Knowledge-Aided Range-Spread Target Detection for Distributed MIMO Radar in Nonhomogeneous Environments

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***Abstract*—This paper deals with the problem of detecting a moving range-spread target in distributed MIMO radar. A new knowledge-aided (KA) model that takes into account the nonhomogenous characteristics of the disturbance (clutter and noise) in distributed MIMO radar is proposed. Specifically, the disturbance covariance matrices corresponding to different transmit-receive (Tx-Rx) pairs are modeled as random matrices. These covariance matrices share a prior covariance matrix structure but with different power levels to model the nonhomogeneous clutter powers across different Tx-Rx pairs. Two cases are considered, involving either no range training (i.e., when the disturbance is highly nonhomogeneous) or some range training data. For the first case, we develop a KA generalized likelihood ratio test (GLRT) for range-spread target detection, along with a simplified version of the KA-GLRT for point-like target detection. For the second case, the KA-GLRT becomes computationally intractable, a simple *ad-doc* KA detector is introduced to take advantage of training data for range-spread target detection. Simulation results are presented to illustrate the performance and effectiveness of the proposed detectors in nonhomogeneous environments.**

***Index Terms*—Range-spread target, knowledge-aided detection, distributed MIMO radar, generalized likelihood ratio test (GLRT).**

I. Introduction

MIMO radar has received considerable attention in recent years [1]–[8]. Unlike the traditional phased-array radar using one transmit antenna with a single probing waveform, MIMO radar is equipped with multiple transmit-receive (Tx-Rx) antennas along with multiple probing waveforms. Extensive studies via theoretical analysis and computer simulation have shown that MIMO radar has many potential advantages, such as enhanced detection performance [4], improved angular resolution and parameter identifiability [1], providing more degrees of freedom, and better spatial coverage [7]. There are two types of MIMO radar in terms of antenna configuration. The first is the co-located MIMO radar with closely spaced antenna elements within both the transmit and receive arrays. The second is the distributed MIMO radar with widely separated antennas. This work addresses the distributed MIMO radar.

Distributed MIMO radars can view a target from different spatial angles, resulting in a spatial diversity that can be exploited to enhance detection performance. In [9], the spatial diversity and the resulting detection diversity gain was examined. Assuming the availability of a set of training data, a sample covariance matrix (SCM)-based detector is considered for detection in cases with homogeneous disturbance signals (clutter, noise, etc.) [2], [10], [11]. Then, a robust SCM (RSCM)-based detector is developed by exploiting a compound Gaussian model for the disturbance [12]. In [13], the authors utilize a persymmetric covariance structure to reduce the training size for detection in distributed MIMO radar. In [14], an auto-regressive model is employed to detect moving targets. The target detection in distributed MIMO radar under phase synchronization mismatch or imperfect signal separation is considered in [15]. Most of the above studies assume the disturbance to be spatially homogeneous, namely, the covariance matrices for all Tx-Rx pairs are identical. As shown in [16], the disturbance is strongly location dependent, i.e., it depends on the geometry of the Tx-Rx pair. Thus, the covariance matrix may vary significantly across resolution cells and is different from one Tx-Rx pair to another.

High range resolution (HRR) radar is of increasing interest for various applications including automatic target recognition (ATR) [17]. For target detection, HRR radar conveys abundant target information and makes the signals backscattered by the range-spread target less fluctuated [18]. However, with high resolution radar systems, a target may occupy multiple range bins. As a result, the detection of range-spread targets has become a critical problem of research in the application of high-resolution radars. For range-spread target detection in white Gaussian noise, a generalized likelihood ratio test (GLRT) detector incorporating the target scatterer density is derived in [19]. Range-spread target detection in Gaussian noise with an unknown covariance matrix is considered in [20], where both a GLRT and a two-step GLRT detectors are proposed, which employ target-free range training signals to estimate the noise covariance matrix. Range-spread target detection involving a multi-rank subspace target signal is examined in [21], [22]. Range-spread target detection with an orthogonal interference rejection ability is discussed in [23]. In [18], the detection of a range-spread target is addressed under the assumption that a properly transformed inverse covariance matrix belongs to a unitary invariant function set. More discussions about range-spread target detection and other related work can be found in [24]–[27]

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and the references therein. To the best of our knowledge, there is little published work addressing distributed MIMO high resolution radar detection in non-homogeneous environments.

In this paper, we consider the problem of knowledge-aided (KA) detection of a range-spread target in distributed MIMO radar. We propose a new KA model that takes into account non-homogeneous characteristics of the disturbance. The covariance matrix for each Tx-Rx pair is modeled as a random matrix which is also different across different Tx-Rx pairs. Moreover, the power level for each stochastic covariance matrix is also different across the many Tx-Rx pairs. Based on the proposed signal model, we consider range-spread target detection in two cases with or without training data. For the range training-free case, we develop a KA-GLRT detector along with a simplified version tailored for detecting a point-like target. When training is available, the exact KA-GLRT becomes computationally intractable. Instead, we develop a simple *ad-hoc* KA detector which can still benefit from both the prior knowledge and the training data. Numerical results show that the proposed detectors can achieve significantly enhanced detection performance over conventional detectors when the amount of training data is limited.

The remainder of this paper is organized as follows. In Section II, we present the signal model. The proposed KA detectors are presented in Section III for the training-free case and, respectively, in Section IV for the case with training. In Section V, simulation results are provided to illustrate the detection performance of the proposed detectors. Finally, conclusions are given in Section VI.

The following notations are used throughout the paper. The conjugate, transpose, and conjugate transpose operations are denoted by $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^\dagger$, respectively. tr$(\cdot)$ indicates the trace of a matrix. etr$(\cdot)$ indicates the exponential function of the trace of a matrix. $|\cdot|$ with a square matrix argument represents the determinant of that matrix, and $|\cdot|$ with a complex number represents the modulus. The notation $\mathcal{CN}$ denotes the complex Gaussian distribution. The notation $\mathcal{CW}$ denotes the complex inverse Wishart distribution.

II. Signal Model

Consider the moving target detection problem involving a high range-resolution distributed MIMO radar with M transmit antennas and N receive antennas that are widely spaced from each other [2], [3]. Suppose each transmitter sends L pulses over a coherent processing interval, and the waveforms from different transmitters are orthogonal to each other. Each receiver uses a bank of M matched filters corresponding to M orthogonal waveforms. Due to high range resolution, we have to address the range-spread issue. Assume a target may spread up to $H \geq 1$ range cells. The matched filter output at the nth receive antenna that is matched to the mth transmit antenna is denoted by $\mathbf{X}_{mn} = [\mathbf{x}_{mn,1}, \cdots, \mathbf{x}_{mn,H}] \in \mathbb{C}^{L\times H}; m = 1, \cdots, M, n = 1, \cdots, N$. Across different Tx-Rx pairs, the primary data vector $\mathbf{X}_{mn}$ can be assumed to be independent of each other due to the use of widely spaced antennas in distributed MIMO radar [13]. The range-spread target detection problem can then be formulated as the following binary hypothesis testing

$$\begin{aligned} H_1 &: \mathbf{X}_{mn} = \mathbf{p}_{mn}\boldsymbol{\alpha}_{mn}^T + \mathbf{N}_{mn}, \\ H_0 &: \mathbf{X}_{mn} = \mathbf{N}_{mn}, \\ & m = 1, \cdots, M, n = 1, \cdots, N, \end{aligned} \tag{1}$$

where $\boldsymbol{\alpha}_{mn} = [\alpha_{mn,1}, \cdots, \alpha_{mn,H}]^T$ contains the unknown amplitude parameters of the range-spread target, $\mathbf{p} \in \mathbb{C}^{L\times 1}$ denotes the target steering vector, and $\mathbf{N}_{mn} = [\mathbf{n}_{mn,1}, \cdots, \mathbf{n}_{mn,H}]$ denotes the disturbance that may include clutter and noise. The target steering vector is given by

$$\mathbf{p}_{mn} = [1\ e^{-j2\pi f_{mn}}\ \cdots\ e^{-j2\pi f_{mn}(L-1)}]^T, \tag{2}$$

where f_{mn} denotes the normalized Doppler frequency for each Tx-Rx pair, and the Doppler frequency is different for different Tx-Rx due to the moving target motion. Each column vector of $\mathbf{N}_{mn}$ is assumed to be independent and identically distributed (i.i.d.) complex zero-mean Gaussian vector with covariance matrix $\mathbf{R}_{mn}$, namely, $\mathbf{n}_{mn,h} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_{mn}), h = 1, \cdots, H$. Note that $\mathbf{R}_{mn}$ is in general different from one Tx-Rx pair to another in distributed MIMO radar.

Remark: It is noted that orthogonal waveforms are standard choices of signalling for MIMO radar. In practice, due to phase noise of oscillators, synchronization errors, and other propagation related issues, strict orthogonality may not be maintained in real MIMO systems. The effects of the above impairments are examined in a number of studies, e.g., [15], [28]. For simplicity and to focus on the problem of interest, we do not consider such effects in this work.

