Cooperative Spectrum Sensing With Location Information

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***Abstract*—In this paper, we address the problem of cooperatively detecting a primary user (PU) among multiple cognitive users (CUs) when their location information is available at a CU base station. For fast detection, each CU reports a power estimate, based on one-snapshot observation of the radio environment, to the CU base station. A generalized likelihood ratio test (GLRT) is developed at the CU base station to first estimate the transmit power of the PU and then form a test variable for detection. The maximum likelihood estimator (MLE) of the unknown transmit power is discussed and analyzed to offer insight into the proposed cooperative spectrum-sensing scheme. In addition, a weighted average estimator (WAE) is proposed, which is computationally more efficient than the MLE. Asymptotic analysis for the proposed GLRT is presented. Performance of the MLE and WAE is examined along with the corresponding Cramer–Rao bound (CRB). Extensive comparisons between the proposed GLRTs and the hard- and soft-decision based spectrum sensing methods are provided, which show the effectiveness of the proposed detector.**

***Index Terms*—Cognitive radio, cooperative spectrum sensing, generalized likelihood ratio test (GLRT), location information, maximum likelihood estimation.**

I. Introduction

COGNITIVE radio [1] is considered a promising technology for efficient spectrum utilization by allowing secondary users to share the spectrum with licensed users (also called primary users (PUs)) without causing harmful interference. A cognitive radio system is an intelligent wireless communication system that is aware of its surrounding environment, learns from the environment, and adapts its operating parameters in real-time [2]. One fundamental requirement of this system is the ability to identify spectrum holes (also known as white space) in the spectrum of interest to provide opportunistic spectrum access. Spectrum sensing should be periodically performed to recognize the operation of the PU systems and other cognitive radio systems [3].

Generally, spectrum sensing techniques are classified into two categories, namely local sensing and cooperative sensing. For local sensing, each cognitive user (CU) performs spectrum sensing based on its local observations. These techniques include matched filter detection [4], energy detection [5]–[7], and cyclostationary feature detection [8] techniques. Each has its advantages and disadvantages. Cyclostationary detection and matched filter based detection require *a priori* knowledge of the PU, e.g., modulation type, pulse shaping, and/or timing/carrier synchronization. Moreover, they both have high implementation complexity [9]. In contrast, energy detection does not require any information of the PU signal and is robust to the unknown channels. However, the detection performance of the energy detection degrades when the signal-to-noise ratio (SNR) is low.

To detect a hidden PU and reduce the sensing time, cooperation among multiple CUs are very useful [10], [11]. In [12], Quan *et al.* developed a linear cooperation framework for spectrum sensing, where the global decision is based on a linear combination of the local statistics from individual nodes. The threshold at the fusion center is jointly determined with the linear combining weights to maximize the probability of detection while satisfying a requirement on the probability of false alarm. In [13], the solutions are given in both the PU's perspective (minimize the false alarm probability for a fixed detection probability) and the CU's perspective (maximize the detection probability for a fixed false alarm probability) using AND and OR fusion rules. In [14], an optimum soft combining (OSC) scheme is developed by maximizing the detection probability for a given false alarm probability. The method is computationally involved and, moreover requires knowledge of the SNR at each CU, which needs to be estimated from local measurements.

In this paper, a CU base station collects sensing information from CUs and performs spectrum sensing. We assume that CUs can employ some positioning mechanisms to acquire their positions, e.g., by using the global positioning system (GPS) [15]. Meanwhile, the PU is a fixed base station, including a TV broadcasting station or cellular base station, etc., whose position is known to the CU base station. As a result, the relative position information of each CU to the PU is also available at the cognitive radio base station. Since most modern wireless communication systems employ power control to adaptively adjust their transmit power [16], the transmit power of the PU is unknown to the cognitive radio system. We develop a generalized likelihood ratio test (GLRT) detector based on the one snapshot measurements of the radio environment obtained by the cooperative CUs. The underlying maximum likelihood estimator (MLE) for the PU's transmit power involves rooting of a high-order polynomial. To avoid the computational burden of the MLE, a simple weighted average estimator (WAE) is introduced. We examine the properties of the estimators including the bias and variance, as well as the associated Cramer–Rao

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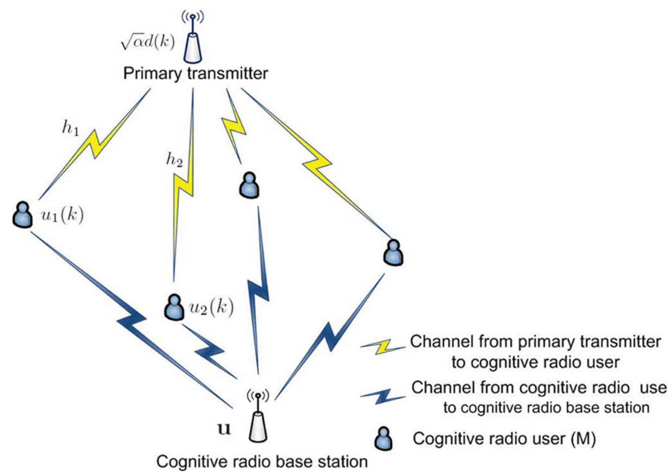

Fig. 1. System model of location-based cooperative spectrum sensing.

bound (CRB). We show that the proposed cooperative detection approach is able to achieve better detection performance than the conventional hard- and soft-decision based cooperative sensing methods.

