Precoding for Decentralized Detection of Unknown Deterministic Signals

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We consider a decentralized detection problem in which a number of sensor nodes collaborate to detect the presence of an unknown deterministic vector signal. To cope with the power/bandwidth constraints inherent in wireless sensor networks (WSNs), each sensor compresses its observations using a linear precoder. The compressed messages are transmitted to the fusion center (FC), where a global decision is made by resorting to a generalized likelihood ratio test (GLRT). The aim of the work presented here is to develop effective linear precoding strategies and study their detection error exponents under the asymptotic regime where the number of sensors tends to infinity. Two precoding strategies are introduced: a random precoding scheme which generates its precoding vectors following a Gaussian distribution, and a sign-assisted random precoding scheme which assumes the knowledge of the plus/minus signs of the signal components and designs its precoding vectors with the aid of this prior knowledge. Performance analysis shows that utilizing the sign information can radically improve the detection performance. Also, it is found that precoding-based schemes are more effective than the energy detector in detecting weak signals that are buried in noise. Specifically, the sign-assisted random precoding scheme outperforms the energy detector when the observation signal-to-noise ratio (SNR) is less than $1/(\pi-2)$. Numerical results are conducted to corroborate our theoretical analysis and to illustrate the effectiveness of the proposed algorithms.

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I. INTRODUCTION

Decentralized detection using wireless sensor networks (WSNs) is an important problem that has attracted much attention over the past decade [1–21]. Such a problem has a variety of applications in environment monitoring, battlefield surveillance, and space missions (satellite monitoring). A sensor network, consisting of a large amount of low-cost battery-powered devices, usually has very stringent energy and bandwidth constraints. To address this issue, each sensor needs to locally process its sensor observations in order to reduce the amount of information being transmitted. A typical processing is to conduct a local Neyman-Pearson test at each node. The local binary decision is then sent to the fusion center (FC), where a global decision is made. A large number of studies [1–15] were carried out in this context. The problem of interest in the above setting is the determination of the optimal local decision rules. It was shown in [2, 3, 5] that for both Bayesian and Neyman-Pearson criteria, the optimal local sensor decision for a binary hypotheses testing problem is a likelihood ratio test (LRT). Nevertheless, the search of optimal local detectors is still exponentially complex because the optimal local thresholds are generally different and need to be jointly determined along with the global fusion rule. In some other studies [16–18], the observations of each sensor are encoded into a real-valued summary message such as the local likelihood ratio. To address the tradeoff between energy efficiency and the detection performance, the send/no send "censoring" problem was studied to decide which sensors should transmit [16–18]. All the above works, however, assume that each sensor has a perfect knowledge of the distributions of its local observations under respective hypotheses. This requires that, under a deterministic model, the signal to be detected is known a priori, or the probability density function (pdf) of the signal is available if a stochastic model is adopted. Nevertheless, in practice, the knowledge of the signal or its pdf may not be available. Consequently one cannot compute the local likelihood ratio and make a local decision at each sensor. Developing effective decentralized detection methods in such scenarios has not received much attention and is the focus of this paper.

We study the problem of detecting the presence of an unknown deterministic vector signal using a WSN. Due to the lack of signal knowledge, previous approaches that conduct a local Neyman-Pearson test at each node are infeasible. Instead, here we consider using a linear precoder to compress each sensor's observations into a single real-valued message. Such linear operator has the advantage of simple implementation and is suitable for low-cost sensors with limited computational resources. Upon receiving the compressed messages from all sensors, a generalized likelihood ratio test (GLRT) detector is used at the FC to form a final decision. We note that linear precoding design for decentralized detection has also been studied in [19, 21]. Nevertheless, in [19], the signal to be detected is modeled as a multivariate Gaussian random variable and the pdf of the signal is assumed available at the FC (although not known by sensors). In another work [21], optimal precoding design was studied by assuming the knowledge of the deterministic signal. The work [21] also touched lightly on the problem of decentralized detection of unknown deterministic signals, in which a sign-assisted random precoding scheme was briefly discussed. Nevertheless, the analysis in [21] is confined to the case where the signal to be detected is known. The results, therefore, are not suitable in evaluating the performance of the GLRT detector developed in this work. As compared with [21], our current work conducts a thorough analysis and provides a fundamental understanding of the behavior of the proposed precoding strategies in detecting unknown deterministic signals.

The aim of this paper is to develop effective linear precoding strategies and study their detection error exponents in the asymptotic regime. Specifically, we propose in this paper two different precoding schemes, namely, a random precoding scheme and a sign-assisted random precoding scheme. The random precoding scheme, randomly generating its precoding vector following a certain distribution, does not need any prior knowledge of the signal. The sign-assisted random precoding scheme, however, assumes the knowledge of the plus/minus signs of the signal components, and utilizes this information for precoding design. We study the performance of the precoding schemes under the asymptotic regime where the number of sensors tends to infinity. Our theoretical analysis leads to two major findings. Firstly, exploiting the sign information has the potential to radically improve the detection accuracy of precoding-based schemes: the sign-assisted random precoding scheme can provide superior performance for scenarios where the random precoding scheme fails. Secondly, precoding schemes are potentially more effective than the energy detector [22] in detecting weak signals that are overwhelmed by noise. Specifically, if the signal to be detected has a balanced energy distribution across its components, the sign-assisted random precoding scheme outperforms the energy detector when the observation signal-to-noise ratio (SNR) is less than $1/(\pi - 2)$. In this low SNR regime, the energy detector performs barely satisfactorily but the sign-assisted random precoding scheme still renders reliable and accurate detection accuracy.

We note that, besides the studies in the framework of parallel (fusion) architectures, there is a rich literature on another class of decentralized detection work where no fusion center is required and sensors reach consensus (in decision) through local interactions with neighboring nodes, e.g. [23–28]. This type of work is attractive due to scalability and improved robustness against link failure. Specifically, [23] considered consensus-based detection operating in two phases, namely, a sensing phase in which sensors collect measurements and a communication phase in which sensors exchange data with their neighboring nodes and run the consensus algorithm to reach consensus. In some other works, e.g. [24, 26, 28], sensing and communication are operating simultaneously. Readers are referred to [25] for a more detailed discussion of this type of work. In addition to the above type of work, the study of decentralized detection in a serial (tandem) network (e.g. [29–31]) has also attracted interest because the simple serial structure could serve as a basis for the analysis of more complicated tree architectures.

The rest of the paper is organized as follows. In Section II, we introduce the data model, basic assumptions, and the decentralized detection problem. Section III develops a GLRT detector at the FC. Two precoding strategies are introduced in Section IV, and their asymptotic performance is analyzed in Section V. Comparison with the energy detector is discussed in Section VI, followed by numerical results in Section VII and concluding remarks in Section VIII.

II. PROBLEM FORMULATION

We consider a binary hypothesis testing problem in which a number of sensors collaborate to detect the presence of an unknown deterministic process. Each sensor collects p noisy samples of the process: $\mathbf{x}_n = [x_n(1) \dots x_n(p)]^T$. The binary hypothesis testing problem is formulated as follows:

$$\begin{aligned} H_0 :& \quad \mathbf{x}_n = \mathbf{w}_n, & \forall n = 1, \dots, N \\ H_1 :& \quad \mathbf{x}_n = \boldsymbol{\theta} + \mathbf{w}_n, & \forall n = 1, \dots, N \end{aligned} \tag{1}$$

where $\boldsymbol{\theta} = [\theta_1 \dots \theta_p]^T$ is the signal vector containing signal samples obtained by sampling the process at different time instances, $\mathbf{w}_n \in \mathbb{R}^p$ denotes the additive multivariate Gaussian noise with zero mean and covariance matrix $\sigma_n^2\mathbf{I}$, and the noise variance σ_n^2 is known a priori. In practice, the knowledge of σ_n^2 may come from sample estimation after a training phase. The noise $\{w_n\}$ is assumed independent across the sensors. In many previous works (e.g. [19]), the signal to be detected is modeled as a random process. Nevertheless, these stochastic methods require knowledge of the pdf of the signal, which may not be available in practice. Also, the inaccuracy of the prior knowledge of the signal's pdf could severely degrade the detection performance. Here we model the signal to be detected as a deterministic signal. Our objective is to detect the presence of the unknown deterministic vector signal $\boldsymbol{\theta}$ from noise-corrupted measurements.