Conventional covariance matrix-based detectors for point target detection [2], [10]–[12] can be extended to solve the range-spread target detection problem. However, they all require a set of i.i.d. training data, which has the same spectral property as the disturbance in the test data, to estimate the unknown covariance matrix $\mathbf{R}_{mn}$. For a distributed MIMO radar with MN Tx-Rx pairs, the total number of required training data is roughly $2LMN$ in order for such a covariance matrix-based detector to obtain a acceptable detection performance [29]. As such, their training requirement is very demanding for large M, N, and/or L. It is often challenging to obtain a large number of homogeneous training data, due to fact that the clutter is inherently non-homogeneous in multi-static configurations [14]. Therefore, there is a need to develop more efficient solutions for range-spread target detection in distributed MIMO radar.

To address the above problem, we introduce a stochastic model for range-spread target detection in this paper. Our proposed model is an extension of the random covariance matrix model of [30]–[34], in order to deal with the non-homogenous clutter power across different Tx-Rx pairs. Specifically, we model $\mathbf{R}_{mn}$ as a complex inverse Wishart random matrix:

$$\mathbf{R}_{mn} \sim \mathcal{CW}^{-1}(v, \lambda_{mn}(v-L)\bar{\mathbf{R}}), \tag{3}$$

where v denotes the degrees of freedom of the inverse Wishart distribution, $(v-L)\bar{\mathbf{R}}$ denotes the prior covariance matrix structure, and λ_{mn} denotes the different power level for different Tx-Rx pairs. Compared with the previous models [30]–[33], we use MN random matrices $\mathbf{R}_{mn}$ to model the clutter observed by the different Tx-Rx pairs. These random matrices have also different power level parameters λ_{mn} in order to model the non-homogeneous clutter powers across different Tx-Rx pairs.

The probability density function (PDF) of $\mathbf{R}_{mn}$ conditioned on λ_{mn} is given by

$$f(\mathbf{R}_{mn}|\lambda_{mn}) = \frac{|\lambda_{mn}(v-L)\bar{\mathbf{R}}|^{v}}{\bar{\Gamma}_L(v)|\mathbf{R}_{mn}|^{v+L}}\mathrm{etr}[-(v-L)\lambda_{mn}\mathbf{R}_{mn}^{-1}\bar{\mathbf{R}}], \tag{4}$$

where

$$\bar{\Gamma}_L(v) = \pi^{L(L-1)/2}\prod_{l=1}^{L}\Gamma(v-l+1) \tag{5}$$

with $\Gamma(\cdot)$ being the Gamma function. The problem of interest is develop solutions to the range-spread target detection problem (1) in distributed MIMO radar by utilizing the proposed covariance matrix model (3).

III. KA Detection Without Range Training

As stated previously, conventional covariance matrix-based detectors require a large number of training signals for covariance matrix estimation [2], [10]–[13]. However, due to the non-homogeneous nature of the disturbance observed by different Tx-Rx pair, it is difficult to obtain such a large amount of training data. In this section, we consider how to exploit the prior knowledge for target detection in distributed MIMO radar when no range training is available, which correspond to the case when the disturbance is highly non-homogeneous. We first present a KA detector for range-spread target detection, which is then simplified for the detection of a point-like target.

A. Range-Spread Target Detection

No uniformly most powerful (UMP) testing exists for the hypothesis testing problem (1), since the parameters $\boldsymbol{\alpha} = [\boldsymbol{\alpha}_{11},\cdots,\boldsymbol{\alpha}_{mn}]$ and $\boldsymbol{\lambda} = [\lambda_{11},\cdots,\lambda_{mn}]$ are unknown. We consider a GLRT approach. The KA-GLRT for range-spread target detection can be expressed as

$$\frac{\max\limits_{\boldsymbol{\alpha},\boldsymbol{\lambda}}\prod\limits_{m,n}\int f_1(\mathbf{X}_{mn}|\boldsymbol{\alpha}_{mn},\mathbf{R}_{mn})f(\mathbf{R}_{mn}|\lambda_{mn})d\mathbf{R}_{mn}}{\max\limits_{\boldsymbol{\lambda}}\prod\limits_{m,n}\int f_0(\mathbf{X}_{mn}|\mathbf{R}_{mn})f(\mathbf{R}_{mn}|\lambda_{mn})d\mathbf{R}_{mn}} \underset{H_0}{\overset{H_1}{\gtrless}} \xi_1, \tag{6}$$

where

$$f_0(\mathbf{X}_{mn}|\mathbf{R}_{mn}) = \frac{\pi^{-LH}}{|\mathbf{R}_{mn}|^{H}}\mathrm{etr}(-\mathbf{R}_{mn}^{-1}\mathbf{X}_{mn}\mathbf{X}_{mn}^{\dagger}) \tag{7}$$

and

$$f_1(\mathbf{X}_{mn}|\boldsymbol{\alpha}_{mn},\mathbf{R}_{mn}) = \frac{\pi^{-LH}}{|\mathbf{R}_{mn}|^{H}}\mathrm{etr}(-\mathbf{R}_{mn}^{-1}\mathbf{Y}_{mn}\mathbf{Y}_{mn}^{\dagger}) \tag{8}$$

are the the conditional PDF under H_0 and H_1, respectively, with $\mathbf{Y}_{mn} = \mathbf{X}_{mn} - \mathbf{p}_{mn}\boldsymbol{\alpha}_{mn}^{T}$, and ξ_1 denotes the detection threshold.

Due to the independent assumption on the test data, the maximization of the left-hand side of (6) can be performed term by term. Thus, (6) can be rewritten as

$$\prod_{m,n}\frac{\max\limits_{\boldsymbol{\alpha}_{mn},\lambda_{mn}}\int f_1(\mathbf{X}_{mn}|\boldsymbol{\alpha}_{mn},\mathbf{R}_{mn})f(\mathbf{R}_{mn}|\lambda_{mn})d\mathbf{R}_{mn}}{\max\limits_{\lambda_{mn}}\int f_0(\mathbf{X}_{mn}|\mathbf{R}_{mn})f(\mathbf{R}_{mn}|\lambda_{mn})d\mathbf{R}_{mn}} \underset{H_0}{\overset{H_1}{\gtrless}} \xi_1. \tag{9}$$

For each Tx-Rx pair, it follows from (4), (7) and (8) that

$$\begin{aligned}\Lambda_{mn} &= \frac{\max\limits_{\boldsymbol{\alpha}_{mn},\lambda_{mn}}\int f_1(\mathbf{X}_{mn}|\boldsymbol{\alpha}_{mn},\mathbf{R}_{mn})f(\mathbf{R}_{mn}|\lambda_{mn})d\mathbf{R}_{mn}}{\max\limits_{\lambda_{mn}}\int f_0(\mathbf{X}_{mn}|\mathbf{R}_{mn})f(\mathbf{R}_{mn}|\lambda_{mn})d\mathbf{R}_{mn}}\\ &= \frac{\max\limits_{\boldsymbol{\alpha}_{mn},\lambda_{mn}}\lambda_{mn}^{Lv}|\mathbf{Y}_{mn}\mathbf{Y}_{mn}^{\dagger}+\lambda_{mn}(v-L)\bar{\mathbf{R}}|^{-(v+H)}}{\max\limits_{\lambda_{mn}}\lambda_{mn}^{Lv}|\mathbf{X}_{mn}\mathbf{X}_{mn}^{\dagger}+\lambda_{mn}(v-L)\bar{\mathbf{R}}|^{-(v+H)}}.\end{aligned} \tag{10}$$

The final test statistics is given by (see Appendix A for details)

$$\prod_{m,n}\frac{\hat{\lambda}_{mn,0}^{-\frac{Lv}{v+H}}|\mathbf{X}_{mn}\mathbf{X}_{mn}^{\dagger}+\hat{\lambda}_{mn,0}(v-L)\bar{\mathbf{R}}|}{\hat{\lambda}_{mn,1}^{-\frac{Lv}{v+H}}|\hat{\mathbf{Y}}_{mn}\hat{\mathbf{Y}}_{mn}^{\dagger}+\hat{\lambda}_{mn,1}(v-L)\bar{\mathbf{R}}|} \underset{H_0}{\overset{H_1}{\gtrless}} \xi_{m1}, \tag{11}$$

where

$$\hat{\mathbf{Y}}_{mn} = \mathbf{X}_{mn} - \frac{\mathbf{p}_{mn}\mathbf{p}_{mn}^{\dagger}\bar{\mathbf{R}}^{-1}\mathbf{X}_{mn}}{\mathbf{p}_{mn}^{\dagger}\bar{\mathbf{R}}^{-1}\mathbf{p}_{mn}}, \tag{12}$$

ξ_{m1} denotes the modified threshold, $\hat{\lambda}_{mn,1}$ and $\hat{\lambda}_{mn,0}$ are the maximum likihood estimate (MLE) of λ_{mn} under H_1 and H_0, respectively, which can be obtained by solving equation (A5) in the Appendix A.