The rest of this paper is organized as follows: The signal model of the cognitive radio system is presented in Section II. Several conventional cooperative sensing techniques are briefly discussed in Section III. Our proposed location based techniques are presented in Section IV. Numerical results are given in Section V. Conclusions are drawn in Section VI.

II. Signal Model

As shown in Fig. 1, a cognitive radio network with M CUs is employed to detect the presence of the PU. For fast detection, a decision is made using only one snapshot of the radio environment. Each CU uses K_m antennas (we may have $K_m = 1$) to collect samples, calculates the local energy, and reports to the CU base station, where a final decision is made. The binary hypothesis test is

$$H_0: \quad x_m(k) = v_m(k)$$

$$H_1: \quad x_m(k) = \sqrt{\alpha L_m} s_m(k) + v_m(k)$$

where $x_m(k)$ denotes the received sample at antenna k of the mth CU, α is the unknown transmitted signal energy, L_m is the path loss factor, and $v_m(k)$ is the complex Gaussian noise with zero mean and variance σ^2, i.e., $v_m(k) \sim \mathcal{CN}(0, \sigma^2)$. We assume the standard path loss model [17]

$$L_m = cD_m^{-r}$$

where r and c denote the path loss exponent and path loss unit constant, respectively, whereas D_m is the distance from CU m to the PU, which are all assumed to be known. The unknown PU signal $s_m(k)$ is

$$s_m(k) = h_m(k)\, d$$

where d is a sample of signal transmitted by the PU (assuming without loss of generality that d has unit energy), and $h_m(k)$ is the complex fading coefficient of the channel from the PU to the kth antenna of CU m. Following the standard Rayleigh fading model, $h_m(k) \sim \mathcal{CN}(0, 1)$, and $s_m(k)$ for different m and k are independent and identically distributed (i.i.d.) Gaussian random variables with zero mean and unit variance. The SNR at CU m is defined as

$$\rho_m = L_m \alpha / \sigma^2.$$

The PU signal energy α is, in general, unknown. However, we assume that D_m is known at the CU base station. This is the case when the PU is a high transmit power base station such as a TV broadcasting station or cellular base station whose location is known; meanwhile, CUs can acquire their positions by using a positioning device, such as the GPS [15].

Let $\mathbf{x}_m = [x_m(1), \ldots, x_m(K_m)]^T$. The local averaged signal energy for user m is

$$u_m = \frac{1}{K_m} \mathbf{x}_m^H \mathbf{x}_m. \tag{1}$$

As shown in Appendix A, u_m is a scaled central chi-square random variable under both hypotheses. As a result, the binary hypothesis test at the CU base station translates to

$$H_0: \quad u_m \sim \frac{\sigma^2}{2K_m} \chi^2_{2K_m}$$

$$H_1: \quad u_m \sim \frac{\sigma^2 + \alpha L_m}{2K_m} \chi^2_{2K_m}$$

where $\chi^2_{2K_m}$ denotes a chi-square probability density function (pdf) with $2K_m$ degrees of freedom.

III. Conventional Cooperative Spectrum Sensing

A number of cooperative solutions have been introduced without location information. These techniques involve either hard or soft combining. Hard combining schemes such as the OR- and AND-rule based methods are easy to implement [13]. Specifically, each CU makes a local decision (0 or 1) by comparing the averaged received signal energy with a threshold τ and reports this decision to the CU base station. The probability of false alarm [18] and detection at CU m are

$$P_{f,m} = \frac{\bar{\Gamma}\left(K_m, \frac{K_m \tau}{\sigma^2}\right)}{\Gamma(K_m)} \tag{2}$$

$$P_{d,m} = \frac{\bar{\Gamma}\left(K_m, \frac{K_m \tau}{L_m \alpha + \sigma^2}\right)}{\Gamma(K_m)} \tag{3}$$

where

$$\bar{\Gamma}(s, x) = \int_0^x t^{s-1} e^{-t}\, dt \tag{4}$$

$$\Gamma(s) = \int_0^\infty t^{s-1} e^{-t}\, dt \tag{5}$$

represent the upper incomplete Gamma function and the Gamma function, respectively [19]. In the OR-rule, when at least one out of M CUs detect the PU, a final decision declares that the PU is present. The detection probability is $P_d = 1 - \prod_{m=1}^{M}(1 - P_{d,m})$, and the false alarm probability is $P_f = 1 - \prod_{m=1}^{M}(1 - P_{f,m})$. In the AND-rule, a final decision declares that the PU is present only when all the M CUs detect the PU. The detection probability is $P_d = \prod_{m=1}^{M} P_{d,m}$, and the false alarm probability is $P_f = \prod_{m=1}^{M} P_{f,m}$.