To meet the stringent bandwidth/power budgets inherent in WSNs, each sensor sends only one real-valued message to the FC, where a global decision is made (see Fig. 1). Let $\mathbf{c}_n \in \mathbb{R}^{q_n}$ denote the precoding vector for sensor n. To focus on the precoding vector design, we

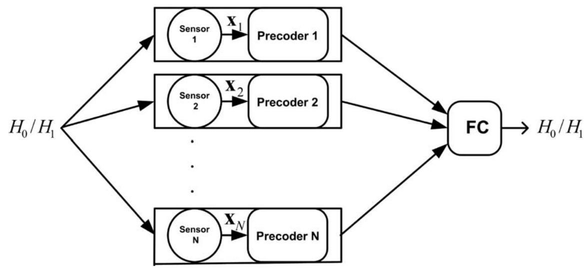

Fig. 1. Decentralized detection of unknown deterministic vector signal. Each node processes its vector observations through linear precoder. Messages are then sent to FC to form final decision.

assume a perfect channel scenario where the real-valued message can be received by the FC without distortion. The signal received from the nth sensor is then given by

$$y_n = \mathbf{c}_n^T \mathbf{x}_n \quad n = 1, \ldots, N \tag{2}$$

The FC, based upon the received data $\{y_n\}_{n=1}^N$, forms a final decision concerning the presence or absence of $\boldsymbol{\theta}$. The problem of interest consists of two aspects: 1) develop a detector to detect $\boldsymbol{\theta}$ and provide an estimate of $\boldsymbol{\theta}$ if the signal is present, 2) develop suitable precoding strategies and analyze their corresponding detection performance. In the following, assuming that the precoding vectors are prespecified, we first develop a GLRT detector at the FC. Precoding strategies are then discussed and analyzed.

III. GLRT DETECTOR

Suppose that the precoding vectors $\{\mathbf{c}_n\}$ are predetermined. We can use a GLRT that replaces the unknown parameters with their maximum likelihood estimates (MLEs). In the case where there are no unknown parameters under H_0, the GLRT decides H_1 if

$$L_G(\mathbf{y}) = \frac{p(\mathbf{y}|\hat{\boldsymbol{\theta}}; H_1)}{p(\mathbf{y}|H_0)} > \eta \tag{3}$$

where $\mathbf{y} \triangleq [y_1\ y_2\ \ldots\ y_N]^T$, and $\hat{\boldsymbol{\theta}}$ is the MLE of $\boldsymbol{\theta}$ found by maximizing

$$p(\mathbf{y}|\boldsymbol{\theta}; H_1) = \frac{1}{(2\pi)^{N/2}|\boldsymbol{\Sigma}|^{1/2}} \times \exp\left\{-\frac{1}{2}(\mathbf{y} - \mathbf{P}\boldsymbol{\theta})^T \boldsymbol{\Sigma}^{-1}(\mathbf{y} - \mathbf{P}\boldsymbol{\theta})\right\} \tag{4}$$

in which $\boldsymbol{\Sigma}$ is a diagonal matrix with its nth diagonal element given by $\sigma_n^2 \mathbf{c}_n^T \mathbf{c}_n$, and

$$\mathbf{P} \triangleq [\mathbf{c}_1\ \mathbf{c}_2\ \ldots\ \mathbf{c}_N]^T \tag{5}$$

The MLE of $\boldsymbol{\theta}$ can be solved by taking the logarithm of $p(\mathbf{y}|\boldsymbol{\theta}; H_1)$ and setting the first derivative with respect to $\boldsymbol{\theta}$ equal to zero, which gives

$$\hat{\boldsymbol{\theta}} = (\mathbf{P}^{\mathbf{T}} \boldsymbol{\Sigma}^{-1} \mathbf{P})^{\dagger} \mathbf{P}^{\mathbf{T}} \boldsymbol{\Sigma}^{-1} \mathbf{y} \tag{6}$$

where † denotes the pseudoinverse of a matrix. Substituting $\hat{\theta}$ back into (3), we have

$$\ln L_G(\mathbf{y}) = \frac{1}{2} \mathbf{y}^T \boldsymbol{\Sigma}^{-1} \mathbf{P}(\mathbf{P}^T \boldsymbol{\Sigma}^{-1} \mathbf{P})^{\dagger} \mathbf{P}^T \boldsymbol{\Sigma}^{-1} \mathbf{y} \tag{7}$$

In other words, we can decide H_1 if

$$2 \ln L_G(\mathbf{y}) = \mathbf{y}^T \boldsymbol{\Sigma}^{-1} \mathbf{P}(\mathbf{P}^T \boldsymbol{\Sigma}^{-1} \mathbf{P})^{\dagger} \mathbf{P}^T \boldsymbol{\Sigma}^{-1} \mathbf{y} > \eta' \tag{8}$$

From [32, Sect. 6.5], we know that when $N \to \infty$, the test statistic $2 \ln L_G(\mathbf{y})$ follows a central chi-squared pdf with p degrees of freedom under the null hypothesis, and a noncentral chi-squared pdf with p degrees of freedom and noncentrality parameter λ under the alternative hypothesis. The noncentrality parameter λ can be computed as

$$\begin{aligned} \lambda &= (\boldsymbol{\theta}_1 - \boldsymbol{\theta}_0)^T \mathbf{I}(\boldsymbol{\theta}_0)(\boldsymbol{\theta}_1 - \boldsymbol{\theta}_0) \\ &= \boldsymbol{\theta}^T \mathbf{P}^T \boldsymbol{\Sigma}^{-1} \mathbf{P} \boldsymbol{\theta} = \boldsymbol{\theta}^T \left(\sum_{n=1}^N \frac{\mathbf{c}_n \mathbf{c}_n^T}{\sigma_n^2 \mathbf{c}_n^T \mathbf{c}_n} \right) \boldsymbol{\theta} \end{aligned} \tag{9}$$

where $\boldsymbol{\theta}_0 = \mathbf{0}$ and $\boldsymbol{\theta}_1 = \boldsymbol{\theta}$ denote the value of $\boldsymbol{\theta}$ under H_0 and H_1, respectively, and $\mathbf{I}(\boldsymbol{\theta}) = \mathbf{P}^T \sum^{-1} \mathbf{P}$ denotes the Fisher information matrix (FIM), which can be readily obtained by computing the expectation of the second derivative of $\ln p(\mathbf{y}|\boldsymbol{\theta})$. We see that the noncentrality parameter λ is a function of the precoding vectors. This suggests that the detection performance fundamentally relies on the choice of the precoding vectors $\{\mathbf{c}_n\}$.