B. Point-Like Target Detection

In this sub-section, we provide a simplified solution for detecting a point-like target ($H = 1$) under the same KA framework. In such a case, the hypothesis testing problem (1) reduces to

$$\begin{aligned}&H_1: \mathbf{x}_{mn} = \beta_{mn}\mathbf{p}_{mn} + \mathbf{n}_{mn},\\ &H_0: \mathbf{x}_{mn} = \mathbf{n}_{mn},\\ &\qquad m = 1,\cdots,M, n = 1,\cdots,N,\end{aligned} \tag{13}$$

where β_{mn} is the unknown amplitude of the target observed by the mth transmit and nth receive antenna.

Denote $\boldsymbol{\beta} = [\beta_{11},\cdots,\beta_{mn}]$ and $\mathbf{y}_{mn} = \mathbf{x}_{mn} - \beta_{mn}\mathbf{p}_{mn}$. The KA-GLRT for the detection problem (13) can be formulated as follows

$$\frac{\max\limits_{\boldsymbol{\beta},\boldsymbol{\lambda}}\prod\limits_{m,n}\int f_1(\mathbf{x}_{mn}|\beta_{mn},\mathbf{R}_{mn})f(\mathbf{R}_{mn}|\lambda_{mn})d\mathbf{R}_{mn}}{\max\limits_{\boldsymbol{\lambda}}\prod\limits_{m,n}\int f_0(\mathbf{x}_{mn}|\mathbf{R}_{mn})f(\mathbf{R}_{mn}|\lambda_{mn})d\mathbf{R}_{mn}} \underset{H_0}{\overset{H_1}{\gtrless}} \xi_2, \tag{14}$$

where

$$f_1(\mathbf{x}_{mn}|\boldsymbol{\beta}_{mn},\mathbf{R}_{mn}) = \frac{\pi^{-L}}{|\mathbf{R}_{mn}|}\mathrm{etr}[-\mathbf{R}_{mn}^{-1}\mathbf{y}_{mn}\mathbf{y}_{mn}^{\dagger}], \tag{15}$$

$$f_0(\mathbf{x}_{mn}|\mathbf{R}_{mn}) = \frac{\pi^{-L}}{|\mathbf{R}_{mn}|}\mathrm{etr}(-\mathbf{R}_{mn}^{-1}\mathbf{x}_{mn}\mathbf{x}_{mn}^{\dagger}), \tag{16}$$

and ξ_2 denotes the detection threshold. Inserting (4), (15) and (16) into (14) yields

$$\prod_{m,n} \frac{\max\limits_{\beta_{mn},\lambda_{mn}} \lambda_{mn}^{Lv} |\mathbf{y}_{mn}\mathbf{y}_{mn}^\dagger + \lambda_{mn}(v-L)\bar{\mathbf{R}}|^{-(v+1)}}{\max\limits_{\lambda_{mn}} \lambda_{mn}^{Lv} |\mathbf{x}_{mn}\mathbf{x}_{mn}^\dagger + \lambda_{mn}(v-L)\bar{\mathbf{R}}|^{-(v+1)}} \underset{H_0}{\overset{H_1}{\gtrless}} \xi_2. \tag{17}$$

As shown in Appendix B, the MLE of λ_{mn} under H_1 and H_0 is given by

$$\hat{\lambda}_{mn,1} = \frac{v+1-L}{L}\mathbf{y}_{mn}^\dagger((v-L)\bar{\mathbf{R}})^{-1}\mathbf{y}_{mn},$$
$$\hat{\lambda}_{mn,0} = \frac{v+1-L}{L}\mathbf{x}_{mn}^\dagger((v-L)\bar{\mathbf{R}})^{-1}\mathbf{x}_{mn}. \tag{18}$$

Inserting (18) into (17) yields

$$\prod_{m,n} \max_{\beta_{mn}} \frac{\hat{\lambda}_{mn,0}^{\frac{-Lv}{v+1}} |\mathbf{x}_{mn}\mathbf{x}_{mn}^\dagger + \hat{\lambda}_{mn,0}(v-L)\bar{\mathbf{R}}|}{\hat{\lambda}_{mn,1}^{\frac{-Lv}{v+1}} |\mathbf{y}_{mn}\mathbf{y}_{mn}^\dagger + \lambda_{mn,1}(v-L)\bar{\mathbf{R}}|} \underset{H_0}{\overset{H_1}{\gtrless}} \xi_{m2}. \tag{19}$$

where ξ_{m2} denotes the modified threshold. From (19), the estimate of β_{mn} can be obtained as follows

$$\hat{\beta}_{mn} = \frac{\mathbf{p}_{mn}^\dagger \bar{\mathbf{R}}^{-1}\mathbf{x}_{mn}}{\mathbf{p}_{mn}^\dagger \bar{\mathbf{R}}^{-1}\mathbf{p}_{mn}}. \tag{20}$$

Substituting (18) and (20) into (17) yields the final test statistic of the knowledge-aided MIMO detector, given by (21) shown at the bottom of this page, where

$$\hat{\mathbf{y}}_{mn} = \mathbf{x}_{mn} - \frac{\mathbf{p}_{mn}^\dagger \bar{\mathbf{R}}^{-1}\mathbf{x}_{mn}\mathbf{p}_{mn}}{\mathbf{p}_{mn}^\dagger \bar{\mathbf{R}}^{-1}\mathbf{p}_{mn}}, \tag{22}$$

and

$$c = \frac{v+1-L}{L}. \tag{23}$$

IV. KA Detection With Range Training

When training data is available, we attempt to exploit it to improve the detection performance. In this section, we assume the availability of non-homogeneous sample support and develop a KA MIMO detector for range-spread target detection in distributed MIMO radar. In this case, the problem (1) is modified as

$$\begin{cases} H_0: \begin{cases} \mathbf{X}_{mn} = \mathbf{p}_{mn}\boldsymbol{\alpha}_{mn}^T + \mathbf{N}_{mn}, \\ \mathbf{x}_{mn}^{(t)} = \mathbf{n}_{mn}^{(t)}, \end{cases} \\ H_1: \begin{cases} \mathbf{X}_{mn} = \mathbf{p}_{mn}\boldsymbol{\alpha}_{mn}^T + \mathbf{N}_{mn}, \\ \mathbf{x}_{mn}^{(t)} = \mathbf{n}_{mn}^{(t)}, \end{cases} \\ \quad m = 1,\cdots,M, n = 1,\cdots,N, \\ \quad t = 1,\cdots,T, \end{cases} \tag{24}$$

where $\mathbf{x}_{mn}^{(t)}, t = 1,\cdots,T$, denotes the training data and T is the number of training data. In (24), the disturbance in the primary data $\mathbf{X}_{mn}$ is modeled as a compound-Gaussian process that can be interpreted as the product of a complex, zero-mean, possibly correlated Gaussian process and a real, non-negative component (referred to as texture) to depict the non-homogeneity, namely, $\mathbf{n}_{mn,h} \sim \mathcal{CN}(\mathbf{0}, \tau_{mn,h}\mathbf{R}_{mn}), h = 1,\cdots,H$, where $\tau_{mn,h}$ denotes the texture.

As the joint ML estimation for the problem (24) using both the primary and training data is computationally intractable, we resort to an *ad-hoc* two-step procedure to solve the problem (24). More precisely, we first derive the test statistic under the assumption that the covariance matrix $\mathbf{R}_{mn}$ is known. Then, $\mathbf{R}_{mn}$ is replaced by a KA-based estimate using both the prior knowledge and the training data to obtain the final test statistic.

