Signal Processing 102 (2014) 201–206

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Fast communication

Sparse signal recovery from one-bit quantized data: An iterative reweighted algorithm ☆

Jun Fang [a], Yanning Shen [a], Hongbin Li [b,*], Zhi Ren [c]

[a] National Key Laboratory of Science and Technology on Communications, University of Electronic Science and Technology of China, Chengdu 611731, China
[b] Department of Electrical and Computer Engineering, Stevens Institute of Technology, Hoboken, NJ 07030, USA
[c] School of Communication and Information Engineering, Chongqing University of Posts and Telecommunications, Chongqing, China

ARTICLE INFO

Article history:
Received 24 September 2013
Received in revised form
23 January 2014
Accepted 19 March 2014
Available online 28 March 2014

Keywords:
Compressed sensing
One-bit quantization
Iterative reweighted algorithm
Surrogate function

ABSTRACT

This paper considers the problem of reconstructing sparse signals from one-bit quantized measurements. We employ a log-sum penalty function, also referred to as the Gaussian entropy, to encourage sparsity in the algorithm development. In addition, in the proposed method, the logistic function is introduced to quantify the consistency between the measured one-bit quantized data and the reconstructed signal. Since the logistic function has the tendency to increase the magnitudes of the solution, an explicit unit-norm constraint is no longer necessary to be included in our optimization formulation. An algorithm is developed by iteratively minimizing a convex surrogate function that bounds the original objective function. This leads to an iterative reweighted process that alternates between estimating the sparse signal and refining the weights of the surrogate function. Numerical results are provided to illustrate the effectiveness of the proposed algorithm.

1. Introduction

Conventional compressed sensing framework recovers a sparse signal $\boldsymbol{x} \in \mathbb{R}^n$ from only a few linear measurements:

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} \tag{1}$$

where $\boldsymbol{y} \in \mathbb{R}^m$ denotes the acquired measurements, $\boldsymbol{A} \in \mathbb{R}^{m \times n}$ is the sampling matrix, and $m \ll n$. Such a problem has been extensively studied and a variety of algorithms that provide consistent recovery performance guarantee were proposed, e.g. [1,2]. In practice, however, measurements have to be quantized before being further processed. Moreover, in distributed systems where data acquisition is limited by bandwidth and energy constraints, aggressive quantization strategies which compress real-valued measurements into one or only a few bits of information are preferred. This has inspired recent interest in studying compressed sensing based on quantized measurements. Specifically, in this paper, we are interested in an extreme case where each measurement is quantized into one bit of information

$$\boldsymbol{b} = \text{sign}(\boldsymbol{y}) = \text{sign}(\boldsymbol{A}\boldsymbol{x}) \tag{2}$$

where "sign" denotes an operator that performs the sign function element-wise on the vector, the sign function returns 1 for positive numbers and -1 otherwise. Clearly, in this case, only the sign of the measurement is retained

☆ This work was supported in part by the National Science Foundation of China under Grant 61172114, and the National Science Foundation under Grant ECCS-0901066.

* Corresponding author. Tel.: +1 201 216 5604; fax: +1 201 216 8264.
E-mail addresses: JunFang@uestc.edu.cn (J. Fang), 201121260110@std.uestc.edu.cn (Y. Shen), Hongbin.Li@stevens.edu (H. Li), renzhi@cqupt.edu.cn (Z. Ren).

http://dx.doi.org/10.1016/j.sigpro.2014.03.026
0165-1684/

while the information about the magnitude of the signal is lost. This makes an exact reconstruction of the sparse signal $\boldsymbol{x}$ impossible. Nevertheless, if we impose a unit-norm on the sparse signal, it has been shown [3,4] that signals can be recovered with a bounded error from one bit quantized data. Besides, in many practical applications such as source localization, direction-of-arrival estimation, and chemical agent detection, it is the locations of the nonzero components of the sparse signal, other than the amplitudes of the signal components, that have significant physical meanings and are of our ultimate concern. Recent results [5] show that asymptotic reliable recovery of the support of sparse signals is possible even with only one-bit quantized data.