IV. PRECODING STRATEGIES

If the knowledge of the signal $\boldsymbol{\theta}$ is available, the optimal precoding vector has been shown to be a matched filter in the conventional centralized scenario [32] or in a power-constrained distributed scenario [21]. Nevertheless, when $\boldsymbol{\theta}$ is unknown, determining the optimal precoding vector is impossible. In this case, effective precoding strategies with appealing detection performance need to be developed. In the following, we introduce two precoding schemes, namely, a random precoding scheme and a sign-assisted random precoding scheme. For the random precoding scheme, sensors design their own precoders without any knowledge of the signal to be detected. The sign-assisted random precoding scheme needs to know the plus/minus signs of the signal components. Nevertheless, as we show later, its detection performance can be radically improved by utilizing the sign knowledge of the unknown signal.

A. Random Precoding

The precoder at each sensor is randomly generated according to a Gaussian distribution with zero mean and unit variance, i.e.,

$$c_{n,i} \sim \mathcal{N}(0, 1) \quad \forall n, i \tag{10}$$

where $\mathbf{c}_{n,i}$ denotes the ith component of the precoding vector $\mathbf{c}_n$. For this precoding scheme, no knowledge of the signal is needed. We note that, instead of employing a common precoder, the proposed scheme independently generates the precoding vector for each sensor. The reason is that a common precoder, although it admits a simple implementation, fails to provide consistent detection accuracy since its performance is critically dependent on the signal being detected. Using a set of randomly generated vectors has an averaging effect which helps improve the detection reliability. In addition, when the number of sensors is far greater than the signal dimension, i.e., $N \gg p$, the independently generated precoders guarantee that the full column rank condition of $\mathbf{P}$ is satisfied with a high probability. In this case, a joint detection and estimation can be accomplished, that is, an effective estimate of the signal can be provided given that the signal is being detected, otherwise the MLE (6) involves an ill-posed inverse problem.

The GLRT detector developed at the FC requires the information of the precoding vectors $\{\mathbf{c}_n\}$ to make a final decision [c.f. (6)]. Hence we need to find a way to share the knowledge of the precoding vectors between sensors and the FC, which inevitably incurs additional communication overhead. Note that other precoding-based decentralized detection methods such as [19] also face the same issue. Clearly, letting each sensor report the compressed message along with the precoding vector to the FC is undesirable since it involves sending the same number of real-valued messages as transmitting the original observations to the FC. A feasible solution to address this problem is to have both the FC and sensors equipped with a common lookup table that consists of a number of randomly generated vectors [the entries are generated according to (10)]. In this case, each sensor can randomly select one item from its lookup table as its precoding vector, and send the item index, along with the compressed message, to the FC. The FC recovers the precoding vector from the lookup table based on the received item index. Another way is to let the FC choose a precoding vector for each sensor from the lookup table, and assign the corresponding item index to the sensor. We see that the above scheme incurs minimum communication overhead for sharing the knowledge of the precoding vectors between the FC and sensors. Of course, this comes at the expense of using a certain amount of memory resources at both sensors and the FC.

B. Sign-Assisted Random Precoding

In practice, the plus/minus sign of each component of the signal $\boldsymbol{\theta}$ may be known a priori. For example, in gas leak detection, the atmospheric concentration of the gas measured at node n is given by [33]

$$s_n(t) = \frac{M}{4\pi Dt} \exp\left(-\frac{\|\boldsymbol{d}_n - \boldsymbol{d}_0\|^2}{4Dt}\right)$$

which are quantities greater than zero; here D is the diffusion coefficient of the gas, M is the mass of the gas released at point $\boldsymbol{d}_0$, $\boldsymbol{d}_n$ is the position vector of sensor n, and t represents the relative time at which the measurement is taken. In some other scenarios, detection is performed based on signal intensity measurements. As an example, when using microphone sensors to detect acoustic sources, the acoustic intensity measured by sensors can be expressed as [34]

$$s_n(t) = g_n \frac{a(t - t_0)}{\|\boldsymbol{d}_n - \boldsymbol{d}_0\|}$$

where g_n denotes the sensor gain factor of the nth acoustic sensor, $a(t-t_0)$ is the intensity of the acoustic source measured 1 m from the source, and t_0 is the propagation delay of the acoustic source from the source to sensor n, $\boldsymbol{d}_n$ and $\boldsymbol{d}_0$ are the position vectors of sensor n and the source, respectively. Again, in this instance, the measurements at sensors have positive values. The prior knowledge about the plus/minus signs of the signal being detected can be utilized for precoding design to improve the detection performance.

Let sgn($\mathbf{x}$) be a sign column vector with its elements given by sgn(x_i), where sgn(x_i) $= 1$ if $x_i > 0$, and sgn(x_i) $= -1$ otherwise. We design the precoding vector for each sensor as follows

$$\mathbf{c}_n = |\mathbf{r}_n| \odot \text{sgn}(\boldsymbol{\theta}) \quad \forall n \tag{11}$$

where $\mathbf{r}_n$ is a column vector whose entries are independently and randomly generated according to a Gaussian distribution with zero mean and unit variance, $|\mathbf{r}_n|$ takes the absolute value of each entry of $\mathbf{r}_n$, and $\odot$ denotes the entry-wise multiplication. The rationale behind this precoding design is to preserve the signal energy as much as possible by aligning the signs of the signal components. This explains the use of the term sgn($\boldsymbol{\theta}$) in (11).

For the sign-assisted random precoding scheme, sharing the knowledge of precoding vectors between sensors and the fusion center can also be achieved by equipping a common lookup table at both the FC and sensors. First, the FC broadcasts the sign information of the signal to all sensors. Each sensor then randomly selects one item (i.e. a random vector) from its lookup table and designs its precoder according to (11) by using the knowledge of signs. This sign-assisted precoding vector can be easily recovered at the FC based on the received item index and signs of the signal components.

V. ASYMPTOTIC PERFORMANCE ANALYSIS

In this section, we examine the exponential rate of decay in the miss probability as the number of sensors N tends to infinity. The decaying rate in miss probability, also referred to as the error exponent, is a natural performance measure for large-scale sensor networks. In some other works (e.g. [35–37]), their proposed decentralized detection strategies were also evaluated by the error exponent metric under the same asymptotic regime $N \to \infty$. The detection error exponent is defined as

$$K \triangleq \lim_{N\to\infty} -\frac{1}{N} \log P_{\mathrm{M}}(N) \tag{12}$$

where P_{M} denotes the miss probability, and N inside () indicates the dependence of the miss probability on N. The error exponent not only gives a rough idea of how many sensors are needed in order to attain a certain level of detection performance, but also reveals how the test power is influenced by other parameters such as the SNR and the energy distribution pattern of the signal to be detected.

Recall that the test statistic $2 \ln L_G(\mathbf{y})$ follows a central or a noncentral chi-squared pdf with p degrees of freedom. The chi-squared pdf arises as a result of summing the squares of p independent Gaussian random variables with zero or nonzero means. According to the central limit theorem, the test statistic $2 \ln L_G(\mathbf{y})$ can be approximated by a Gaussian distribution for a sufficiently large p. As pointed out in [22], in practice, this approximation is accurate even for a moderately large p, say, $p \geq 10$. It can be easily verified that $2 \ln L_G(\mathbf{y})$ is asymptotically normally distributed as

$$2 \ln L_G(\mathbf{y}) \overset{a}{\sim} \begin{cases} \mathcal{N}(p, 2p) & H_0 \\ \mathcal{N}(p+\lambda, 2p+4\lambda) & H_1 \end{cases} \quad (13)$$

Based on (13), a closed-form expression of the detection probability can be derived. Suppose that P_{FA} is the specified false alarm probability. We have

$$P_{\text{FA}} = \mathbf{Pr}(2 \ln L_G(\mathbf{y}) > \eta; H_0) = Q\left(\frac{\eta - p}{\sqrt{2p}}\right) \quad (14)$$

which gives $\eta = \sqrt{2p}Q^{-1}(P_{\text{FA}}) + p$. The miss probability can therefore be written as

$$\begin{aligned} P_{\text{M}} &= 1 - \mathbf{Pr}(2 \ln L_G(\mathbf{y}) > \eta; H_1) \\ &= 1 - Q\left(\frac{\sqrt{2p}Q^{-1}(P_{\text{FA}}) - \lambda}{\sqrt{2p+4\lambda}}\right) \end{aligned} \quad (15)$$

We see that the detection performance of the GLRT detector is determined by the noncentrality parameter λ. The expression inside $Q(\cdot)$, i.e. $(\sqrt{2p}Q^{-1}(P_{\text{FA}}) - \lambda)/\sqrt{2p+4\lambda}$ can be proved to be a monotonically decreasing function of λ (proof is provided in Appendix A). Hence a larger λ results in better detection performance.