In step 1, the disturbance covariance matrix $\mathbf{R}_{mn}$ is assumed to be known. Let $\boldsymbol{\tau} = [\boldsymbol{\tau}_{11},\cdots,\boldsymbol{\tau}_{mn},\cdots,\boldsymbol{\tau}_{MN}]$ with $\boldsymbol{\tau}_{mn} = [\tau_{mn,1},\cdots,\tau_{mn,H}]$. The GLRT using only the primary data $\mathbf{X}_{mn}$ is given by

$$\frac{\max\limits_{\boldsymbol{\alpha},\boldsymbol{\tau}} \prod\limits_{m,n} f_1(\mathbf{x}_{mn,1},\cdots,\mathbf{x}_{mn,H}|\boldsymbol{\alpha}_{mn},\boldsymbol{\tau}_{mn})}{\max\limits_{\boldsymbol{\tau}} \prod\limits_{m,n} f_0(\mathbf{x}_{mn,1},\cdots,\mathbf{x}_{mn,H}|\boldsymbol{\tau}_{mn})} \underset{H_0}{\overset{H_1}{\gtrless}} \xi_3, \tag{25}$$

where

$$f_1(\mathbf{x}_{mn,1},\cdots,\mathbf{x}_{mn,H}|\boldsymbol{\alpha}_{mn},\boldsymbol{\tau}_{mn}) = \prod_{h=1}^{H} \frac{1}{(\tau_{mn,h}\pi)^L|\mathbf{R}_{mn}|}$$
$$\times\exp\Big[-\frac{(\mathbf{x}_{mn}-\alpha_{mn,h}\mathbf{p}_{mn})^\dagger \mathbf{R}_{mn}^{-1}(\mathbf{x}_{mn}-\alpha_{mn,h}\mathbf{p}_{mn})}{\tau_{mn,h}}\Big], \tag{26}$$

$$f_0(\mathbf{x}_{mn,1},\cdots,\mathbf{x}_{mn,H}|\boldsymbol{\tau}_{mn}) = \prod_{h=1}^{H} \frac{1}{(\tau_{mn,h}\pi)^L|\mathbf{R}_{mn}|}$$
$$\times\exp\left(-\frac{\mathbf{x}_{mn}^\dagger \mathbf{R}_{mn}^{-1}\mathbf{x}_{mn}}{\tau_{mn,h}}\right), \tag{27}$$

and ξ_3 denotes the threshold. The MLEs of $\boldsymbol{\alpha}$ and $\boldsymbol{\tau}$ are easy to obtain and omitted for simplicity. Using these MLEs in (25), we can show that the detector reduces to

$$\prod_{m,n}\prod_{h=1}^{H}\Big(1-\frac{||\mathbf{x}_{mn,h}^\dagger \mathbf{R}_{mn}^{-1}\mathbf{p}_{mn}||^2}{\mathbf{p}_{mn}^\dagger \mathbf{R}_{mn}^{-1}\mathbf{p}_{mn}\sum\limits_{h=1}^{H}\mathbf{x}_{mn,h}^\dagger \mathbf{R}_{mn}^{-1}\mathbf{x}_{mn,h}}\Big)^{-L} \underset{H_0}{\overset{H_1}{\gtrless}} \xi_{m3}, \tag{28}$$

where ξ_{m3} denotes the modified threshold.

For step 2, we consider using the maximum a posteriori (MAP) estimate of $\mathbf{R}_{mn}$:

$$\widehat{\mathbf{R}}_{mn,\mathrm{MAP}} = \arg\max_{\mathbf{R}_{mn},\lambda_{mn}} f(\mathbf{x}_{mn,1},\cdots,\mathbf{x}_{mn,T}|\mathbf{R}_{mn})f(\mathbf{R}_{mn}), \tag{29}$$

where $f(\mathbf{x}_{mn}^{(1)},\cdots,\mathbf{x}_{mn}^{(T)}|\mathbf{R}_{mn})$ denotes the multivariate Gaussian PDF conditioned on $\mathbf{R}_{mn}$ for the training data. Thus,

$$\prod_{m,n}\frac{(\mathbf{p}_{mn}^\dagger \bar{\mathbf{R}}^{-1}\mathbf{p}_{mn})(\mathbf{x}_{mn}^\dagger \bar{\mathbf{R}}^{-1}\mathbf{x}_{mn})}{(\mathbf{p}_{mn}^\dagger \bar{\mathbf{R}}^{-1}\mathbf{p}_{mn})(\mathbf{x}_{mn}^\dagger \bar{\mathbf{R}}^{-1}\mathbf{x}_{mn}) - |\mathbf{p}_{mn}^\dagger \bar{\mathbf{R}}^{-1}\mathbf{x}_{mn}|^2}\frac{|\mathbf{x}_{mn}\mathbf{x}_{mn}^\dagger + c\mathbf{x}_{mn}^\dagger \bar{\mathbf{R}}^{-1}\mathbf{x}_{mn}\bar{\mathbf{R}}|}{|\hat{\mathbf{y}}_{mn}\hat{\mathbf{y}}_{mn}^\dagger + c\hat{\mathbf{y}}_{mn}^\dagger \bar{\mathbf{R}}^{-1}\hat{\mathbf{y}}_{mn}\bar{\mathbf{R}}|} \underset{H_0}{\overset{H_1}{\gtrless}} \xi_{m2}, \tag{21}$$

we have

$$f(\mathbf{x}_{mn}^{(1)},\cdots,\mathbf{x}_{mn}^{(T)}|\mathbf{R}_{mn})f(\mathbf{R}_{mn}) \propto \frac{\lambda^{Lv}}{|\mathbf{R}_{mn}|^{T+v+L}}\text{etr}\Big\{-\mathbf{R}_{mn}^{-1}\left[\mathbf{S}_{mn}+(v-N)\lambda_{mn}\bar{\mathbf{R}}\right]\Big\}, \quad (30)$$

where $\mathbf{S}_{mn}=\sum_{t=1}^{T}\mathbf{x}_{mn}^{(t)}\mathbf{x}_{mn}^{(t)\dagger}$. It follows from (30) that

$$\max_{\mathbf{R}_{mn},\lambda_{mn}} \ (T+v+L)\log(|\mathbf{R}_{mn}^{-1}|) - \text{tr}\{\mathbf{R}_{mn}^{-1}[\mathbf{p}_{mn}+(v-N)\lambda_{mn}\bar{\mathbf{R}}]\}. \quad (31)$$

Using the following algebraic inequalities,

$$1/n\sum_{i=1}^{n}x_i \geq \Big(\prod_{i=1}^{n}x_i\Big)^{1/n},$$
$$a\log(x)-Nx^{1/N} \leqslant a(N\log(a)-N),$$
$$\text{tr}(\mathbf{A})=\sum_i \varsigma_i,$$
$$|\mathbf{A}|=\prod_i \varsigma_i, \quad (32)$$

where ς_i denotes the ith eigen value of a matrix $\mathbf{A}$, we have

$$(T+v+L)\log(|\mathbf{R}_{mn}^{-1}|)-\text{tr}\{\mathbf{R}_{mn}^{-1}[\mathbf{S}_{mn}+(v-N)\lambda_{mn}\bar{\mathbf{R}}]\} \leqslant (T+v+L)[N\log(T+v+L)-L - \log(|\mathbf{S}_{mn}+(v-L)\lambda_{mn}\bar{\mathbf{R}}|)]. \quad (33)$$

Equation (33) reduces to an equality when

$$\widehat{\mathbf{R}}_{mn,\text{MAP}}(\lambda_{mn})=\frac{1}{T+v+L}[\mathbf{S}_{mn}+(v-N)\lambda_{mn}\bar{\mathbf{R}}]. \quad (34)$$

Then, the estimate of λ can be obtained as follows

$$\widehat{\lambda}_{mn}=\arg\max_{\lambda_{mn}}\frac{\lambda_{mn}^{Lv}}{|(v-N)\lambda_{mn}\bar{\mathbf{R}}|^{T+v+L}}. \quad (35)$$

Similar to (A4), $\widehat{\lambda}_{mn}$ can be obtained as the unique positive solution of the equation

$$\sum_{i=1}^{r}\frac{\gamma_i}{\gamma_i+\lambda_{mn}}=L-\frac{Lv}{v+L+T}, \quad (36)$$

where γ_i is the eigen-value of $\sum_{t=1}^{T}\mathbf{x}_{mn}^{(t)\dagger}((v-L)\bar{\mathbf{R}})^{-1}\mathbf{x}_{mn}^{(t)}$. Thus, we have

$$\widehat{\mathbf{R}}_{mn,\text{MAP}}=\frac{1}{T+v+L}[\mathbf{S}_{mn}+(v-N)\widehat{\lambda}_{mn}\bar{\mathbf{R}}]. \quad (37)$$

Substituting $\widehat{\mathbf{R}}_{mn,map}$ into (28) yields the proposed detector

$$\prod_{m,n}\prod_{h=1}^{H}\left(1-\frac{||\mathbf{x}_{mn,h}^{\dagger}\widehat{\mathbf{R}}_{mn,\text{MAP}}^{-1}\mathbf{p}_{mn}||^2}{\mathbf{p}_{mn}^{\dagger}\widehat{\mathbf{R}}_{mn,\text{MAP}}^{-1}\mathbf{p}_{mn}\sum_{h=1}^{H}\mathbf{x}_{mn,h}^{\dagger}\widehat{\mathbf{R}}_{mn,\text{MAP}}^{-1}\mathbf{x}_{mn,h}}\right) \underset{H_0}{\overset{H_1}{\gtrless}} \xi_{m3}. \quad (38)$$

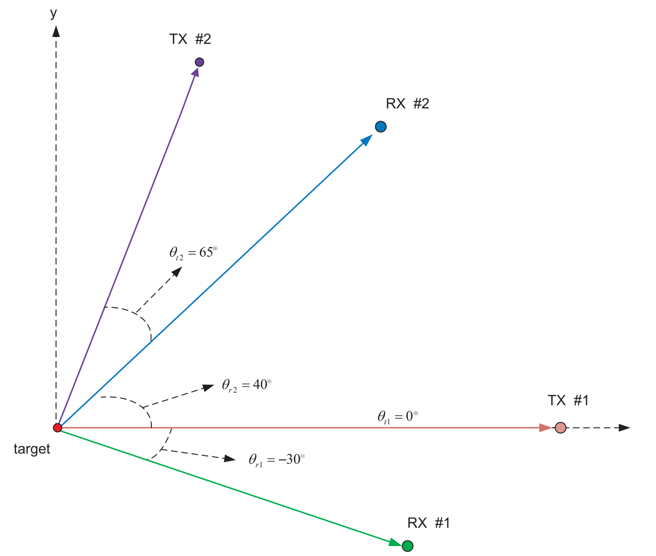

Fig. 1. Distributed MIMO radar configuration.