The problem of recovering a sparse or compressible signal from one-bit measurements was first introduced by Boufounos and Baraniuk in their work [6]. Following that, the reconstruction performance from one-bit measurements was more thoroughly studied [3–5,7,8] and a variety of one-bit compressed sensing algorithms such as binary iterative hard thresholding (BIHT) [3,9], matching sign pursuit (MSP) [10], l_1 minimization-based linear programming (LP) [4], and restricted-step shrinkage (RSS) [11] were proposed. Although achieving good reconstruction performance, these algorithms either require the knowledge of the sparsity level [3,10] or are l_1-type methods that often yield solutions that are not necessarily the sparsest [4,11]. In this paper, we study a new method that uses the log-sum penalty function for sparse signal recovery. The log-sum penalty function has the potential to be much more sparsity-encouraging than the l_1 norm. By resorting to a bound optimization approach, we develop an iterative reweighted algorithm that successively minimizes a sequence of convex surrogate functions. The proposed algorithm has the advantage that it does not need the cardinality of the support set, K, of the sparse signal. Moreover, numerical results show that the proposed algorithm outperforms existing methods in terms of both the mean squared error and the support recovery accuracy metrics.

2. Problem formulation

Since the only information we have about the original signal is the sign of the measurements, we hope that the reconstructed signal $\hat{\boldsymbol{x}}$ yields estimated measurements that are consistent with our knowledge, that is

$$\text{sign}(\boldsymbol{a}_i^T\hat{\boldsymbol{x}}) = b_i \quad \forall i \tag{3}$$

or in other words

$$b_i\boldsymbol{a}_i^T\hat{\boldsymbol{x}} \geq 0 \quad \forall i \tag{4}$$

where $\boldsymbol{a}_i$ denotes the transpose of the ith row of the sampling matrix $\boldsymbol{A}$, b_i is the ith element of the sign vector $\boldsymbol{b}$. This consistency can be enforced by hard constraints [4,11] or can be quantified by a well-defined metric which is meant to be maximized/minimized [3,10,12]. In this paper, we introduce the logistic function to quantify the consistency between the measurements and the estimates. The metric is defined as

$$\phi(\boldsymbol{x}) \triangleq \sum_{i=1}^{m} \log(\sigma(b_i\boldsymbol{a}_i^T\boldsymbol{x})) \tag{5}$$

where $\sigma(x) \triangleq 1/(1+\exp(-x))$ is the logistic function. The logistic function, with an 'S' shape, approaches one for positive x and zero for negative x. Hence it is a useful tool to measure the consistency between b_i and $\boldsymbol{a}_i^T\boldsymbol{x}$. Also, the logistic function, differentiable and log-concave, is more amiable for algorithm development than the indicator function adopted in [3,10,12]. Note that the logistic function, also referred to as the logistic regression model, has been widely used in statistics and machine learning to represent the posterior class probability [13].

Naturally our objective is to find $\boldsymbol{x}$ to maximize the consistency between the acquired data and the reconstructed measurements, i.e.

$$\max_{\boldsymbol{x}} \quad \phi(\boldsymbol{x}) = \sum_{i=1}^{m} \log(\sigma(b_i\boldsymbol{a}_i^T\boldsymbol{x})) \tag{6}$$

This optimization, however, does not necessarily lead to a sparse solution. To obtain sparse solutions, a sparsity-encouraging term needs to be incorporated to encourage sparsity of the signal coefficients. The most commonly used sparsity-encouraging penalty function is l_1 norm. An attractive property of the l_1 norm is its convexity, which makes the l_1-based minimization a well-behaved numerical problem. Despite its popularity, l_1 type methods suffer from the drawback that the global minimum does not necessarily coincide with the sparsest solution, particularly when only a few measurements are available for signal reconstruction [14,15]. In this paper, we consider the use of an alternative sparsity-encouraging penalty function for sparse signal recovery. This penalty function, referred to as the Gaussian entropy, is defined as

$$h_G(\boldsymbol{x}) = \sum_{i=1}^{n} \log(x_i^2+\epsilon) \tag{7}$$

where x_i denotes the ith component of the vector $\boldsymbol{x}$, and $\epsilon > 0$ is a small parameter to ensure that the function is well-defined. Such a log-sum penalty function was first introduced in [16] for basis selection and later more extensively investigated in [15,17–20]. This penalty function behaves more like the l_0 norm than the l_1 norm [15,21]. It can be readily shown that each individual log term $\log(x_i^2+\epsilon)$, when $\epsilon \to 0$, has infinite slope at $x_i = 0$, $\forall i$, which implies that a relatively large penalty is placed on small nonzero coefficients to drive them to zero. Using this penalty function, the problem of finding a sparse solution to maximize the consistency can be formulated as follows:

$$\begin{aligned} \hat{\boldsymbol{x}} &= \arg\min L(\boldsymbol{x}) \\ &= \arg\min_{\boldsymbol{x}} -\sum_{i=1}^{m} \log(\sigma(b_i\boldsymbol{x}^T\boldsymbol{a}_i)) + \lambda\sum_{i=1}^{n}\log(x_i^2+\epsilon) \end{aligned} \tag{8}$$

where λ is a parameter controlling the trade-off between the quality of consistency and the degree of sparsity. Note that for most state-of-the-art one-bit compressed sensing algorithms (e.g. [4,10,11]), a unit-norm constraint has to be imposed on the solution, otherwise the algorithms yield a trivial all-zero solution. Nevertheless, such a unit-norm constraint is non-convex [4,11]. To deal with the unit-norm constraint, sophisticated optimization techniques [11] or alternative constraints [4] need to be used. For our formulation, such a unit-norm constraint is no longer necessary. This is because the logistic function that is used to measure the sign consistency has a tendency to increase the magnitudes of the solution: note that the logistic function $\sigma(b_i \boldsymbol{a}_i^T \boldsymbol{x})$ achieves its maximum value when $b_i \boldsymbol{a}_i^T \boldsymbol{x}$ goes to infinity. Hence all-zero is not a minimizer of the new cost function, and the all-zero trivial solution can be prevented without imposing the unit-norm constraint.

3. One-bit compressed sensing

3.1. Proposed algorithm

We develop our algorithm based on the bound optimization approach [22]. The idea is to construct a surrogate function $Q(\boldsymbol{x}|\hat{\boldsymbol{x}}^{(t)})$ such that

$$Q(\boldsymbol{x}|\hat{\boldsymbol{x}}^{(t)}) - L(\boldsymbol{x}) \geq 0 \tag{9}$$

and the minimum is attained when $\boldsymbol{x} = \hat{\boldsymbol{x}}^{(t)}$, i.e. $Q(\hat{\boldsymbol{x}}^{(t)}|\hat{\boldsymbol{x}}^{(t)}) = L(\hat{\boldsymbol{x}}^{(t)})$. In the following, we show that optimizing $L(\boldsymbol{x})$ can be replaced by minimizing the surrogate function $Q(\boldsymbol{x}|\hat{\boldsymbol{x}}^{(t)})$ iteratively. Suppose that

$$\hat{\boldsymbol{x}}^{(t+1)} = \min_{\boldsymbol{x}} Q(\boldsymbol{x}|\hat{\boldsymbol{x}}^{(t)})$$

We have

$$\begin{aligned} L(\hat{\boldsymbol{x}}^{(t+1)}) &= L(\hat{\boldsymbol{x}}^{(t+1)}) - Q(\hat{\boldsymbol{x}}^{(t+1)}|\hat{\boldsymbol{x}}^{(t)}) + Q(\hat{\boldsymbol{x}}^{(t+1)}|\hat{\boldsymbol{x}}^{(t)}) \\ &\leq L(\hat{\boldsymbol{x}}^{(t)}) - Q(\hat{\boldsymbol{x}}^{(t)}|\hat{\boldsymbol{x}}^{(t)}) + Q(\hat{\boldsymbol{x}}^{(t+1)}|\hat{\boldsymbol{x}}^{(t)}) \\ &\leq L(\hat{\boldsymbol{x}}^{(t)}) - Q(\hat{\boldsymbol{x}}^{(t)}|\hat{\boldsymbol{x}}^{(t)}) + Q(\hat{\boldsymbol{x}}^{(t)}|\hat{\boldsymbol{x}}^{(t)}) \\ &= L(\hat{\boldsymbol{x}}^{(t)}) \end{aligned} \tag{10}$$

where the first inequality follows from the fact that $Q(\boldsymbol{x}|\hat{\boldsymbol{x}}^{(t)}) - L(\boldsymbol{x})$ attains its minimum when $\boldsymbol{x} = \hat{\boldsymbol{x}}^{(t)}$; the second inequality comes by noting that $Q(\boldsymbol{x}|\hat{\boldsymbol{x}}^{(t)})$ is minimized at $\boldsymbol{x} = \hat{\boldsymbol{x}}^{(t+1)}$. We see that, through minimizing the surrogate function iteratively, the objective function $L(\boldsymbol{x})$ is guaranteed to be non-increasing at each iteration.