To compute the exponential rate of decay in miss probability, we need to analyze the asymptotic behavior of λ when $N \to \infty$. For simplicity, we first assume a homogeneous scenario where $\sigma_n^2 = \sigma^2$ for all n (the extension to the inhomogeneous scenario is discussed later).

In this case, λ can be expressed as

$$\lambda = \frac{1}{\sigma^2} \sum_{n=1}^{N} \frac{\boldsymbol{\theta}^T \mathbf{c}_n \mathbf{c}_n^T \boldsymbol{\theta}}{\mathbf{c}_n^T \mathbf{c}_n} \quad (16)$$

Define β_n as a metric measuring the correlation between the two vectors $\mathbf{c}_n$ and $\boldsymbol{\theta}$

$$\beta_n \triangleq \frac{(\mathbf{c}_n^T \boldsymbol{\theta})^2}{\|\mathbf{c}_n\|^2 \|\boldsymbol{\theta}\|^2} \quad (17)$$

Note that for both precoding schemes, precoding vectors are independently generated according to a common distribution. Hence $\{\beta_n\}$ are independent and identically distributed (IID) random variables. As $N \to \infty$, the ratio of the noncentrality parameter λ (16) to the number of sensors N therefore approaches

$$\frac{\lambda}{N} = \frac{p\gamma}{N} \sum_{n=1}^{N} \beta_n \overset{N\to\infty}{\to} p\gamma E[\beta_n] \quad (18)$$

where $\gamma \triangleq \|\boldsymbol{\theta}\|^2/(p\sigma^2)$ is the average SNR of sensor observations, and the last equality comes from the strong law of large numbers (LLN).

A. Asymptotic Behavior of λ for Random Precoding

To compute (18), we first analyze the distribution of β_n. Let $\bar{\boldsymbol{\theta}}$ be the normalized vector of $\boldsymbol{\theta}$, i.e., $\bar{\boldsymbol{\theta}} \triangleq \boldsymbol{\theta}/\|\boldsymbol{\theta}\|_2$. Then β_n can be expressed as

$$\beta_n = \frac{(\mathbf{c}_n^T \bar{\boldsymbol{\theta}})^2}{\mathbf{c}_n^T \mathbf{c}_n} = \frac{\mathbf{c}_n^T \bar{\boldsymbol{\theta}} \bar{\theta}^T \mathbf{c}_n}{\mathbf{c}_n^T \mathbf{c}_n} \quad (19)$$

Consider the eigenvalue decomposition (EVD) of $\bar{\boldsymbol{\theta}}\bar{\boldsymbol{\theta}}^T$: $\bar{\boldsymbol{\theta}}\bar{\boldsymbol{\theta}}^T = \mathbf{QDQ}^T$, where $\mathbf{Q} = [\bar{\boldsymbol{\theta}}\ \mathbf{q}_2\ \dots\ \mathbf{q}_p]$ is orthogonal, and $\mathbf{D} = \text{diag}\ \{1\ 0 \dots 0\}$, then the numerator and the denominator of β_n can be written, respectively, as

$$\begin{aligned} \mathbf{c}_n^T \bar{\boldsymbol{\theta}}\bar{\boldsymbol{\theta}}^T \mathbf{c}_n &= e_1^2 \\ \mathbf{c}_n^T \mathbf{c}_n &= e_1^2 + e_2^2 \dots + e_p^2 \end{aligned} \quad (20)$$

where $\mathbf{e} = [e_1\ e_2 \dots e_p]^T = \mathbf{Q}^T\ \mathbf{c}_n$. It can be easily verified that $\mathbf{e}$, the rotation of $\mathbf{c}_n$, contains IID normalized Gaussian entries as well. Furthermore, β_n can be written as

$$\beta_n = \frac{e_1^2}{e_1^2 + e_2^2 \dots + e_p^2} = \frac{1}{1 + \sum_{i=2}^{p} e_i^2/e_1^2} \quad (21)$$

where $\sum_{i=2}^{p} e_i^2 \sim \chi_{p-1}^2$ follows a central chi-squared distribution with $p - 1$ degrees of freedom, $e_1^2 \sim \chi_1^2$ follows a central chi-squared distribution with one degree of freedom, and the two chi-squared random variables $\sum_{i=2}^{p} e_i^2$ and e_1^2 are independent. From [32], we know that

$$f \triangleq \frac{\sum_{i=2}^{p} e_i^2}{(p-1)e_1^2} \sim F_{p-1,1} \quad (22)$$

That is, f follows an F distribution with $p - 1$ numerator degrees of freedom and one denominator degrees of freedom. We can express β_n in terms of f as

$$\beta_n = \frac{1}{1 + (p-1)f} \quad (23)$$

According to [38] (also see [39, Prop. 5]), β_n follows a Beta distribution with parameters 1/2 and $(p - 1)/2$, i.e.,

$$\beta_n \sim \beta_{1/2,(p-1)/2} \quad (24)$$

Recalling the properties of the Beta distribution, we have

$$E[\beta_n] = \frac{1}{p} \quad (25)$$

Combining (18) and (25), we arrive at

$$\frac{\lambda_{\text{RP}}}{N} \overset{N\to\infty}{\to} \gamma \quad (26)$$

where the subscript "RP" stands for the random precoding scheme.

B. Asymptotic Behavior of λ for Sign-Assisted Precoding

For the sign-assisted random precoding, previous analyses for random precoding are no longer applicable because the design of the precoding vectors $\{\mathbf{c}_n\}$ is dependent on the signal $\boldsymbol{\theta}$. We, instead, examine the expectation of β_n directly. Clearly, we have

$$E[\beta_n] = E\left[\frac{(\mathbf{c}_n^T\boldsymbol{\theta})^2}{\|\mathbf{c}_n\|^2\|\boldsymbol{\theta}\|^2}\right] \overset{(a)}{=} E\left[\frac{\left(\sum_{i=1}^{p}|r_{n_i}\|\theta_i|\right)^2}{\|\mathbf{r}_n\|^2\|\boldsymbol{\theta}\|^2}\right] = \frac{1}{\|\boldsymbol{\theta}\|^2}\sum_{i=1}^{p}\sum_{j=1}^{p}|\theta_i\|\theta_j|E\left[\frac{|r_{n_i}\|r_{n_j}|}{\|\mathbf{r}_n\|^2}\right] \quad (27)$$