V. Performance Assessment

A. *Extensions of Conventional Covariance Matrix-Based Detectors*

For comparison purposes, we consider extended versions of the covariance matrix-based detectors [10], [12], which were originally introduced for point-like target detection, to detect a range-spread target in distributed MIMO radar. If $\mathbf{R}_{mn}$ is known, the matched filter (MF) for range-spread target detection is given by

$$\frac{\max_{\boldsymbol{\alpha}}\prod_{m,n}f(\mathbf{x}_{mn,1},\cdots,\mathbf{x}_{mn,H}|\boldsymbol{\alpha}_{mn})}{\prod_{m,n}f(\mathbf{x}_{mn,1},\cdots,\mathbf{x}_{mn,H})}, \quad (39)$$

where $f(\mathbf{x}_{mn,1},\cdots,\mathbf{x}_{mn,H}|\boldsymbol{\alpha}_{mn})$ and $f(\mathbf{x}_{mn,1},\cdots,\mathbf{x}_{mn,H})$ are the multivariate Gaussian density functions under under H_0 and H_1, respectively. It is not difficult to derive (39). The MF test statistic is given by (see [29])

$$T_{\text{MF}}=\prod_{m,n}\sum_{h=1}^{H}\frac{|\mathbf{x}_{mn,h}^{\dagger}\mathbf{R}_{mn}^{-1}\mathbf{p}_{mn}|^2}{\mathbf{p}_{mn}^{\dagger}\mathbf{R}_{mn}^{-1}\mathbf{p}_{mn}}. \quad (40)$$

The SCM range-spread target detector is obtained by replacing $\mathbf{R}_{mn}$ in (40) by the sample covariance matrix

$$\hat{\mathbf{R}}_{mn}=\frac{1}{T}\sum_{t=1}^{T}\mathbf{x}_{mn}^{(t)}\mathbf{x}_{mn}^{(t)\dagger}, \quad (41)$$

which yields

$$T_{\text{SCM}}=\prod_{m,n}\sum_{h=1}^{H}\frac{|\mathbf{x}_{mn,h}^{\dagger}\hat{\mathbf{R}}_{mn}^{-1}\mathbf{p}_{mn}|^2}{\mathbf{p}_{mn}^{\dagger}\hat{\mathbf{R}}_{mn}^{-1}\mathbf{p}_{mn}}. \quad (42)$$

The RSCM detector for range-spread target detection can be described by the following two-step procedure. First, the test statistics with a known $\mathbf{R}_{mn}$ is obtained in the same way as (28). Second, replacing $\mathbf{R}_{mn}$ in (28) by the robust covariance

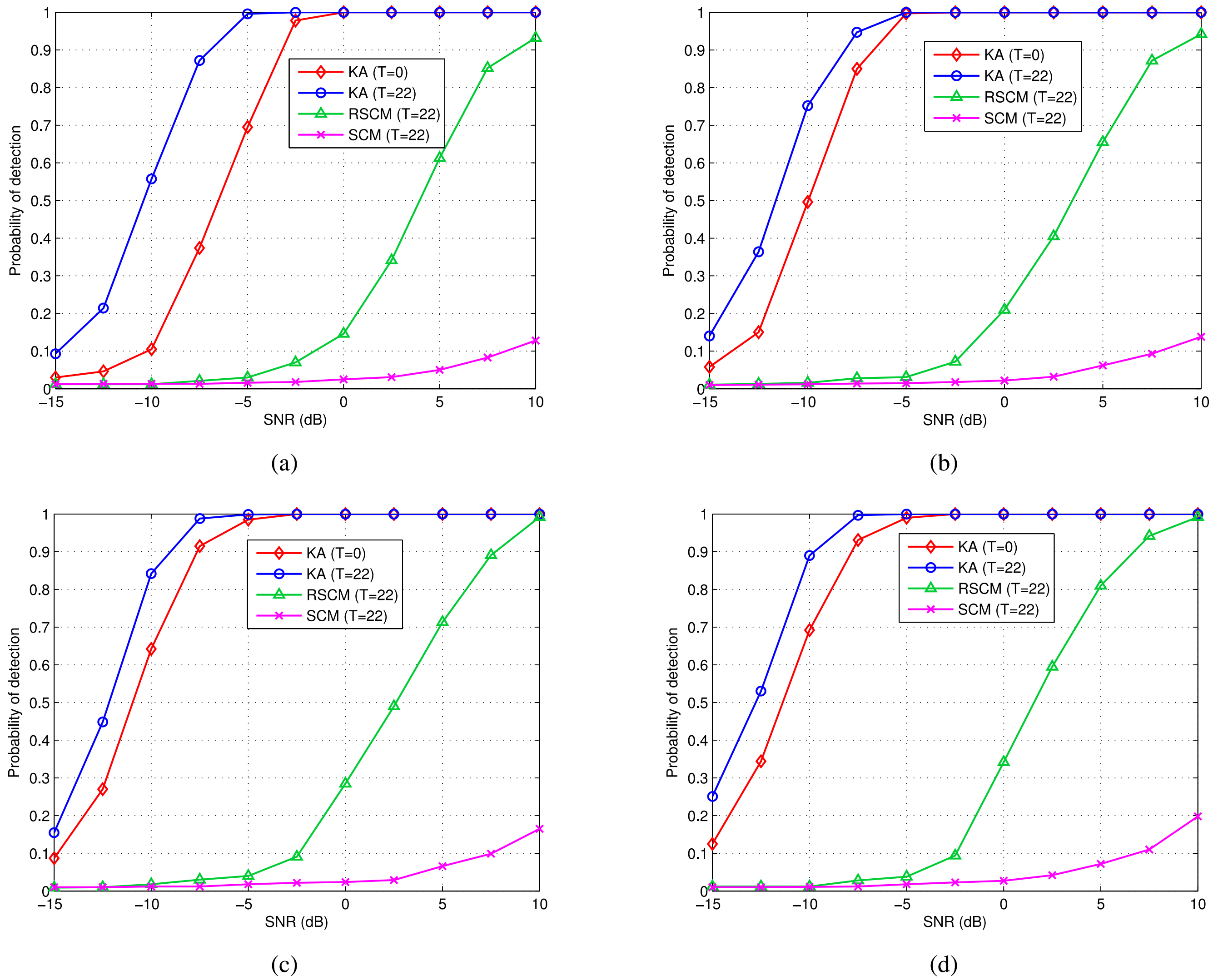

Fig. 2. Probability of detection versus SNR with a non-fluctuating range-spread target. (a) $H = 4$; (b) $H = 6$; (c) $H = 8$; (d) $H = 10$.

matrix estimate $\hat{\mathbf{M}}_{mn}$ yields the RSCM detector

$$T_{\text{RSCM}} = \prod_{m,n}\prod_{h=1}^{H}\left(1 - \frac{||\mathbf{x}_{mn,h}^{\dagger}\hat{\mathbf{M}}_{mn}^{-1}\mathbf{p}_{mn}||^2}{\mathbf{p}_{mn}^{\dagger}\hat{\mathbf{M}}_{mn}^{-1}\mathbf{p}_{mn}\sum_{h=1}^{H}\mathbf{x}_{mn,h}^{\dagger}\hat{\mathbf{M}}_{mn}^{-1}\mathbf{x}_{mn,h}}\right)^{-L}, \tag{43}$$

where $\hat{\mathbf{M}}_{mn}$ is a fixed point estimate (FPE) of the covariance matrix by solving [35]

$$\hat{\mathbf{M}}_{mn} = \frac{L}{T}\sum_{t=1}^{T}\frac{\mathbf{x}_{mn}^{(t)}\mathbf{x}_{mn}^{(t)\dagger}}{\mathbf{x}_{mn}^{(t)\dagger}\mathbf{M}_{mn}^{-1}\mathbf{x}_{mn}^{(t)}}. \tag{44}$$

B. Numerical Results

In this section, numerical examples are provided to assess the performance of the proposed detectors, referred to as the KA detectors, which are compared with the SCM detector (42) and the RSCM (43), for range-spread target detection. For point-like target detection, the proposed KA detector (21) is compared with the SCM detector in [10] and the RSCM detector in [12].