A soft combing scheme, which is referred to here as the OSC, was recently introduced in [14], whereby local unquantized energy measurements are sent to the CU base station, i.e.,

$$T = \frac{1}{\sigma^2} \sum_{m=1}^{M} K_m w_m u_m.$$

The combining weights w_m are selected by maximizing the probability of detection, given a probability false alarm P_{fa}, i.e.,

$$P_d = Q\left(\frac{Q^{-1}(P_{\text{fa}})\sqrt{\sum_{m=1}^{M} w_m^2} - \sqrt{\frac{K}{2}}\sum_{m=1}^{M} w_m \rho_m}{\sqrt{\sum_{m=1}^{M} w_m^2 (1 + 2\rho_m)}}\right)$$

where $Q^{-1}(x)$ is the inverse function of $Q(y) = \int_y^\infty 1/\sqrt{2\pi} e^{-t^2/2} dt$. Finding the optimum combining weights requires multidimensional searches over the parameter space. Furthermore, the OSC requires the local SNRs ρ_m, which need to be estimated from local measurements. When the local SNRs are unknown, a simplified strategy is to use equal gain combining (EGC): $w_m = 1/\sqrt{M}$. It is known that EGC yields inferior performance, compared with the OSC [14].

IV. Proposed Method

In this section, a GLRT for cooperative spectrum sensing is developed by exploiting location information of the PU and CUs. We first outline the cooperative GLRT. Then, the underlying maximum likelihood parameter estimation is discussed along with the CRB. Moreover, a computationally efficient WAE is proposed and compared with the MLE. Finally, the test statistic of the proposed GLRT is given, and its asymptotic performance is provided as a baseline to benchmark the performance of the proposed spectrum sensing technique.

The general form of the GLRT for our problem is given by

$$T_{\text{GLR}} = \frac{\max_\alpha p_1(u_1, \ldots, u_M|\alpha)}{p_0(u_1, \ldots, u_M)} \tag{6}$$

where $p_1(u_1, \ldots, u_M|\alpha)$ and $p_0(u_1, \ldots, u_M)$ denote the joint pdfs under H_1 and H_0, respectively. Since the CUs are independent

$$p_1(u_1, \ldots, u_M|\alpha) = \prod_{m=1}^{M} p_1(u_m|\alpha) \tag{7}$$

$$p_0(u_1, \ldots, u_M) = \prod_{m=1}^{M} p_0(u_m) \tag{8}$$

where $p_1(u_m|\alpha)$ and $p_0(u_m)$ are given in (23) and (26), respectively. The GLRT requires the MLE of α of the PU, which is examined next.

A. MLE

From (7), the log-likelihood function under H_1 is

$$-\sum_{m=1}^{M} K_m \ln(\sigma^2 + \alpha L_m) - \sum_{m=1}^{M} \frac{K_m u_m}{\sigma^2 + \alpha L_m} + A$$

where A includes terms independent of α. Taking the derivative of the log-likelihood function with respect to α and equating it to zero, we have

$$\sum_{m=1}^{M} \frac{K_m L_m (u_m - \sigma^2 - \alpha L_m)}{(\sigma^2 + \alpha L_m)^2} = 0. \tag{9}$$

Note that α denotes the power of the PU and has to be nonnegative. The MLE of α is a positive solution to the aforementioned equation. In general, (9) is a high-order equation whose solutions cannot be found in closed form. However, a searching process can be employed to find the real nonnegative solution. To gain more insight of the MLE of α, we consider several cases of interest.

1) Case 1—Single CU: The MLE in this case is

$$\hat{\alpha} = \max\{0, \alpha^*\} \tag{10}$$

where

$$\alpha^* = \frac{1}{L}(u - \sigma^2). \tag{11}$$

Proof of (10): Clearly, α^* is the unique solution to

$$\frac{d \ln p_1(u|\alpha)}{d\alpha} = \frac{KL(u - \sigma^2 - \alpha L)}{(\sigma^2 + \alpha L)^2} = 0.$$

Furthermore, we have $d \ln p_1(u|\alpha)/d\alpha \leq 0$ for $\alpha \geq \alpha^*$, which implies that the log-likelihood is a nonincreasing function for $\alpha \geq \alpha^*$. Hence, (10) follows. ■

The statistical behaviors of $\hat{\alpha}$ are examined next. Let P_0 denote the probability that $\hat{\alpha} = 0$. From Appendix B, we have

$$P_0 = \frac{\bar{\gamma}\left(K, \frac{K}{1+\rho}\right)}{\Gamma(K)}.$$

As shown in Fig. 2, P_0 exponentially decreases as the SNR increases. α^* is unbiased, i.e.,

$$\mathbb{E}[\alpha^*] = \frac{(\sigma^2 + L\alpha - \sigma^2)}{L} = \alpha.$$

The variance of α^* is [see (24) and (25)]

$$\text{Var}(\alpha^*) = \mathbb{E}\left[(\alpha^* - \mathbb{E}(\alpha^*))^2\right] = \frac{\alpha^2}{K}\left(\frac{1}{\rho} + 1\right)^2 \tag{12}$$

where $\rho = \alpha L/\sigma^2$ is the SNR at the CU. Using the aforementioned results, it is shown in (27) and (28) in Appendix B that the mean and variance of the ML estimate $\hat{\alpha}$ are given by

$$\mathbb{E}(\hat{\alpha}) = \alpha(1 - P_0)$$

$$\text{Var}(\hat{\alpha}) = \frac{\alpha^2}{K}\left(\frac{1}{\rho} + 1\right)^2 + \alpha^2 P_0 \left[(1 - P_0) - \frac{1}{K}\left(\frac{1}{\rho} + 1\right)^2\right].$$