where (a) comes from (11), r_{n_i} denotes the ith entry of $\mathbf{r}_n$. To calculate $E[\beta_n]$, we need to compute $E[|r_{n_i}\|r_{n_j}|/\|\mathbf{r}_n\|^2]$. An exact computation seems difficult as the numerator and the denominator are correlated. Nevertheless, their correlation is very mild as long as p is not too small. To see this, notice that the two random variables r_{n_i} and $\|\mathbf{r}_n\|^2$ are uncorrelated, i.e., their correlation coefficient is equal to zero (details are provided in Appendix B). Since r_{n_i} follows a Gaussian distribution and $\|\mathbf{r}_n\|^2$ can be accurately approximated as a Gaussian random variable even for a moderately large p (say $p \geq 10$), the random variable r_{n_i} is therefore approximately independent of $\| \mathbf{r}_n \|^2$. Consequently, $|r_{n_i}|$ and $\| \mathbf{r}_n \|^2$ are independent. Hence we have

$$E\left[\frac{|r_{n_i}\|r_{n_j}|}{\|\mathbf{r}_n\|^2}\right] \approx E[|r_{n_i}\|r_{n_j}|]E\left[\frac{1}{\|\mathbf{r}_n\|^2}\right] \overset{(a)}{=} \frac{E[|r_{n_i}\|r_{n_j}|]}{p-2} \overset{(b)}{=} \begin{cases} \dfrac{2}{\pi(p-2)} & i \neq j \\ \dfrac{1}{p-2} & i = j \end{cases} \quad (28)$$

where (a) comes from the fact that $1/\|\mathbf{r}_n\|^2$ follows an inverse-chi-squared distribution with p degrees of freedom; (b) comes by noting that $|r_{n_i}|$ and $|r_{n_i}|^2$ are random variables following a half-normal distribution with $E[|r_{n_i}|] = \sqrt{2/\pi}$, and a chi-squared distribution with one degree of freedom, respectively.

Combining (27)–(28), we arrive at

$$E[\beta_n] = \frac{\pi + 2(\varphi - 1)}{\pi(p-2)} \quad (29)$$

where

$$\varphi \triangleq \frac{(\sum_{i=1}^{p}|\theta_i|)^2}{\boldsymbol{\theta}^T\boldsymbol{\theta}} \quad (30)$$

is a factor characterizing the energy distribution pattern of the signal $\boldsymbol{\theta}$. Using the Cauchy-Schwarz inequality, we derive that φ is in the range $1 \leq \varphi \leq p$, in which φ reaches its upper bound p when the absolute values of the entries $\{\theta_i\}$ are identical, and attains its lower bound 1 if there is only one nonzero entry in $\boldsymbol{\theta}$.

Substituting $E[\beta_n]$ into (18), the ratio of the asymptotic λ to the number of sensors N is given by

$$\frac{\lambda_{\text{SAP}}}{N} \overset{N\to\infty}{\to} p\gamma\frac{\pi + 2(\varphi - 1)}{\pi(p-2)} \quad (31)$$

where the subscript "SAP" represents the sign-assisted random precoding scheme.

REMARKS Since a larger λ results in better detection performance, the sign-assisted random precoding scheme is most effective when the signal has a uniform energy distribution across its components, i.e., the components $\{\theta_i\}$ have identical amplitudes, in which case φ achieves its upper bound p. On the other hand, if the energy distribution is highly unbalanced, exploiting sign information does not help much. In particular, when there is only one nonzero component in $\boldsymbol{\theta}$, we have $\varphi = 1$ and the sign-assisted random precoding performs similarly as the random precoding scheme. Nevertheless, many natural signals such as the concentration of a chemical due to gas leak are persistent over a certain period of time and have a balanced time-domain power distribution. The quantity φ of these signals can be very close to the upper bound p. In addition, numerical results show that the value φ associated with a sinusoidal signal is usually no less than $0.8p$. As we show later, the sign-assisted random precoding scheme presents a significant performance advantage over the random precoding scheme in detecting those "well-shaped" signals.

C. Error Exponents

We now analyze the error exponents associated with the two precoding schemes. Since the noncentrality parameter λ for both precoding schemes is proportional to N, λ becomes a dominant factor as $N \to \infty$. Hence the limit of the miss probability (15) as $N \to \infty$ is

$$P_{\text{M}} \overset{N\to\infty}{\to} Q\left(\frac{\sqrt{\lambda}}{2}\right) \quad (32)$$

Using the Chernoff bound of Q-function: $Q(x) \leq \frac{1}{2}\exp\left(-\frac{x^2}{2}\right), \forall x > 0$,the miss probability is upper bounded by

$$P_{\text{M}} \leq \frac{1}{2}\exp\left(-\frac{\lambda}{8}\right) \quad (33)$$

This bound is tight when λ becomes large. With the derived results (26) and (31), the error exponents for both precoding schemes are, respectively, given by

$$K_{\text{RP}} \triangleq \lim_{N\to\infty} -\frac{1}{N}\log P_{\text{M}-\text{RP}} = \frac{\gamma}{8}$$
$$K_{\text{SAP}} \triangleq \lim_{N\to\infty} -\frac{1}{N}\log P_{\text{M}-\text{SAP}} = \frac{p\gamma(\pi+2(\varphi-1))}{8\pi(p-2)} \quad (34)$$

We see that for both precoding strategies, their miss probabilities converge exponentially to zero as the number of sensors increases. The sign-assisted random precoding scheme achieves a larger error exponent than the random precoding scheme. In particular, the best error exponent is attained when the signal components $\{\theta_i\}$ are of equal power. In this case, we have $\varphi = p$, and the error exponent of the sign-assisted random precoding scheme is approximately equal to

$$K_{\mathrm{SAP}} \approx \frac{p\gamma}{4\pi} \tag{35}$$

which is about $2p/\pi$ times the error exponent of the random precoding scheme.

We conduct experiments to illustrate the effectiveness of the error exponent in predicting the behavior of the error rate for finite N. To this objective, we introduce the notion "finite-N exponential error rate" which was originally proposed in [36] to quantify the accuracy of the error exponent in finite sample approximation. The finite exponential rate is defined as

$$K(N) \triangleq -\frac{1}{N}\log P_{\mathrm{M}}(N)$$

where $\lim_{N\to\infty} K(N) = K$. In our experiments, we set $p = 20$, and $P_{\mathrm{FA}} = 0.05$. The signal is assumed to be a sample cosine signal $\alpha\cos(2\pi fk)$, where $k = 1, \ldots, p, f = 1/(2\pi)$, and α is the amplitude of the cosine signal. Also, we assume a homogeneous case where all sensors have identical noise variances $\sigma_n^2 = 1, \forall n$. In Fig. 2, we plot the error exponents and the finite exponential rates as a function of sensor SNR, γ, in which the error exponents K_{RP} and K_{SAP} are determined from (34). To compute the finite exponential rates, for each realization of $\{\mathbf{c}_n\}$, we compute λ using (9) and then calculate the miss probabilities of the two precoding schemes according to the chi-squared distributions. The finite exponential rates are then obtained by averaging over 10^4 independent realizations of $\{\mathbf{c}_n\}$. From Fig. 2, we see that the theoretical error exponents provide a good approximation to the finite-sample behavior. For both precoding schemes, the finite exponential rate has a tendency to approach the error exponent when the number of sensors increases.