We consider two cases without and, respectively, with range training. When training is available, the training signals are non-homogeneous and is compound-Gaussian distribution with a scaling factor of 5 and a shape factor of 0.2. The configuration of the distributed MIMO is shown in Fig. 1, which consists of two transmitters at 0° and 65° relative to the target and two receivers at -30° and 40°. It is noted that the configuration is the same as the one in [2], [10]. The pulse repetition frequency is 500 Hz, the carrier frequency is 1 GHz, the target velocity is 108 km/h. The above parameters lead to a normalized target Doppler frequency 0.2. In all simulations, we set $L = 20$ and $v = 24$, $\lambda_{11} = 1, \lambda_{12} = 2, \lambda_{21} = 3, \lambda_{22} = 4$, unless stated otherwise. For simplicity, $\alpha_{mn,h}$ is set to the same value among all the H range cells cells but $\boldsymbol{\alpha}_{mn}$ is different for different Tx-Rx pairs. As to $\bar{\mathbf{R}}$, we assume an exponentially correlation covariance matrix with one-lag correlation coefficient $\rho = 0.9$, namely, the (i,j)th element of $\bar{\mathbf{R}}$ is given by $\rho^{|i-j|}$. To decrease the computational load, the probability of false alarm P_{fa} is chosen to be 10^{-2} and the number of independent trials is $100/P_{fa}$.

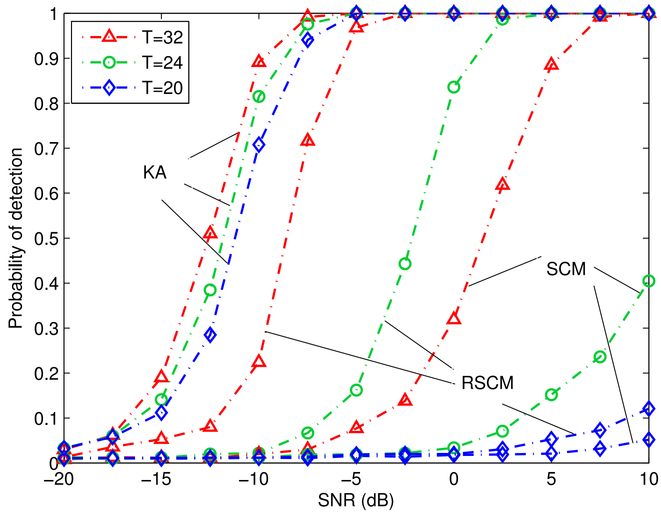

Fig. 3. Probability of detection versus SNR with a non-fluctuating range-spread target and different numbers of training data.

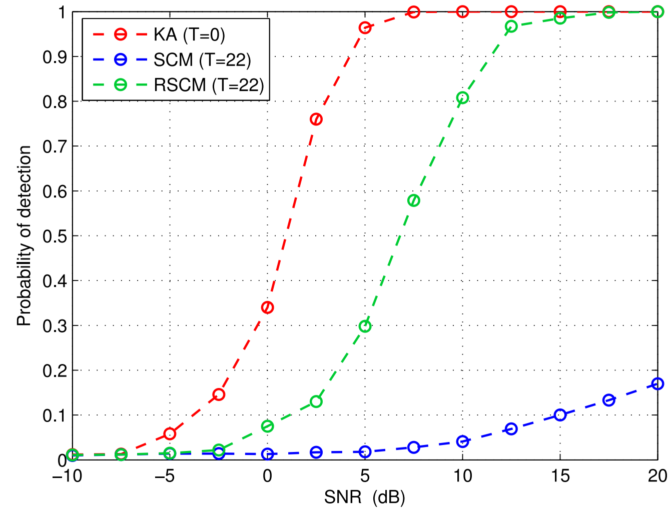

Fig. 4. Probability of detection versus SNR with a non-fluctuating point-like target.

First, we consider a non-fluctuating target model with fixed target amplitudes from trial to trial. In this case, the signal-to-noise ratio (SNR) is defined as

$$\text{SNR} = L\sum_{mn}\frac{\sum_{h=1}^{H}|\alpha_{mn,h}|^2}{\lambda_{mn}}. \tag{45}$$

The detection probability versus SNR is shown in Fig. 2, where $H = 4, 6, 8$ and 10, respectively. The number of training data for each Tx-Rx pair of the covariance matrix-based detectors is 22. KA (T = 0) represents the proposed detector (11) without range training, whereas KA (T = 22) represents the proposed detector (38) with 22 training data samples. As shown in Fig. 2, the proposed KA detector with training performs the best due to the exploitation of both the *a priori* knowledge and the training data. Without training, the KA detector experiences some degradation but still significantly outperforms the other conventional detectors. It is also observed that increasing the radar resolution capabilities, which corresponding to increasing H, can produce a significant detection gain.

Next, we study the effect of different numbers of training data on the detection performance. Fig. 3 shows the detection probability of the KA detector (38), RSCM, and SCM for different training sizes, where $H = 8$. It is seen that the larger the number of training data, the better the detection performance. Additionally, the performance of the proposed KA-MIMO significantly outperforms the RSCM and SCM, especially when the number of training signals is small.

Fig. 4 shows the detection probability curves for detecting a point-like target when the training size for each Tx-Rx pair is 22. The proposed KA-MIMO (21) does not use any training data, but for the SCM and RSCM detectors, the total amount of training data is $4 \times 22 = 88$. As shown in Fig. 4, the proposed KA-MIMO significantly outperforms the traditional solutions.

We now examine the case of detecting a fluctuating target, where the target amplitude $\alpha_{mn,h}$ is generated as a complex Gaussian random variable with zero mean and variance $\sigma^2_{\alpha_{mn,h}} = 1$. In this case, the SNR is defined as

$$\text{SNR} = L\sum_{mn}\frac{\sum_{h=1}^{H}|\sigma_{\alpha_{mn,h}}|^2}{\lambda_{mn}}. \tag{46}$$

All other parameters remain the same as before.

The detection performance for detecting a fluctuating range-spread target are illustrated in Fig. 5. It is seen that the relationships among the different detectors are similar to those in Fig. 2. The performances of the proposed KA detector without training (11) and KA detector with training (38) are better than the RSCM and the SCM detectors. Comparing these results with different H, we see that increasing the radar resolution capabilities also improves the detection performance. In addition, a comparison between Fig. 2 and Fig. 5 reveals that with the fluctuating target model, all detectors experience some loss in detection performance.

The influence of different numbers of training data on the detection performance is illustrated in Fig. 6. This figure implies that increasing the number of training data yields a better the detection performance. A comparison between Fig. 6 and Fig. 3 shows that the fluctuating target model also leads to some loss in detection performance.

The detection performance for a point-like fluctuating target is shown in Fig. 7. It is seen that the proposed KA detector without any training significantly outperforms the conventional SCM and RSCM detectors.

We now consider a case when the powers λ_{mn} are Gamma distributed random variables. Note that in our data model [e.g., (3)], λ_{mn} is treated as a deterministic unknown parameter, and our detectors were developed based on this assumption. Nevertheless, for performance evaluation, it is standard to test the detectors with random powers, which are widely employed to model spatial power heterogeneity. For fair comparison with the fixed-valued case, e.g., Fig. 2(a), the shape and scale parameters of the Gamma distributed λ_{mn} are set to appropriate values such