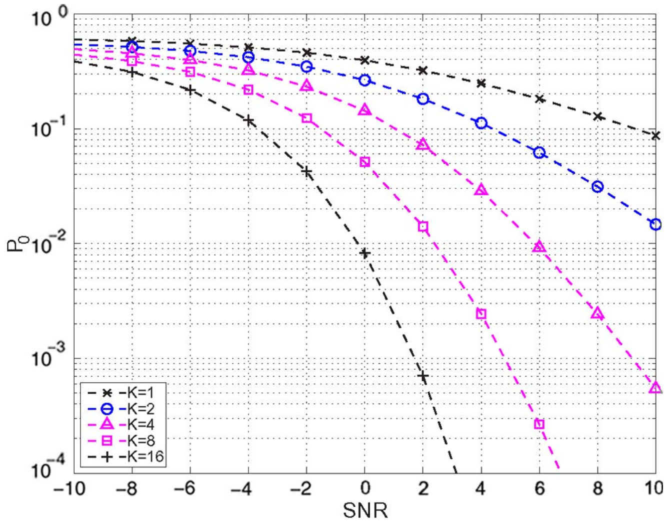

Fig. 2. P_0 versus SNR for different K.

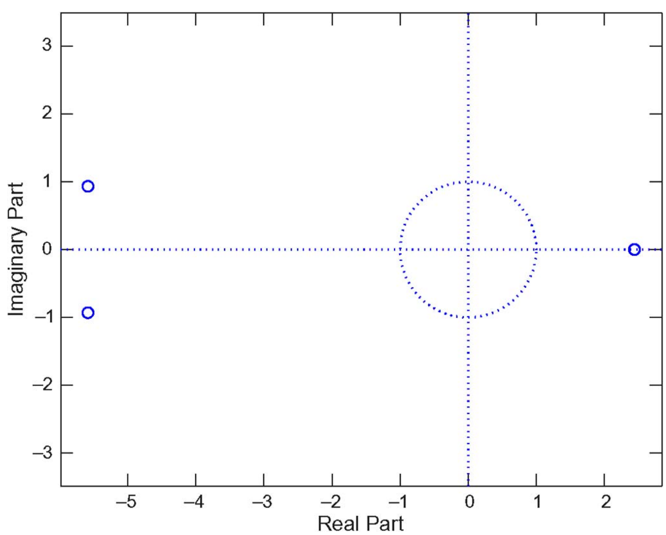

Fig. 3. Roots of the MLE cost function with two CUs at different distances to the PU.

Thus, the MLE $\hat{\alpha}$ is biased; however, the bias αP_0 vanishes as the SNR increases. The variance of $\hat{\alpha}$ is a function of SNR ρ and the number of antennas K and approaches 0 when both ρ and K are large. For finite ρ and K, particularly when ρ is small (the SU is far from the PU), the single CU estimate is inaccurate, and user cooperation is needed to reduce the estimation error.

2) Case 2—Two CUs at Different Distances to the PU: Another case of interest is that two CUs are located at different distances to the PU. The MLE $\hat{\alpha}$ is the nonnegative solution to the following third-order equation:

$$\frac{K_1 L_1(u_1-\sigma^2-\alpha L_1)}{(\sigma^2+\alpha L_1)^2}+\frac{K_2 L_2(u_2-\sigma^2-\alpha L_2)}{(\sigma^2+\alpha L_2)^2}=0.$$

As an illustration, the roots are shown in Fig. 3 when $D_1 = 150$ m, $D_2 = 180$ m, $K_1 = 2$, $K_2 = 4$, and SNR $= 0$ dB. In general, we usually have one real root and two complex conjugate roots. However, for small K_m, the real root is not guaranteed to be positive (although a negative estimate rarely happens in our simulations).

B. WAE

Since the multi-user MLE requires rooting a high-order polynomial, which may be computationally intensive, a more efficient estimator is of interest. By exploiting the single-user MLE of Section IV-A1, a simple solution is developed following the framework of the best linear unbiased estimation (BLUE). Specifically, let $\hat{\alpha}_m$ denote the single-user MLE corresponding to CU m, α can be estimated by linearly combining $\hat{\alpha}_m$

$$\tilde{\alpha}=\sum_{m=1}^{M} a_m\hat{\alpha}_m. \tag{13}$$

The BLUE combining coefficients are obtained in Appendix C, i.e.,

$$a_m^*=\frac{K_m\left(\frac{1}{\rho_m}+1\right)^{-2}}{\sum_{i=1}^{M}K_i\left(\frac{1}{\rho_i}+1\right)^{-2}}$$

which, however, depend on the unknown α and are infeasible.