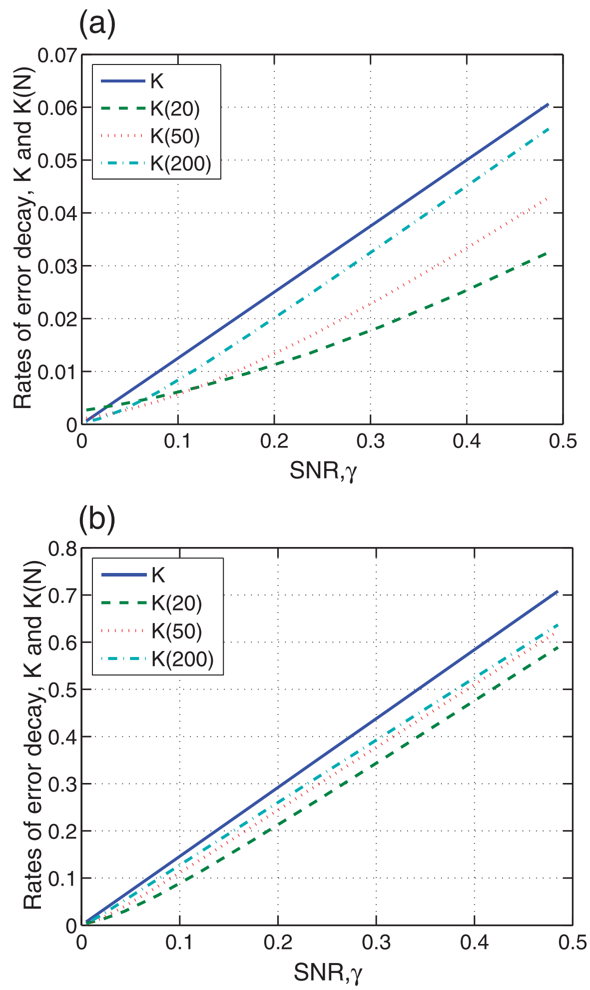

Fig. 2. Error exponents and finite error exponential rates vs. SNR. (a) Random precoding. (b) Sign-assisted random precoding.

D. Extension to the Inhomogeneous Case

The above results can be readily extended to the inhomogeneous scenario where the noise variances across sensors are different, i.e., $\mathbf{R}_{w_n} = \sigma_n^2\mathbf{I}$. In this case, the ratio of the noncentrality parameter λ (9) to the number of sensors N becomes

$$\begin{aligned}\frac{\lambda}{N} &= \frac{1}{N}\sum_{n=1}^{N}\frac{\boldsymbol{\theta}^T\mathbf{c}_n^T\mathbf{c}_n\boldsymbol{\theta}}{\sigma_n^2\mathbf{c}_n\mathbf{c}_n^T} = \frac{1}{N}\sum_{n=1}^{N}\frac{(\mathbf{c}_n^T\boldsymbol{\theta})^2}{\|\mathbf{c}_n\|^2\|\boldsymbol{\theta}\|^2}\frac{\|\boldsymbol{\theta}\|^2}{\sigma_n^2}\\ &\overset{(a)}{=} \frac{p}{N}\sum_{n=1}^{N}\beta_n\gamma_n \overset{a.s}{\to} pE\left[\frac{1}{N}\sum_{n=1}^{N}\beta_n\gamma_n\right]\\ &= \frac{p}{N}E[\beta_n]\sum_{n=1}^{N}\gamma_n \triangleq pE[\beta_n]\bar{\gamma}\end{aligned} \tag{36}$$

where in (a), $\gamma_n \triangleq \|\boldsymbol{\theta}\|^2/(p\sigma_n^2)$ denotes nth sensor's local observation SNR, the approximation in the second line comes from the LLN for independent but nonidentically distributed random variables: the sample average converges almost surely to the expected value, i.e., $\bar{X} \triangleq \frac{1}{n}(X_1 + \ldots + X_n) \overset{a.s}{\to} E[\bar{X}]$. Note that $\{\beta_n\}$ are IID, hence $\{\gamma_n\beta_n\}$ are independent but nonidentically distributed. Therefore our analyses in previous subsections hold valid except with γ replaced by the average SNR $\bar{\gamma}$.

VI. COMPARISON WITH EXISTING METHODS

In this section, we compare our precoding-based detection schemes with the energy detector. Energy detector is an effective approach to detect unknown deterministic signals in practice. It measures the energy of the observed signal and then uses a single threshold to determine the presence or absence of the signal. In [22], the conventional energy detector for a single sensor system is extended to the multisensor system where multiple sensors (cognitive radios) collaborate to detect a common signal. We briefly review this work [22] before the comparison.

A. Review of [22]

In [22], a binary hypothesis testing problem that has the same problem formulation as ours was examined in the context of cognitive radio systems, where a number of secondary users cooperate to detect the presence of a primary user. An energy detector was developed in such a decentralized framework. Each sensor computes the energy of the observed signal x_n

$$u_n = \|\mathbf{x}_n\|^2$$

The test statistics $\{u_n\}$ are then sent to the FC for reaching a global decision. Clearly, u_n follows a central (noncentral) chi-squared distribution under hypothesis H_0 (H_1). To facilitate the analysis, u_n is approximated by a Gaussian random variable in [22] (this approximation is accurate for a moderately large p, say, $p \geq 10$). At the FC, an LRT-based detector is optimal and desirable. However, as pointed out by [22], the LRT-based fusion rule has a quadratic form. Finding the probability distribution of the likelihood ratio is numerically difficult since it involves many integrals. To overcome this difficulty, [22] linearly combines the received test statistics $\{u_n\}$ to form a global test statistic

$$y = \sum_{n=1}^{N} w_n u_n = \mathbf{w}^T \mathbf{u} \quad (37)$$

where $\mathbf{w}$ is the weight vector used to represent every sensor's contribution to the global decision. The detection probability of this linear cooperative method is given as [22]

$$P_{\text{D}} = Q\left(\frac{Q^{-1}(P_{\text{FA}})\sqrt{\mathbf{w}^T \boldsymbol{\Sigma}_{H_0}\mathbf{w}} - E_s \mathbf{1}^T \mathbf{w}}{\sqrt{\mathbf{w}^T \boldsymbol{\Sigma}_{H_1}\mathbf{w}}}\right) \quad (38)$$

where $\boldsymbol{\Sigma}_{H_0}$ and $\boldsymbol{\Sigma}_{H_1}$ represent the covariance matrices of $\mathbf{u}$ under hypotheses H_0 and H_1, respectively, and E_s denotes the signal energy. The design of the weight vector $\mathbf{w}$ was examined in [22], aiming at minimizing the miss probability under a false alarm constraint. Nevertheless, as indicated in [22], an analytical optimal solution of $\mathbf{w}$ is generally difficult to obtain, which obstructs a subsequent performance evaluation. For simplicity, we hereby consider a homogeneous scenario where sensors have identical noise variances. In this case, sensors are of equal importance in making the global decision. Thus we should have $\mathbf{w} = \mathbf{1}, \mathbf{1}$ denotes a column vector with its entries all equal to one. The detection probability can therefore be simplified as

$$P_{\text{D}} = Q\left(\frac{Q^{-1}(P_{\text{FA}})\sqrt{2p} - \sqrt{N}\gamma p}{\sqrt{2p + 4\gamma p}}\right) \quad (39)$$

where $\gamma = \boldsymbol{\theta}^T \boldsymbol{\theta}/(p\sigma^2)$.