We now discuss how to find a surrogate function for (8). Ideally, we hope that the surrogate function is differentiable and convex so that the minimization of the surrogate function is a well-behaved numerical problem. Since the consistency evaluation term is convex, our objective is to find a convex surrogate function for the log-sum function defined in (7). An appropriate choice of such a surrogate function has a quadratic form (see Fig. 1) and is given by

$$f(\boldsymbol{x}|\hat{\boldsymbol{x}}^{(t)}) \triangleq \sum_{i=1}^{n} \left(\frac{x_i^2 + \epsilon}{(\hat{x}_i^{(t)})^2 + \epsilon} + \log\left((\hat{x}_i^{(t)})^2 + \epsilon \right) - 1 \right) \tag{11}$$

We have

$$\begin{aligned} f(\boldsymbol{x}|\hat{\boldsymbol{x}}^{(t)}) - h_G(\boldsymbol{x}) = \sum_{i=1}^{n} &\left(\frac{x_i^2 + \epsilon}{(\hat{x}_i^{(t)})^2 + \epsilon} + \log\left((\hat{x}_i^{(t)})^2 + \epsilon \right) \right. \\ &\left. - 1 - \log(x_i^2 + \epsilon) \right) \triangleq \sum_{i=1}^{n} g(x_i) \end{aligned} \tag{12}$$

Note that $g(x_i)$ is a symmetric function with respect to the origin. Examining the first derivative of $g(x_i)$ for $x_i > 0$, we find that the first derivative is a monotonically increasing function of x_i (for $x_i > 0$) and equal to zero at $x_i = |\hat{x}_i^{(t)}|$, which suggests that $g(x_i)$ for $x_i > 0$ is non-negative and attains its minimum 0 when $x_i = |\hat{x}_i^{(t)}|$. Since $g(x_i)$ is symmetric, $g(x_i)$ also achieves its minimum when $x_i = \hat{x}_i^{(t)}$. Therefore we have

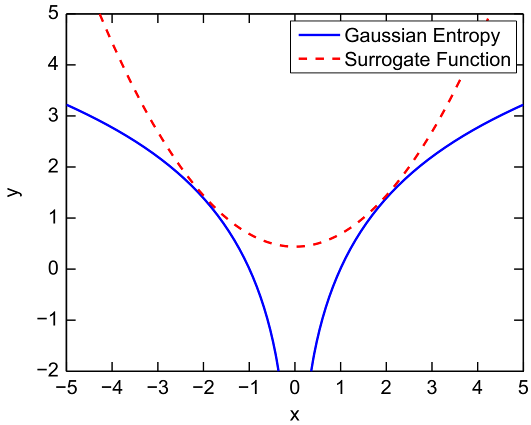

Fig. 1. The log-sum penalty function and its surrogate function, $n=1$, $\epsilon = 0.01$.

$$f(\boldsymbol{x}|\hat{\boldsymbol{x}}^{(t)}) - h_G(\boldsymbol{x}) \geq 0 \tag{13}$$

with the minimum 0 attained when $\boldsymbol{x} = \hat{\boldsymbol{x}}^{(t)}$. The convex function $f(\boldsymbol{x}|\hat{\boldsymbol{x}}^{(t)})$ is thus a desired surrogate function for the Gaussian entropy $h_G(\boldsymbol{x})$. As a consequence, the surrogate function for the objective function $L(\boldsymbol{x})$ is given by

$$\begin{aligned} &Q(\boldsymbol{x}|\hat{\boldsymbol{x}}^{(t)}) \\ &= -\sum_{i=1}^{m} \log\left(\sigma(b_i \boldsymbol{x}^T \boldsymbol{a}_i)\right) + \lambda \sum_{i=1}^{n} \frac{x_i^2 + \epsilon}{(\hat{x}_i^{(t)})^2 + \epsilon} + \text{constant} \\ &= -\sum_{i=1}^{m} \log(\sigma(b_i \boldsymbol{x}^T \boldsymbol{a}_i)) + \lambda \boldsymbol{x}^T \boldsymbol{D}(\hat{\boldsymbol{x}}^{(t)}) \boldsymbol{x} + \text{constant} \end{aligned} \tag{14}$$

where

$$\boldsymbol{D}(\hat{\boldsymbol{x}}^{(t)}) \triangleq \text{diag}\{((\hat{x}_1^{(t)})^2 + \epsilon)^{-1}, \ldots, ((\hat{x}_n^{(t)})^2 + \epsilon)^{-1}\}$$

Optimizing $L(\boldsymbol{x})$ now reduces to minimizing the surrogate function $Q(\boldsymbol{x}|\hat{\boldsymbol{x}}^{(t)})$ iteratively. For clarity, the iterative algorithm is briefly summarized as follows.