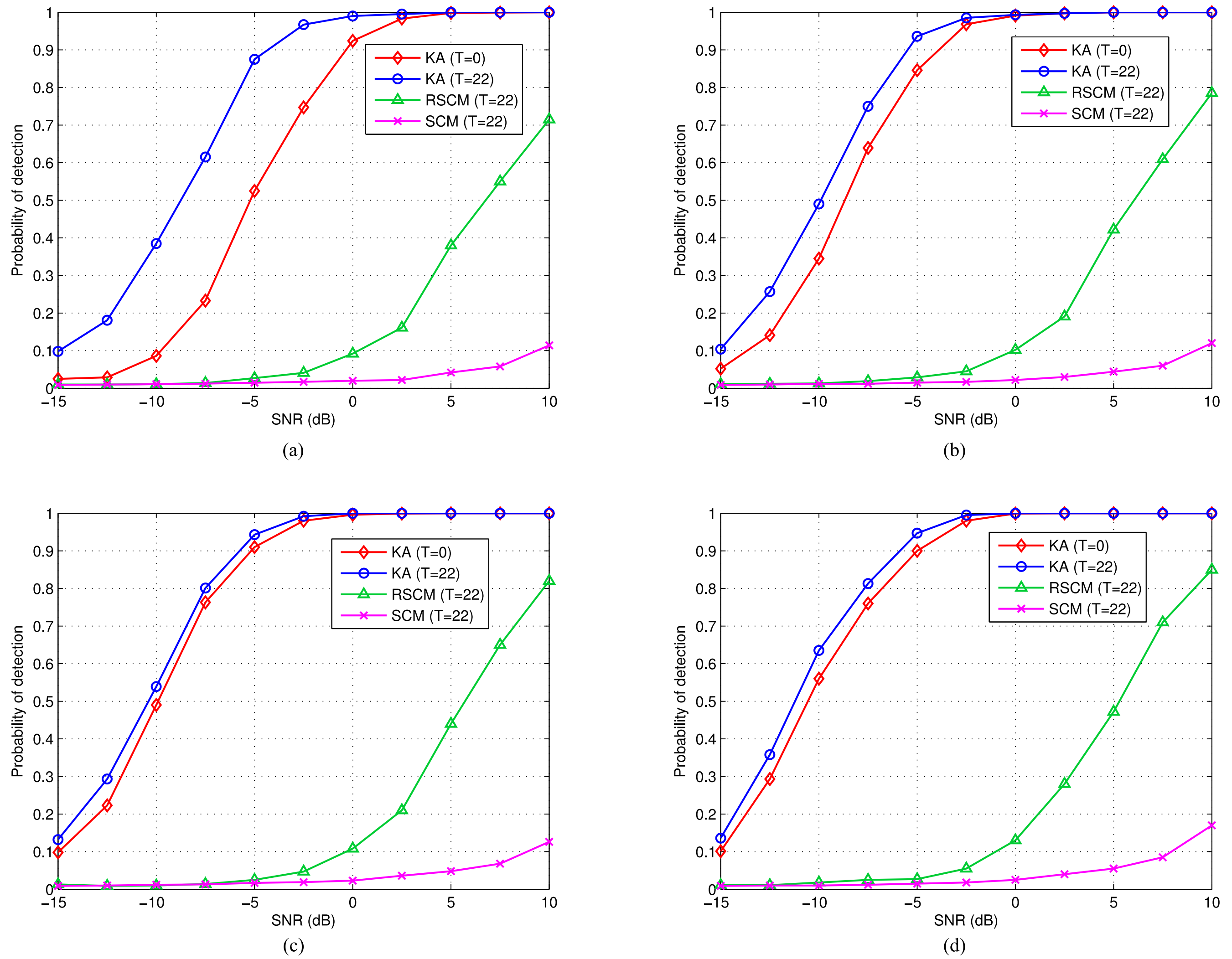

Fig. 5. Probability of detection versus SNR for fluctuating range-spread target. (a) $H = 4$; (b) $H = 6$; (c) $H = 8$; (d) $H = 10$.

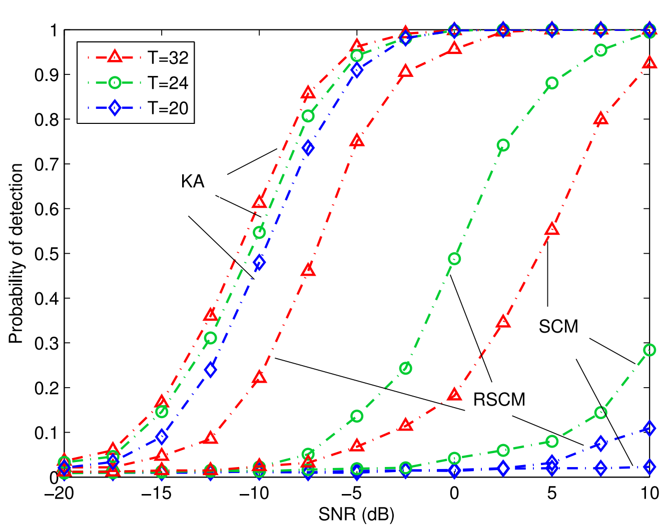

Fig. 6. Probability of detection versus SNR with a fluctuating range-spread target and different numbers of training data.

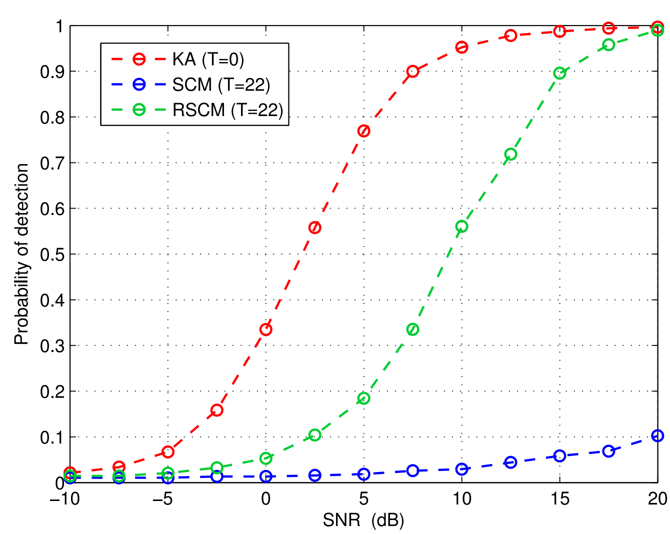

Fig. 7. Probability of detection versus SNR with a fluctuating point-like target.

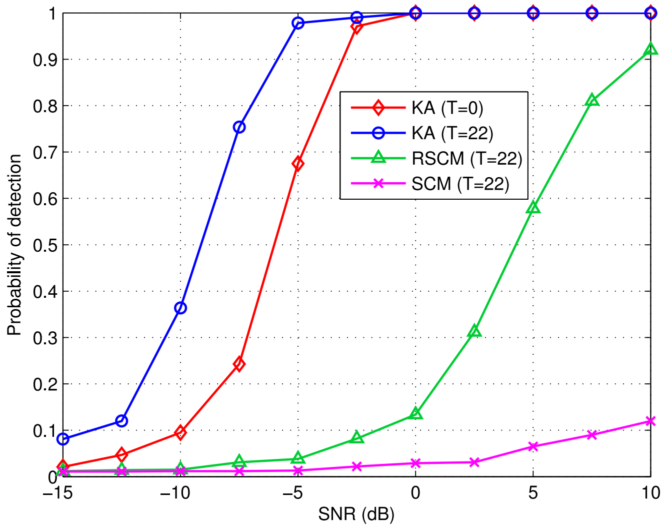

Fig. 8. Probability of detection versus SNR with random powers.

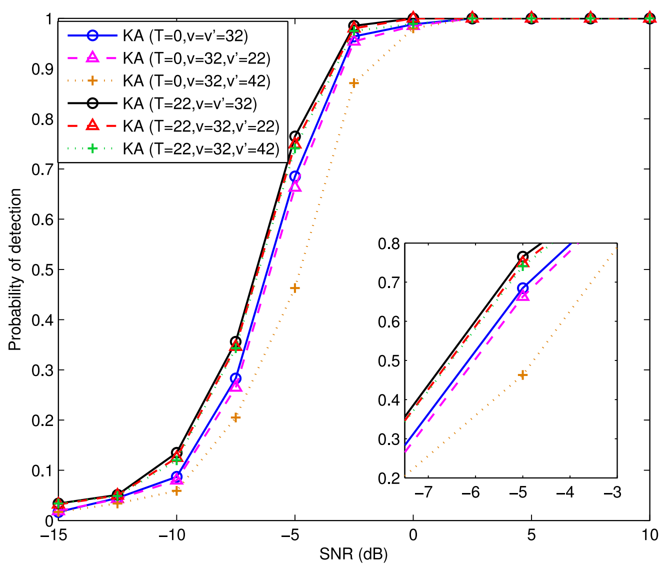

Fig. 9. Probability of detection versus SNR with a mismatch in v.

that its mean equals the fixed λ_{mn} in Fig. 2(a). Other parameters remain the same as Fig. 2(a). The results are shown in Fig. 8. It is seen that there is some degradation for all detectors with random powers compared with the fixed power case.

So far, we have assumed that the degree of freedom v in the proposed statistical model (4) for the disturbance covariance matrix $\mathbf{R}_{mn}$ is known. In practice, v has to be estimated from prior observations, and there can be a mismatch between the estimated v and its actual value. The impact of such a mismatch between the estimated and the actual v is now examined. We consider a case where the target is non-fluctuating and $H = 4$. Specifically, the actual value used for data generation is $v = 32$, while the detectors use two different values, $v' = 22$ or 42 to simulate an underestimated and, respectively, overestimated scenario. The results of the simulation are shown in Fig. 9. Based on the simulation results, we see that the proposed methods are in general not very sensitive to the mismatch, although the proposed KA-MIMO without training appears to be slightly more sensitive to an overestimated v than using an underestimated value.

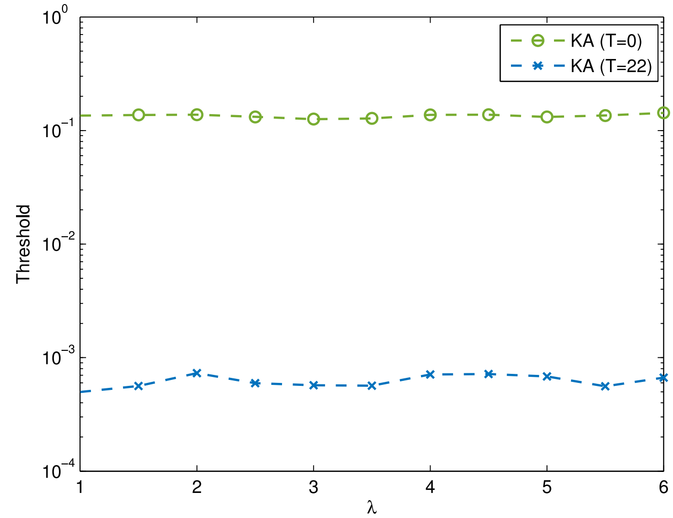

Fig. 10. Threshold versus the power.