B. Asymptotic Regime $N \to \infty$

When N tends to infinity, the miss probability of the energy detector asymptotically approaches

$$P_{\text{M}-\text{ED}} \overset{N\to\infty}{\to} Q\left(\frac{\sqrt{N}\sqrt{p}\gamma}{\sqrt{2+4\gamma}}\right) \quad (40)$$

Using the Chernoff bound of the Q-function, the error exponent of the energy detector is given by

$$K_{\text{ED}} \triangleq \lim_{N\to\infty} -\frac{1}{N} \log P_{\text{M}-\text{ED}} = \frac{p\gamma^2}{4+8\gamma} \quad (41)$$

Comparing (34) and (41), it can be derived that the error exponents associated with the precoding-based schemes are greater than that of the energy detector when the SNR of the sensors' observations γ are less than a certain threshold. Specifically, we have

$$\begin{aligned} &K_{\text{RP}} > K_{\text{ED}} \quad \text{if } \gamma < \frac{1}{2p-2} \triangleq \tau_1 \\ &K_{\text{SAP}} > K_{\text{ED}} \quad \text{if } \gamma < \frac{\pi+2(\varphi-1)}{2[\pi(p-3)-2(\varphi-1)]} \triangleq \tau_2 \end{aligned} \quad (42)$$

This suggests that precoding-based schemes are more effective than the energy detector in the low SNR regime. This theoretical result can be explained as follows. The accuracy and sensitivity of the energy detector are critically dependent on the energy of the received signals. For signals that are completely overwhelmed by the noise, the energy detector has difficulty in detecting them. Precoding-based schemes, on the contrary, allow us to exploit the inherent features of the signal to be detected. The signal is learned first and the learned signal is then used for detection. Therefore precoding-based schemes are potential to achieve better performance than the energy detector in detecting weak signals.

Recall that φ is a value within [1 p]. When $\varphi = 1$, we have $\tau_1 \approx \tau_2$. On the other hand, if the signal has a uniform energy distribution among its components, φ achieves its upper bound p, and τ_2 is simplified as

$$\begin{aligned} \tau_2 &= \frac{\pi + 2(p-1)}{2[\pi(p-3) - 2(p-1)]} > \frac{2(p-1)}{(2\pi-4)(p-1)} \\ &= \frac{1}{\pi - 2} \end{aligned} \quad (43)$$

In this case we have

$$K_{\text{SAP}} > K_{\text{ED}} \quad \text{if } \gamma < \frac{1}{\pi-2} \quad (44)$$

which means that the sign-assisted random precoding scheme is more powerful than the energy detector when the SNR is less than $1/(\pi - 2)$.

VII. NUMERICAL RESULTS

We now carry out experiments to corroborate our previous analysis and to illustrate the performance of the proposed precoding-based GLRT detectors.

We first examine the efficiency of the proposed schemes in detecting a weak sinusoidal signal in the presence of noise. The signal is assumed to be a sample cosine signal $\alpha \cos(2\pi fk)$, where $k = 1, \ldots, p, f = 1/(2\pi)$, and α is the amplitude of the cosine signal. Also, we

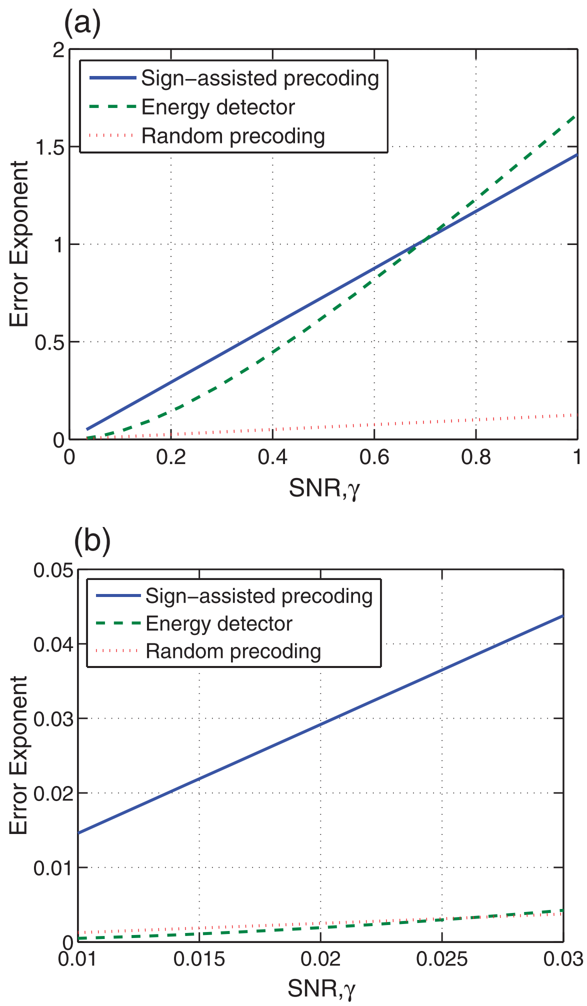

Fig. 3. Error exponents of respective schemes vs. SNR. (a) Error exponents of respective schemes. (b) Enlarged segment of Fig. 3(a).

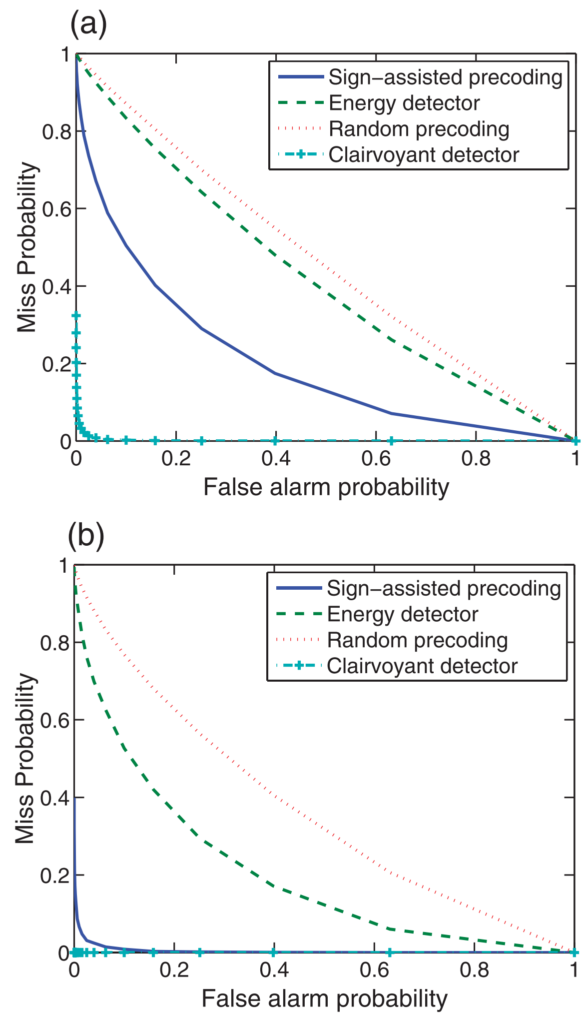

Fig. 4. Miss probabilities vs. false alarm probability for different α. (a) $\alpha = 0.15$. (b) $\alpha = 0.3$.

assume a homogeneous case where all sensors have identical noise variances $\sigma_n^2 = 1, \forall n$. Fig. 3 depicts the error exponents of respective detection schemes as a function of the sensor observation SNR, where we set $p = 20$, and $P_{\mathrm{FA}} = 0.05$. Note that Fig. 3(b) is an enlarged segment of Fig. 3(a) in order to allow a comparison between the energy detector and the random precoding scheme. From Fig. 3(a), we see that the energy detector outperforms precoding schemes when the signal is dominant or comparable to the noise level. Nevertheless, when it comes to the low SNR regime where the signal is overwhelmed by noise, precoding-based detectors are more effective and achieve higher error decaying rates. Specifically, the sign-assisted random precoding scheme is superior to the energy detector when $\gamma < 0.7$ [see Fig. 3(a)]. Note that for sinusoidal signals, the parameter φ is approximately equal to $0.8p$, which is close but not exactly equal to its upper bound p. Hence the point $\gamma = 0.7$ at which the sign-assisted precoding surpasses the energy detector is slightly lower than the threshold $1/(\pi - 2)$ calculated in (43) which is obtained by assuming φ achieves its upper bound. Also, from Fig. 3(b), we observe that the random precoding scheme attains a higher error exponent than the energy detector for $\gamma < 1/(2p - 2) \approx 0.025$, which coincides with the result (42).