1. Given an initialization $\hat{\boldsymbol{x}}^{(0)}$.
2. At iteration $t > 0$, minimize $Q(\boldsymbol{x}|\hat{\boldsymbol{x}}^{(t)})$, which yields a new estimate $\hat{\boldsymbol{x}}^{(t+1)}$. Based on this new estimate, construct a new surrogate function $Q(\boldsymbol{x}|\hat{\boldsymbol{x}}^{(t+1)})$.
3. Go to Step 2 if $\|\hat{\boldsymbol{x}}^{(t+1)} - \hat{\boldsymbol{x}}^{(t)}\|_2 > \omega$, where ω is a prescribed tolerance value; otherwise stop.

3.2. Discussions

The second step in our algorithm involves optimization of the surrogate function $Q(\boldsymbol{x}|\hat{\boldsymbol{x}}^{(t)})$. Since the surrogate function is differentiable and convex, minimizing $Q(\boldsymbol{x}|\hat{\boldsymbol{x}}^{(t)})$ is a well-behaved numerical problem. Also, the gradient

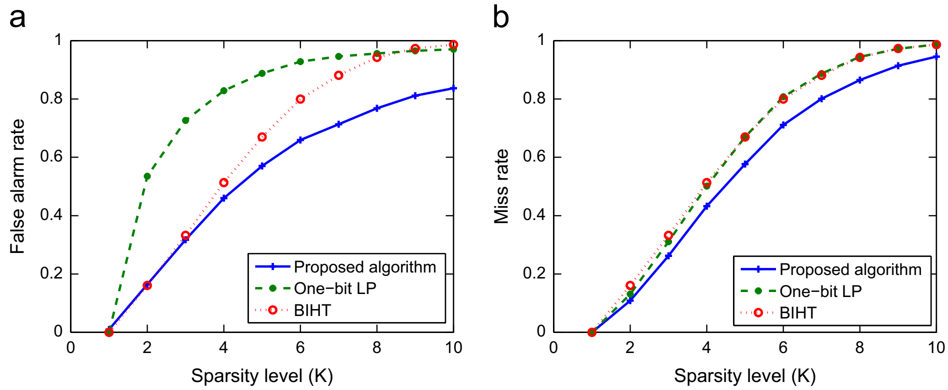

Fig. 2. False alarm and miss rates of respective algorithms, $m=100$, $n=50$. (a) False alarm rates of respective algorithms, (b) Miss rates of respective algorithms.

and the Hessien matrix of the surrogate function $Q(\boldsymbol{x}|\hat{\boldsymbol{x}}^{(t)})$ have analytical expressions which are respectively given as

$$\boldsymbol{g} = -\sum_{i=1}^{m}(1-\sigma(b_i\boldsymbol{x}^T\boldsymbol{a}_i))b_i\boldsymbol{a}_i + 2\lambda\boldsymbol{D}(\hat{\boldsymbol{x}}^{(t)})\boldsymbol{x}$$

$$\boldsymbol{H} = \sum_{i=1}^{m}\sigma(b_i\boldsymbol{x}^T\boldsymbol{a}_i)(1-\sigma(b_i\boldsymbol{x}^T\boldsymbol{a}_i))\boldsymbol{a}_i\boldsymbol{a}_i^T + 2\lambda\boldsymbol{D}(\hat{\boldsymbol{x}}^{(t)})$$

Hence Newton's method which has a fast convergence rate can be used and is guaranteed to converge to the global minimum.

As mentioned earlier, the proposed algorithm results in a non-increasing objective function value and eventually converges to a stationary point of $L(\boldsymbol{x})$. It should be emphasized that the cost function $L(\boldsymbol{x})$ is non-convex. Hence convergence to the global minimum is not guaranteed by any gradient-based search methods. Nevertheless, numerical results demonstrate that the proposed algorithm usually converges to a stationary point that is close to the true solution. Note that the proposed algorithm does not require the knowledge of the sparsity level K. For a pre-specified λ and ϵ, the iterative process determines the sparsity level of the signal in an automatic manner. Although the choice of λ and ϵ has an influence on the sparsity level of the estimated signal, our experiments suggest that the proposed algorithm delivers robust and consistent signal recovery performance as long as λ and ϵ are set in a reasonable range.