In a low SNR environment where collaboration is needed, $1/\rho_m \gg 1$. Thus, the weights are approximated as

$$a_m=\frac{K_m L_m^2}{\sum_{i=1}^{M}K_i L_i^2}.$$

Using these weights in (13), we obtain our WAE of α, i.e.,

$$\tilde{\alpha}=\frac{\sum_{m=1}^{M}K_m L_m^2\hat{\alpha}_m}{\sum_{m=1}^{M}K_m L_m^2}. \tag{14}$$

Remark: Even though the WAE was derived based on a low SNR assumption, it can be applied in any SNR regime, as considered in Section V, where its performance is numerically examined. It is noted that the WAE is in closed form and is computationally much more efficient than the multi-user MLE.

The single-user MLEs $\hat{\alpha}_m$ are independent. The mean and variance of the WAE $\tilde{\alpha}$ are easy to determine, i.e.,

$$\mathbb{E}(\hat{\alpha})=\frac{\alpha\sum_{m=1}^{M}K_m\rho_m^2(1-P_{0m})}{\sum_{m=1}^{M}K_m\rho_m^2}$$

$$\mathrm{Var}(\tilde{\alpha})=\frac{\alpha^2\sum_{m=1}^{M}K_m^2\rho_m^4\left[\frac{1}{K_m}\left(1+\frac{1}{\rho_m}\right)^2+P_{0m}\right](1-P_{0m})}{\left(\sum_{m=1}^{M}K_m\rho_m^2\right)^2}.$$

Fig. 4 shows the bias and standard deviation of the WAE. The single-user case (the WAE and MLE are identical) is also shown. It is seen that the bias is more than one magnitude

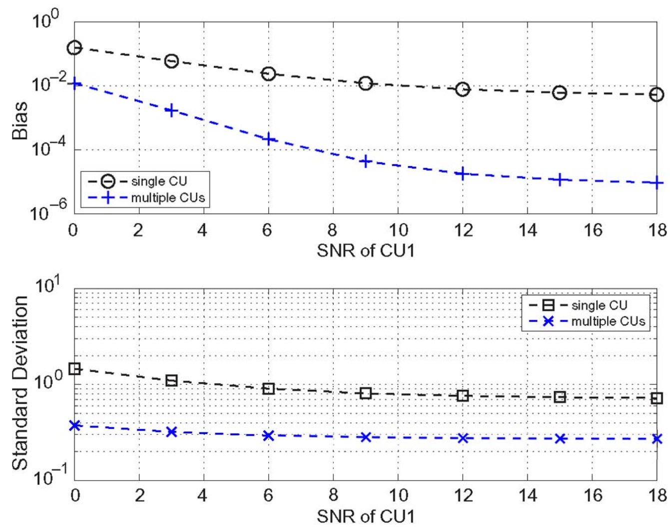

Fig. 4. Bias and standard deviation of the WAE versus the SNR of CU1 when $K = 8$ for each CU.

smaller than the standard deviation, and the WAE can be considered as unbiased.

C. CRB

The CRB specifies a lower bound on the variance of any unbiased estimator, thus offering a baseline for comparison. The CRB for α is derived in Appendix D, which is given by

$$\mathrm{Var}(\hat{\alpha}) \geq \alpha^2 \left[\sum_{m=1}^{M} \frac{K_m}{\left(\frac{1}{\rho_m} + 1\right)^2} \right]^{-1}. \qquad (15)$$

1) Case 1—Single CU: For $M = 1$, the CRB becomes

$$\mathrm{CRB}_m = \frac{\alpha^2}{K_m} \left(\frac{1}{\rho_m} + 1 \right)^2$$

which is identical to the variance of the MLE $\hat{\alpha}$.

2) Case 2—Multiple CUs: Let CRB_c denote the CRB of the estimation using M cooperative CUs. We have

$$\frac{1}{\mathrm{CRB}_c} = \sum_{m=1}^{M} \frac{1}{\mathrm{CRB}_m} \geq \frac{1}{\mathrm{CRB}_m} \qquad \forall m.$$

As a result, $\mathrm{CRB}_c \leq \mathrm{CRB}_m \ \forall m$, meaning that cooperation helps all CUs, regardless of their distances to the PU.

When the distances from the CUs to the PU are identical, the CRB reduces to

$$\mathrm{CRB}_c = \frac{\alpha^2}{\sum_{m=1}^{M} K_m} \left(\frac{1}{\rho} + 1 \right)^2$$

which can be shown to be identical to the MSE of the MLE for the equal-distance case.