To further corroborate our analysis, we conduct Monte Carlo experiments to evaluate the detection performance of the three schemes. For each Monte Carlo run, we generate observations according to the data model (1) (supposing that the signal is present), and the precoding vectors for our proposed precoding schemes according to (10) and (11). We then calculate the test statistic and compare it with a specified threshold (the threshold is determined by the specified false alarm probability) to check whether the detector succeeds or fails in detecting the signal. The miss probability can be computed as the ratio of the number of failures to the total number of independent runs. Fig. 4 depicts the miss probability vs. the false alarm probability of the three schemes, where we set $p = 20$, $N = 80$, and the value of α is set to be 0.15 and 0.3, respectively. The performance of a "clairvoyant" detector, which assumes the knowledge of the signal to be detected, is also included for the comparison. For the clairvoyant detector, since the knowledge of the signal is

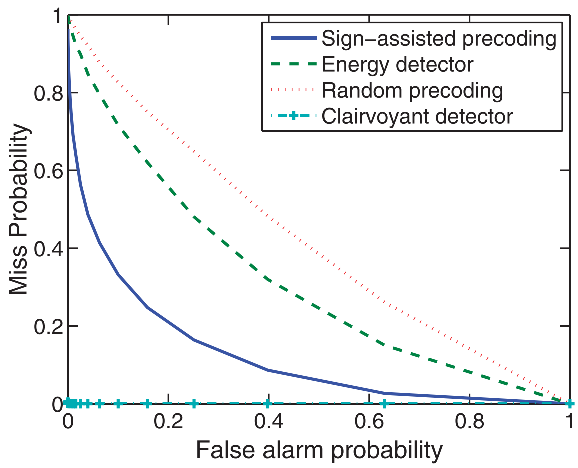

Fig. 5. Miss probabilities of respective schemes vs. false alarm probability for chemical concentration detection.

available, optimal precoding vectors for sensors can be computed (details can be referred to [21]) and an LRT can be employed at the FC to make a global decision. Results are averaged over 10^5 independent runs. We see that for both choices of α whose corresponding SNRs γ are less than $1/(\pi - 2)$, the sign-assisted random precoding scheme presents a clear performance advantage over the energy detector. In particular, when $\alpha = 0.3$, the signal to be detected is completely buried in the noise. In this situation, the energy detector yields barely satisfactory performance. This is not surprising since the energy detector detects the signal based on the energy of the observations. If the signal is relatively small, the energy detector has difficulty in accurately distinguishing the signal from the noise. In contrast, the sign-assisted random precoding still achieves superior detection performance. Also, as expected, the clairvoyant detector, which exploits the knowledge of the signal, outperforms both the precoding-based schemes and the energy detector.

Chemical Concentration Detection: Suppose a number of sensors collaborate to detect a chemical substance of interest. Each sensor measures the atmospheric concentration of the substance in its vicinity. The atmospheric concentration is assumed to have a uniform distribution in space. This assumption holds valid in open environments where the effects of wind or air turbulence help the concentration reach spatial equilibrium in a finite area very quickly. In our simulations, the concentration measurement at each sensor is given by

$$x_n(k) = \alpha \exp(-ck) + w_n(k)$$

where the chemical concentration is assumed to decay exponentially over time; $w_n(k)$ denotes the additive Gaussian noise with zero mean and variance $\sigma^2 = 1$. In practice, the concentration decaying model may not be known to us. Hence we treat $\boldsymbol{\theta} = [\alpha \exp(-ck_1)\ \alpha \exp(-ck_2) \ldots \alpha \exp(-ck_p)]^T$ as an unknown deterministic signal. Note that since the concentration always has a quantity greater than zero, the sign-assisted random precoding scheme can be readily adopted here. We set $a = 0.5$ and $c = 0.2$ in our simulations. Fig. 5 plots the miss probability as a function of the false alarm probability for our proposed schemes, the energy detector, and the clairvoyant detector that assumes the knowledge of the signal. Results are averaged over 10^5 Monte Carlo runs. In the figure, we can see that the sign-assisted random precoding scheme achieves a significant performance improvement as compared with the energy detector in detecting the chemical concentration, which is consistent with our theoretical analysis.

VIII. CONCLUSIONS

We considered a decentralized detection problem in which a number of sensors collaborate to detect the presence of an unknown deterministic vector signal. Due to inherent power/bandwidth constraints, the observations of each sensor are encoded into a real-valued message using a linear precoder. The compressed messages are then transmitted to the FC, where a GLRT detector is employed to reach a final decision. We introduced two precoding strategies in this paper, namely, a random precoding strategy and a sign-assisted random precoding strategy. The detection error exponents associated with these two precoding schemes were analyzed in an asymptotic regime where the number of sensors tends to infinity. Theoretical analysis found that the detection performance of the precoding-based schemes can be radically enhanced by exploiting the knowledge of the plus/minus signs of the signal components. Also, the sign-assisted random precoding scheme provides a substantial performance advantage over the energy detector in the low SNR regime, specifically, when the observation SNR is less than $1/(\pi - 2)$. Numerical results were provided to corroborate our theoretical analysis, and to show the effectiveness of the sign-assisted precoding scheme in detecting weak signals buried in noise.

APPENDIX A. PROOF OF MONOTONICITY OF (15) WRT λ

To prove the monotonicity of (15) with respect to λ, we only need to show that the term inside the Q-function is a monotonically decreasing function of λ. We consider two different cases:

$0 < \lambda \leq \sqrt{2p}Q^{-1}(P_{\rm FA})$: In this case, the numerator and the denominator are both positive. It is easy to verify that the term inside the Q-function decreases with an increasing λ.

$\lambda > \sqrt{2p}Q^{-1}(P_{\rm FA})$: Let $f(\lambda)$ denote the term inside the Q-function (15). The first derivative of $f(\lambda)$ with respect to λ is given by

$$\frac{\partial f(\lambda)}{\partial \lambda} = -\frac{1}{\sqrt{2p + 4\lambda}}\left[1 - \frac{\sqrt{2p}Q^{-1}(P_{\rm FA}) - \lambda}{2p + 4\lambda}\right] \quad (45)$$

Since $\lambda > \sqrt{2p}Q^{-1}(P_{\text{FA}})$, it is easy to see that the first derivative is negative. Therefore $f(\lambda)$ is a monotonically decreasing function of λ.

The proof is completed here.

APPENDIX B. CORRELATION COEFFICIENT BETWEEN r_{n_i} AND $\|\mathbf{r}_n\|^2$

The correlation coefficient between the two random variables $x \triangleq r_{n_i}$ and $y \triangleq \|\mathbf{r}_n\|^2$ can be easily computed as follows

$$\begin{aligned}\rho &= \frac{E\left[r_{n,i}\sum_{i=1}^{p} r_{n,i}^2\right] - E[r_{n,i}]E\left[\sum_{i=1}^{p} r_{n,i}^2\right]}{\sigma_x\sigma_y}\\ &\overset{(a)}{=} \frac{E\left[r_{n,i}\sum_{i=1}^{p} r_{n,i}^2\right]}{\sigma_x\sigma_y}\\ &= \frac{E\left[r_{n,i}^3 + r_{n,i}\sum_{j=1,j\neq i}^{p} r_{n,j}^2\right]}{\sigma_x\sigma_y}\\ &\overset{(b)}{=} 0\end{aligned} \tag{46}$$

where (a) comes from the fact that $\{r_{n,i}\}$ are IID Gaussian random variables with zero mean and unit variance; (b) follows from the fact that the odd-order moments of a standard normal random variable are zero.

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