The proposed iterative algorithm can be considered as consisting of two alternating steps. First, we estimate $\boldsymbol{x}$ through minimizing the current surrogate function $Q(\boldsymbol{x}|\hat{\boldsymbol{x}}^{(t)})$. Second, based on the estimate of $\boldsymbol{x}$, we update the weights of the weighted l_2 norm penalty of the surrogate function. This alternating process finally results in a sparse solution. To see this, note that the weighted l_2 norm of $\boldsymbol{x}$ has their weights specified as $\{((\hat{x}_i^{(t)})^2+\epsilon)^{-1}\}$. When ϵ is small, say $\epsilon=10^{-3}$, the weighted l_2 norm penalty term, i.e. $\boldsymbol{x}^T\boldsymbol{D}(\hat{\boldsymbol{x}}^{(t)})\boldsymbol{x}$ has the tendency to decrease these entries in $\boldsymbol{x}$ whose corresponding weights are large, i.e. whose current estimates $\{\hat{x}_i^{(t)}\}$ are already small. This negative feedback mechanism keeps suppressing these entries until they become negligible, while leaving only a few prominent nonzero entries survived to meet the consistency requirement. We notice that the proposed method is similar to the iterative reweighted least squares algorithm discussed in [19,23]. Nevertheless, our proposed method is developed in the framework of one-bit compressed sensing, while the other two works deal with the conventional compressed sensing problem. In addition, through using the surrogate function, a connection between the log-sum penalty function and the iterative reweighted algorithm is established. This provides a new perspective on the iterative reweighted algorithm.

4. Numerical results

We now carry out experiments to illustrate the performance of our proposed one-bit compressed sensing algorithm.[1] In our simulations, the K-sparse signal is randomly generated with the support set of the sparse signal randomly chosen according to a uniform distribution. The signals on the support set are independent and identically distributed (i.i.d.) Gaussian random variables with zero mean and unit variance. The measurement matrix $\boldsymbol{A}\in\mathbb{R}^{m\times n}$ is randomly generated with each entry independently drawn from Gaussian distribution with zero mean and unit variance, and then each column of $\boldsymbol{A}$ is normalized to unit norm for algorithm stability. We compare our proposed algorithm with the other two algorithms, namely, the l_1 minimization-based linear programming (LP) algorithm [4] (referred to as "one-bit LP") and the binary iterative hard thresholding algorithm [3] (referred to as "BIHT").

Two metrics are used to evaluate the recovery performance, namely, mean squared error (MSE) and support recovery accuracy. Support recovery accuracy is measured by the false alarm (misidentified) rate and the miss rate. A false alarm event represents the case where coefficients that are zero in the original signal are misidentified as nonzero after reconstruction, while a miss event stands for the case where the nonzero coefficients are missed and determined to be zero. Throughout our experiments, we set $\lambda=0.2$, and $\epsilon=0.002$ for our proposed algorithm.

[1] Matlab codes are available at http://www.junfang-uestc.net/codes/OnebitCS.rar.

As mentioned earlier in the paper, λ controls the trade-off between the consistency and the degree of sparsity. Empirical results suggest that a moderate λ in the range (0.1 1) usually renders a reliable estimate. For our proposed algorithm, some of the estimated coefficients of $\boldsymbol{x}$ keep decreasing each iteration, but will not exactly equal to zero. We regard those coefficients in $\hat{\boldsymbol{x}}$ whose values are less than $10^{-7}/\|\hat{\boldsymbol{x}}\|_2$ as zero, where $\hat{\boldsymbol{x}}$ denotes the final estimate of the sparse signal. Fig. 2 depicts the false alarm and miss rates of respective algorithms as a function of the sparsity level K, where we set $m=100$, and $n=50$ in our simulations. Results are averaged over 10^4 independent runs. We see that the proposed algorithm is more effective in identifying the true support set: as compared with the other two algorithms, it presents a higher detection rate (lower miss rate) at a lower false alarm rate. Fig. 3 depicts the MSEs of the three algorithms. Since the information about the magnitude of the signal is lost due to quantization, the norm of the original signal and the estimated signal is normalized to unity in computing the MSEs. The proposed algorithm achieves the smallest MSE among all three algorithms. We also provide results for an underdetermined system, where we set $m=100$ and $n=150$. Figs. 4 and 5 show the support recovery accuracy and MSEs of the three algorithms. Results again validate the superiority of the proposed algorithm: it outperforms the other two algorithms in terms of both metrics. In Fig. 6, we plot one realization of the original signal and the reconstructed signals by respective algorithms. It can be seen that the proposed algorithm provides reconstructed coefficients that are closest to the groundtruth.