D. GLRT Test Statistic and Asymptotic Analysis

By using the MLE or WAE of α in (7), it is easy to show that the GLRT (6) can be expressed as

$$T_{\log-\mathrm{GLR}} = \sum_{m=1}^{M} \frac{K_m \hat{\alpha} L_m u_m}{\sigma^2(\sigma^2 + \hat{\alpha} L_m)} + \sum_{m=1}^{M} K_m \ln \frac{\sigma^2}{\sigma^2 + \hat{\alpha} L_m}. \qquad (16)$$

An exact analysis of the GLRT test variable (16) is too involved due to the nonlinear nature. We provide an asymptotic expression of the test statistic here. As shown in Appendix E, the asymptotic distribution of the GLRT statistic in (16) is given by

$$T_{\log-\mathrm{GLR}} \overset{a}{\sim} \begin{cases} \chi_1^2, & \text{under } H_0 \\ \chi_1'^2(\lambda), & \text{under } H_1 \end{cases} \qquad (17)$$

where $\chi_1'^2$ denotes the noncentral chi-squared distribution with 1 degree of freedom and noncentrality parameter λ given by

$$\lambda = \sum_{m=1}^{M} K_m \rho_m^2.$$

As a result, we can write the asymptotic detection and false alarm probabilities as

$$P_d = Q_{0.5}(\sqrt{\lambda}, \sqrt{\tau_{\mathrm{GLR}}}) \qquad (18)$$

$$P_f = \frac{\bar{\gamma}\,(0.5, \tau_{\mathrm{GLR}})}{\Gamma(0.5)} \qquad (19)$$

where $Q_m(a, b)$ is the generalized Marcum-Q function, and $\bar{\gamma}(s, x)$ is the lower incomplete Gamma function [19].

V. Numerical Examples

In this section, simulation results are presented to illustrate the performance of the proposed cooperative spectrum sensing techniques and to compare them with conventional cooperative methods. In our simulation, the path loss exponent c is set to 2, and the PU transmits independent binary phase-shift keying (BPSK) signals with $\alpha = 2$.

A. Estimation Performance

The MLE and WAE, along with the CRB (15), are examined in Figs. 5 and 6 for both the single-user and multiple-CU cases. Fig. 5 considers a case when the CUs have different distances to the PU. Specifically, CU1 has $K_1 = 4$ antennas and is $D_1 = 100$ m from the PU, and for CU2, $K_2 = 2$ $D_2 = 150$ m. As a result, the SNR of CU1 is approximately 6 dB lower than the SNR of CU2. The horizontal axis shows the SNR for CU1. For comparison, we include the single-user MLE for CU1 or CU2, along with the cooperative MLE and WAE schemes by using both CU1 and CU2 observations. It is seen that, by cooperating, the estimation accuracy is improved by using either MLE or WAE. Although the improvement for CU1 is limited, it is important to note that cooperation is not jeopardizing the more advantageous user. Furthermore, we note that the cooperative

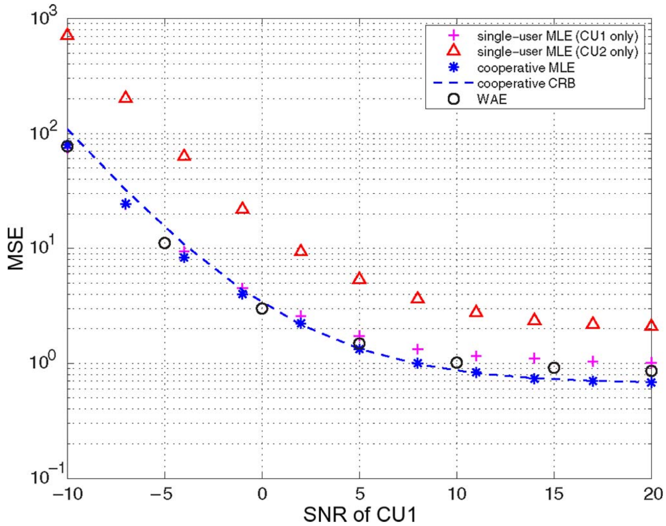

Fig. 5. MSE for a single-user noncooperative and proposed two-user cooperative estimators with $K_1 = 4$ and $K_2 = 2$.

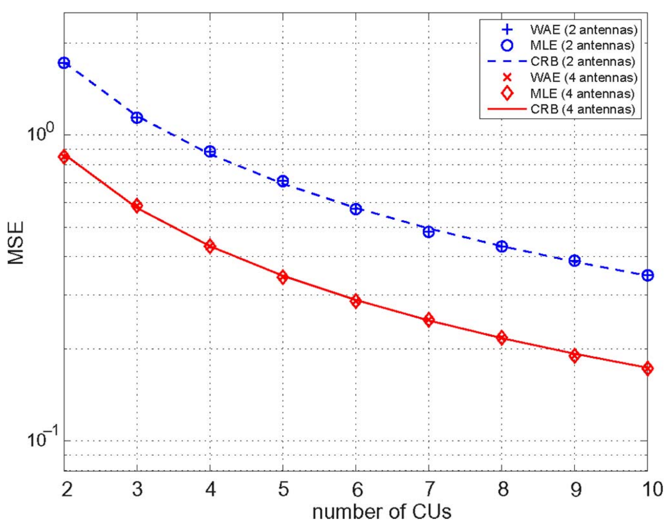

Fig. 6. MSE with SNR = 5 dB and identical distances D_m from the CUs to the PU.

MLE is close to the CRB and that the performance of the WAE is slightly worse. As shown in Fig. 6, the MLE becomes more accurate when either the number of cooperative CUs or the number of antennas increases.

