigma_n^2}{p}\mathbf{I}_{M_t} \triangleq \mathbf{C}_0, m = 1, \ldots, M. \tag{24}$$

It remains to determine the rank M of $\mathbf{R}_{s\star}$. From (22) and (24), we have

$$\mathbf{C}_0^{-\frac{1}{2}}\mathbf{a}_t(\theta_0)\mathbf{a}_t^H(\theta_0)\mathbf{C}_0^{-\frac{1}{2}}\mathbf{C}_0^{\frac{1}{2}}\mathbf{u}_m = \eta_\star\mathbf{C}_0^{\frac{1}{2}}\mathbf{u}_m$$

which indicates that the normalized form of $\mathbf{C}_0^{\frac{1}{2}}\mathbf{u}_m$ is the eigenvector of the rank-one matrix $\mathbf{C}_0^{-\frac{1}{2}}\mathbf{a}_t(\theta_0)\mathbf{a}_t^H(\theta_0)\mathbf{C}_0^{-\frac{1}{2}}$. Hence, we have $\mathbf{C}_0^{\frac{1}{2}}\mathbf{u}_m \propto \mathbf{C}_0^{-\frac{1}{2}}\mathbf{a}_t(\theta_0)$ and more specifically, we can write

$$\mathbf{C}_0^{\frac{1}{2}}\mathbf{u}_m = \mathbf{C}_0^{-\frac{1}{2}}\mathbf{a}_t(\theta_0)\left[\mathbf{a}_t^H(\theta_0)\mathbf{C}_0^{-2}\mathbf{a}_t(\theta_0)\right]^{-\frac{1}{2}}.$$

it follows that all $\{\mathbf{u}_m\}$ are identical, given by

$$\mathbf{u}_m = \mathbf{C}_0^{-1}\mathbf{a}_t(\theta_0)\left[\mathbf{a}_t^H(\theta_0)\mathbf{C}_0^{-2}\mathbf{a}_t(\theta_0)\right]^{-\frac{1}{2}} \triangleq \mathbf{v}.$$

Moreover, since $\{\mathbf{u}_m\}$ are by definition the eigenvectors of the semidefinite matrix $\mathbf{R}_{s\star}$, they must be different. Therefore, we must have $M = 1$ and $\mathbf{R}_{s\star}$ is rank-one, given by

$$\mathbf{R}_{s\star} = \gamma_1\mathbf{v}\mathbf{v}^H = p\mathbf{v}\mathbf{v}^H.$$

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