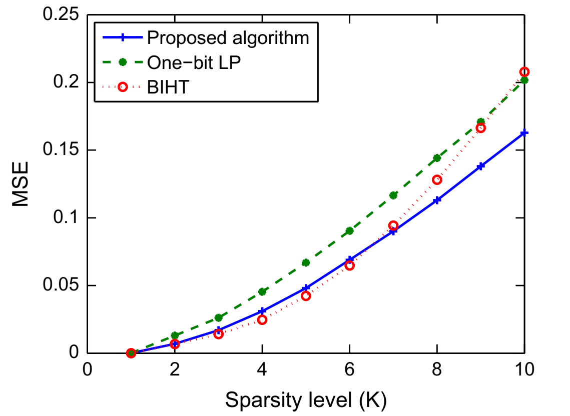

Fig. 5. Mean squared error versus sparsity level K, $m=100$, $n=150$.

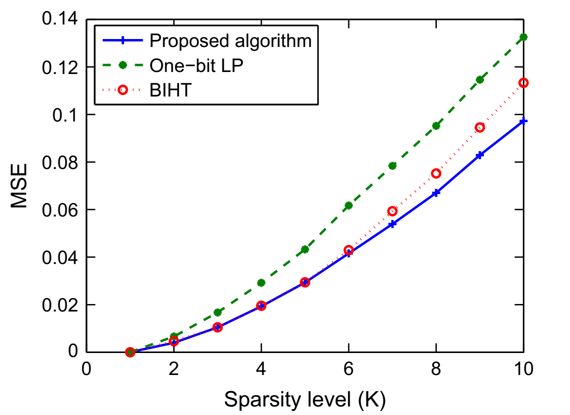

Fig. 3. Mean squared error versus sparsity level K, $m=100$, $n=50$.

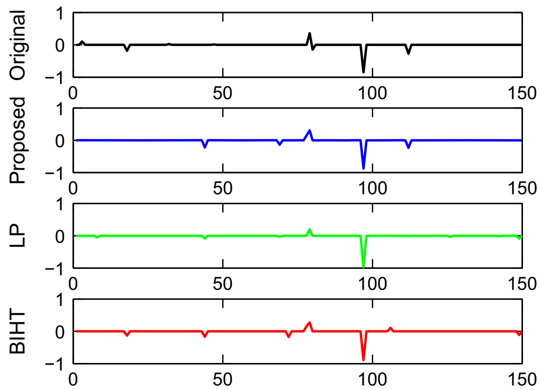

Fig. 6. The original signal and the reconstructed signals by respective algorithms, $m=100$, $n=150$.

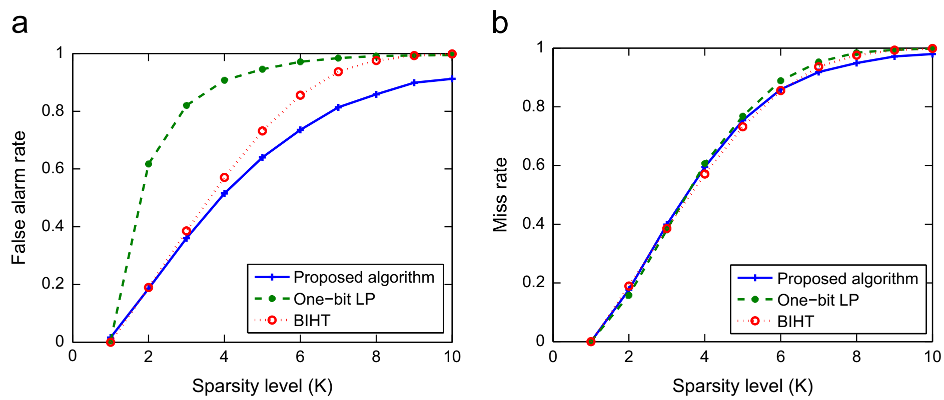

Fig. 4. False alarm and miss rates of respective algorithms, $m=100$, $n=150$. (a) False alarm rates of respective algorithms, (b) Miss rates of respective algorithms.

5. Conclusions

We studied the problem of recovering sparse signals from one-bit measurements. The proposed method introduced the logistic function to quantify the sign consistency between the measurements and the estimates. By resorting to the bound optimization technique, we developed an iterative reweighted algorithm which consists of solving a sequence of convex differentiable minimization problems. Numerical results show that the proposed algorithm outperforms existing methods in terms of the mean squared error and the support recovery accuracy metrics.

References

[1] E. Candés, T. Tao, Decoding by linear programming, IEEE Trans. Inf. Theory 12 (December) (2005) 4203–4215.