B. Comparison With Other Cooperative Detectors

In this section, the proposed GLRT with the MLE and WAE are compared with the hard decision (OR-rule and AND-rule) [13] and soft decision OSC [14] based cooperative detectors. The OSC detector requires the local SNR. We consider two cases when the SNR is exactly known and estimated from the received signal, respectively.

Figure 7 shows the detection performance in the case of 8 CUs with different distances to the PU. Each CU employs 1 antenna. The SNR for the CU located at 100 m is used as the reference and shown as the horizontal axis. We see that the AND-rule detector is the worst, whereas the GLRT provides the best detection performance. The GLRT and the OSC detector with exact SNR are almost identical and better than that of the OSC with estimated SNR. It should be note that the proposed WAE-GLRT as given by (14) and (16) is in closed form, whereas the OSC detector requires a multidimensional search to find the optimum combining weights and is significantly more involved. Fig. 7 also includes the performance of a single-user detector using energy detection for a reference CU located at 100 m, which is considerably worse than everything, except for the AND-rule cooperative detectors.

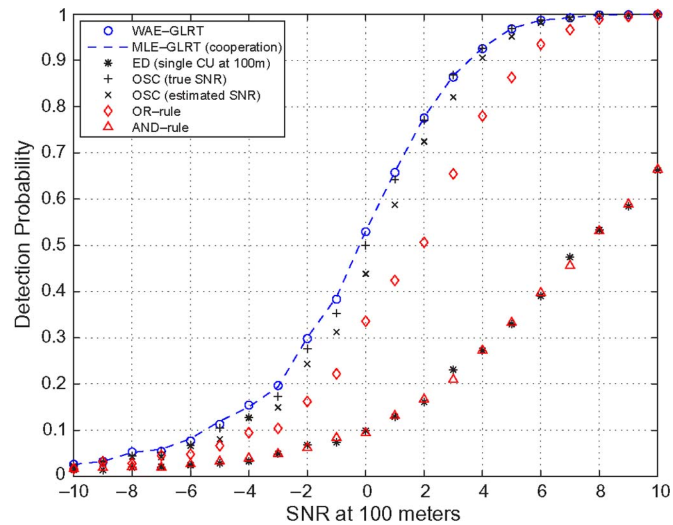

Fig. 7. Detection probability with $P_{\mathrm{fa}} = 0.01$ when $K_m = 1\ \forall m$ and the distances D_m between the CUs and the PU are [100, 80, 110, 70, 115, 120, 100, 105] m.

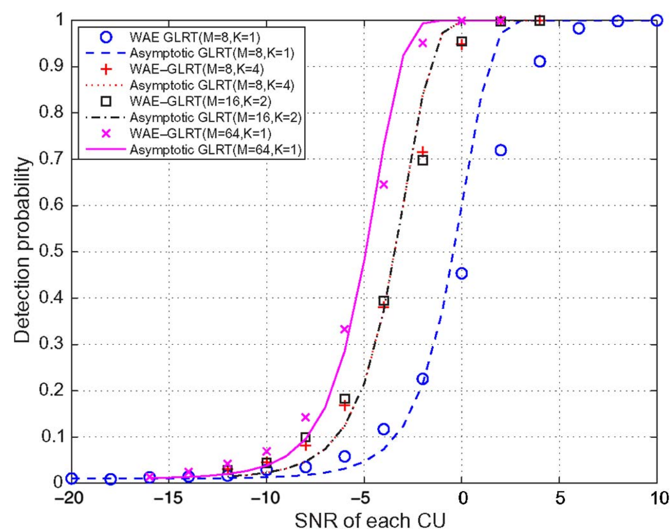

Fig. 8. Comparison with the asymptotic detection probability with $P_{\mathrm{fa}} = 0.01$ and $D_m = 100$ m.

Fig. 8 further shows the detection performance of the proposed detectors when the CU number is large. It is noted that, as the number of CUs increases, the proposed detector approaches the asymptotic performance given in (15).

C. Effect of Diverse Path-Loss Exponents

For simplicity, we assumed that the path loss exponents are identical. The proposed detector can deal with the case when

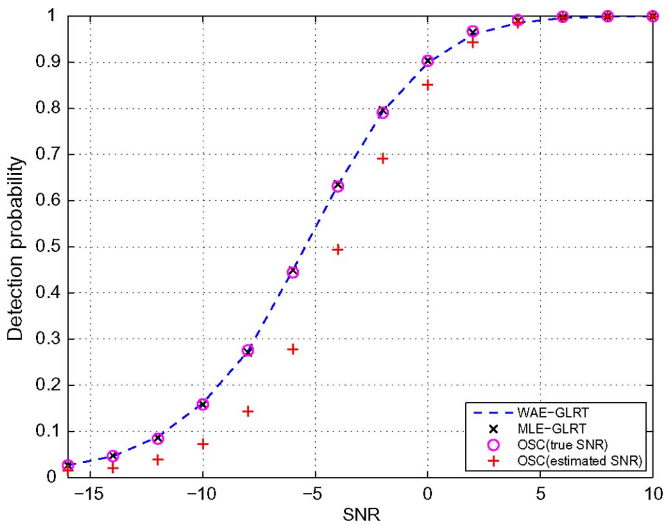

Fig. 9. Detection performance with eight users and diverse path loss exponents: $c = [2, 2.5, 1.5, 2.5, 2, 2.5, 3, 2.5]$, $K_m = 2$, and $D_m = 100$ m $\forall m$.