The CFAR property of the proposed detectors with respect to the power is examined in Fig. 10, where SNR $= -5$ dB, $T = 24$, $H = 4$, and $\lambda_{11} = \lambda_{12} = \lambda_{21} = \lambda_{22} = \lambda$. The simulation result indicates that the proposed methods are approximately CFAR with respect to the disturbance power.

VI. Conclusion

In this paper, we investigated the problem of detecting a range-spread target in distributed MIMO radar. A stochastic KA model involving random matrices with complex inverse Wishart distribution was introduced, where the disturbance covariance matrix exhibits non-homogeneity across different Tx-Rx antenna pairs. Moreover, the power level for each stochastic covariance matrix is also different across different Tx-Rx pairs. Under this framework, we adopted the GLRT approach and developed several KA detectors for two cases with either no training or a limited number of training data. Simulation results showed that the proposed detectors offer significant improvement over the conventional detectors in non-homogeneous and training-deficient environments. A number of future directions may be pursued, including development of the Rao or Wald test based detectors, and subspace processing based detectors.

Appendix A
Proof of (11)

From the numerator of (10), the ML estimate of $\boldsymbol{\alpha}_{mn}$ can be derived by

$$\begin{aligned}\hat{\boldsymbol{\alpha}}_{mn} &= \arg\max_{\boldsymbol{\alpha}_{mn}} |\mathbf{Y}_{mn}\mathbf{Y}_{mn}^{\dagger} + \lambda_{mn}(v-L)\bar{\mathbf{R}}|^{-(v+H)} \\ &= \arg\min_{\boldsymbol{\alpha}_{mn}} |\lambda_{mn}(v-L)\bar{\mathbf{R}}||\mathbf{Y}_{mn}^{\dagger}(\lambda_{mn}(v-L)\bar{\mathbf{R}})^{-1}\mathbf{Y}_{mn} + \mathbf{I}_L| \\ &= \arg\min_{\boldsymbol{\alpha}_{mn}} |(\mathbf{X}_{mn} - \mathbf{p}_{mn}\boldsymbol{\alpha}_{mn})^{\dagger}(\lambda_{mn}(v-L)\bar{\mathbf{R}})^{-1} \\ &\qquad \times (\mathbf{X}_{mn} - \mathbf{p}_{mn}\boldsymbol{\alpha}_{mn}) + \mathbf{I}_L|. \quad \text{(A1)}\end{aligned}$$

Unlike many authors who use several equations to solve the minimum of the determinant of the sum of two matrix, we preset a simpler way. Let $\mathbf{G}_\theta$ denotes a full-rank non-zero matrix function of θ. If $\mathbf{G}_\theta^{-1}$ exists, $|\mathbf{G}_\theta|\mathbf{G}_\theta^{-1} \neq \mathbf{0}$ holds. Then, from

$$\frac{d|\mathbf{G}_\theta|}{d\theta} = |\mathbf{G}_\theta|\mathrm{tr}\left(\mathbf{G}_\theta^{-1}\frac{d\mathbf{G}}{d\theta}\right) = \mathrm{tr}\left(|\mathbf{G}_\theta|\mathbf{G}_\theta^{-1}\frac{d\mathbf{G}_\theta}{d\theta}\right) = 0, \tag{A2}$$

we have $\frac{d\mathbf{G}_\theta}{d\theta} = \mathbf{0}$.

Thus, taking the derivative of $(\mathbf{X}_{mn} - \mathbf{p}_{mn}\boldsymbol{\alpha}_{mn})^\dagger(\lambda_{mn}(v - L)\bar{\mathbf{R}})^{-1}(\mathbf{X}_{mn} - \mathbf{p}_{mn}\boldsymbol{\alpha}_{mn})$ with respect to $\boldsymbol{\alpha}_{mn}$ yields

$$\hat{\boldsymbol{\alpha}}_{mn} = \frac{\mathbf{p}_{mn}^\dagger \bar{\mathbf{R}}^{-1}\mathbf{X}_{mn}}{\mathbf{p}_{mn}^\dagger \bar{\mathbf{R}}^{-1}\mathbf{p}_{mn}}. \tag{A3}$$

Applying $\hat{\boldsymbol{\alpha}}_{mn}$ into (10) results in

$$\Lambda_{mn} = \frac{\min\limits_{\lambda_{mn}} \lambda_{mn}^{-\frac{Lv}{v+H}}|\mathbf{X}_{mn}\mathbf{X}_{mn}^\dagger + \lambda_{mn}(v-L)\bar{\mathbf{R}}|}{\min\limits_{\lambda_{mn}} \lambda_{mn}^{-\frac{Lv}{v+H}}|\hat{\mathbf{Y}}_{mn}\hat{\mathbf{Y}}_{mn}^\dagger + \lambda_{mn}(v-L)\bar{\mathbf{R}}|}, \tag{A4}$$

where $\hat{\mathbf{Y}}_{mn} = \mathbf{X}_{mn} - \frac{\mathbf{p}_{mn}\mathbf{p}_{mn}^\dagger \bar{\mathbf{R}}^{-1}\mathbf{X}_{mn}}{\mathbf{p}_{mn}^\dagger \bar{\mathbf{R}}^{-1}\mathbf{p}_{mn}}$. As shown in [20], the minimum of the function $f(\lambda) = \lambda^a|\mathbf{A} + \lambda\mathbf{B}|$ in (A4) has the unique positive solution of the following equation

$$\sum_{i=1}^{r} \frac{\gamma_i}{\gamma_i + \lambda_{mn}} = L - \frac{Lv}{v+H}, \tag{A5}$$

where γ_i is the eigen-value of $\hat{\mathbf{Y}}_{mn}^\dagger(\lambda_{mn}(v-L)\bar{\mathbf{R}})^{-1}\hat{\mathbf{Y}}_{mn}$ for $\hat{\lambda}_{mn,1}$ under H_1 or the eigen-value of $\mathbf{X}_{mn}^\dagger(\lambda_{mn}(v - L)\bar{\mathbf{R}})^{-1}\mathbf{X}_{mn}$ for $\hat{\lambda}_{mn,0}$ under H_0. Therefore, we have (11) as the test statistic for the proposed KA detector for range-spread target detection.

Appendix B
The ML Estimate of λ_{mn}

From (17), the ML estimate of λ_{mn} under H_1 is obtained by

$$\begin{aligned}
&\min_{\lambda_{mn}} \lambda_{mn}^{-vL}|\mathbf{y}_{mn}\mathbf{y}_{mn}^\dagger + \lambda_{mn}(v-L)\bar{\mathbf{R}}|^{v+1} \\
&= \min_{\lambda_{mn}} \lambda_{mn}^{\frac{L}{v+1}}|\lambda_{mn}^{-1}\mathbf{y}_{mn}\mathbf{y}_{mn}^\dagger + (v-L)\bar{\mathbf{R}}| \\
&= \min_{\lambda_{mn}} \lambda_{mn}^{\frac{L}{v+1}}(1 + \lambda_{mn}^{-1}\mathbf{y}_{mn}((v-L)\bar{\mathbf{R}})^{-1}\mathbf{y}_{mn}^\dagger),
\end{aligned} \tag{B1}$$

where in the second equality, we used the following equation [36]

$$|\mathbf{C} - \mathbf{z}\mathbf{z}^\dagger| = |\mathbf{C}|(1 - \mathbf{z}^\dagger\mathbf{C}^{-1}\mathbf{z}). \tag{B2}$$

Taking the derivative of the log of the right-side of (B1) with respect to λ_{mn} and equating the results to zero, we obtain the MLE the ML estimate of λ_{mn}

$$\hat{\lambda}_{mn,1} = \frac{v+1-L}{L}\mathbf{y}_{mn}^\dagger((v-L)\bar{\mathbf{R}})^{-1}\mathbf{y}_{mn}. \tag{B3}$$

Similarly, the ML estimate of λ_{mn} under H_0 can be shown to be

$$\hat{\lambda}_{mn,0} = \frac{v+1-L}{L}\mathbf{x}_{mn}^\dagger((v-L)\bar{\mathbf{R}})^{-1}\mathbf{x}_{mn}. \tag{B4}$$

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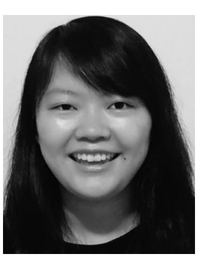

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