[2] J.A. Tropp, A.C. Gilbert, Signal recovery from random measurements via orthogonal matching pursuit, IEEE Trans. Inf. Theory 53 (December (12)) (2007) 4655–4666.

[3] L. Jacques, J.N. Laska, P.T. Boufounos, R.G. Baraniuk, Robust 1-bit compressive sensing via binary stable embeddings of sparse vectors, Trans, Inf. Theory 59 (April (4)) (2013) 2082–2102.

[4] Y. Plan, R. Vershynin, One-bit compressed sensing by linear programming, Communications on Pure and Applied Mathematics 66 (2013) 1275–1297.

[5] T. Wimalajeewa, P.K. Varshney, Performance bounds for sparsity pattern recovery with quantized noisy random projections, IEEE J. Sel. Top. Signal Process. 6 (February (1)) (2012) 43–57.

[6] P.T. Boufounos, R.G. Baraniuk, One-bit compressive sensing, in: Proceedings of the 42nd Annual Conference on Information Sciences and Systems (CISS 08), Princeton, NJ, 2008.

[7] Y. Plan, R. Vershynin, Robust 1-bit compressed sensing and sparse logistic regression: a convex programming approach, July 2012 [online]. Available: arxiv.org/abs/1202.1212.

[8] J.N. Laska, R.G. Baraniuk, Regime change: bit-depth versus measurement-rate in compressive sensing, IEEE Trans. Signal Process. 60 (July (7)) (2012) 3496–3505.

[9] L. Jacques, J.N. Laska, P.T. Boufounos, R.G. Baraniuk, Robust 1-bit compressive sensing via binary stable embeddings of sparse vectors, IEEE Trans. Inf. Theory 59 (April (4)) (2013) 2082–2102.

[10] P.T. Boufounos, Greedy sparse signal reconstruction from sign measurements, in: Proceedings of the 43rd Asilomar Conference on Signals, Systems, and Computers (Asilomar 09), Pacific Grove, CA, 2009.

[11] J.N. Laska, Z. Wen, W. Yin, R.G. Baraniuk, Trust, but verify: fast and accurate signal recovery from 1-bit compressive measurements, IEEE Trans. Signal Process. 59 (November (11)) (2011) 5289–5301.

[12] M. Yan, Y. Yang, S. Osher, Robust 1-bit compressive sensing using adaptive outlier pursuit, IEEE Trans. Signal Process. 60 (July (7)) (2012) 3868–3875.

[13] M. Tipping, Sparse Bayesian learning and the relevance vector machine, J. Mach. Learn. Res. 1 (2001) 211–244.

[14] D.P. Wipf, B.D. Rao, Sparse Bayesian learning for basis selection, IEEE Trans. Signal Process. 52 (August (8)) (2004) 2153–2164.

[15] E. Candés, M. Wakin, S. Boyd, Enhancing sparsity by reweighted l_1 minimization, J. Fourier Anal. Appl. 14 (December) (2008) 877–905.

[16] R.R. Coifman, M. Wickerhauser, Entropy-based algorithms for best basis selection, IEEE Trans. Inf. Theory IT-38 (March) (1992) 713–718.

[17] I.F. Gorodnitsky, B.D. Rao, Sparse signal reconstructions from limited data using focuss: a re-weighted minimum norm algorithm, IEEE Trans. Signal Process. 45 (March (3)) (1997) 616–699.

[18] B.D. Rao, K. Kreutz-Delgado, An affine scaling methodology for best basis selection, IEEE Trans. Signal Process. 47 (January (1)) (1999) 187–200.

[19] R. Chartrand, W. Yin, Iterative reweighted algorithm for compressive sensing, in: International Conference on Acoustics, Speech, and Signal Processing, Las Vegas, NV, USA, 2008.

[20] D. Wipf, S. Nagarajan, Iterative reweighted ℓ_1 and ℓ_2 methods for finding sparse solutions, IEEE J. Sel. Top. Signal Process. 4 (April (2)) (2010) 317–329.

[21] Y. Shen, J. Fang, H. Li, Exact reconstruction analysis of log-sum minimization for compressed sensing, IEEE Signal Process. Lett. 20 (December) (2013) 1223–1226.

[22] K. Lange, D. Hunter, I. Yang, Optimization transfer using surrogate objective functions, J. Comput. Graph. Stat. 9 (March (1)) (2000) 1–20.

[23] I. Daubechies, R. Devore, M. Fornasier, C.S. Gunturk, Iteratively reweighted least squares minimization for sparse recovery, Commun. Pure Appl. Math. LXIII (2010) 1–38.