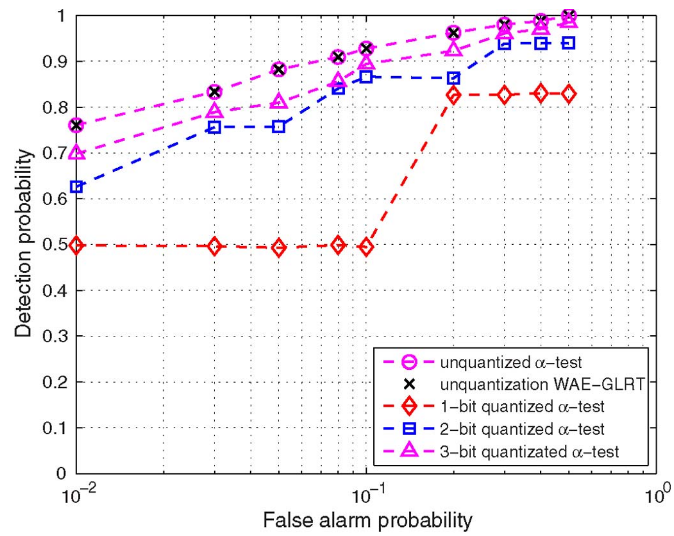

Fig. 10. Effect of quantization when SNR $= 0$ dB, $K_m = 2$, and $D_m = 100$ m $\forall m$. The α-test is a modified version of the GLRT to facilitate quantized transmission.

the path loss exponent of the link from the PU to each CU is different, as shown in Fig. 9. It is seen that the WAE-GLRT outperforms the OSC with estimated SNR and also achieves similar detection performance as that of the MLE-GLRT and the OSC with known SNR, whereas the OCS with estimated SNR experiences noticeable degradation.

D. Effect of Quantization

The GLRT assumes that local energy measurements $\{u_m\}$ are sent to the CU base station in analog format (without quantization). Here, we consider the case when the local transmission is quantized via a uniform quantizer. Since $\{u_m\}$ have different dynamic ranges (because of the difference in locations), it is not appropriate to use an identical quantizer for all $\{u_m\}$. Instead, each local CU can quantize the local estimate of the power α of the PU, which is identical to all CU, and send the quantized estimate.

Specifically, first, each CU calculates u_m by (1). Then, it forms a local estimate $\hat{\alpha}_m$ by (10), quantizes $\hat{\alpha}_m$ to several bits based on a given bandwidth constraint, and forwards the quantized version, denoted by $\bar{\hat{\alpha}}_m$, to the CU base station. The cognitive radio base station uses the WAE to form an estimate of α, i.e.,

$$\bar{\hat{\alpha}} = \frac{\sum_{m=1}^{M} K_m L_m^2 \bar{\hat{\alpha}}_m}{\sum_{m=1}^{M} K_m L_m^2}. \tag{20}$$

Finally, $\bar{\hat{\alpha}}$ is compared with a threshold corresponding to a given probability of false alarm. The aforementioned detector, which is a modified version of the GLRT to facilitate quantized transmission, is referred to as the α-test.

Figure 10 compares the GLRT and α-test with and without quantization. When quantization is applied, we consider the case when $\bar{\hat{\alpha}}_m$ are obtained with 1-, 2-, and 3-bit quantizer, respectively. The quantization thresholds are uniformly spaced across a dynamic range [0,8] $(\alpha = 2)$. We see from Fig. 10 that, without quantization, the GLRT and α-test are identical. As expected, 1-bit quantization is not adequate to recover a good estimate of α. However, even with 2- or 3-bit quantization, the performance loss incurred by quantization is relatively small.

E. Effect of Noisy Report Channel

So far, the results are based on the assumption that the transmission of the local energy measurements to the CU base station is lossless. Our final result considers the effect when the report channel is lossy. In particular, the energy measurement u_m submitted by each CU is subject to fading. The signal received at the CU base station is

$$z_m = h_m u_m + w_m, \qquad m = 1, \ldots, M \tag{21}$$

where h_m is the fading coefficient of the channel between CU m and the CU base station, and w_m is the noise with zero mean and variance σ_1^2. We assume that the CU base station knows the fading channel coefficients with sufficient accuracy via channel estimation. Error in channel estimation is not considered since it can be absolved in the channel noise w_n. For simplicity, we use a simple estimate of u_m by channel inversion, i.e.,

$$\hat{u}_m = \left| \frac{z_m}{h_m} \right|. \tag{22}$$

The estimated energy from each CU are used to calculate the test variable of the detector. Fig. 11 shows the effect of the report channel, where the average SNR is 10 dB and the channels are changing independently from one trial to another. It is observed that, compared with the case when the report channel is lossless, both our proposed GLRT and the OSC detector suffer some degradation caused by the noisy report channel. It is also noted that our GLRT still outperforms the OSC detector with lossy report channel.

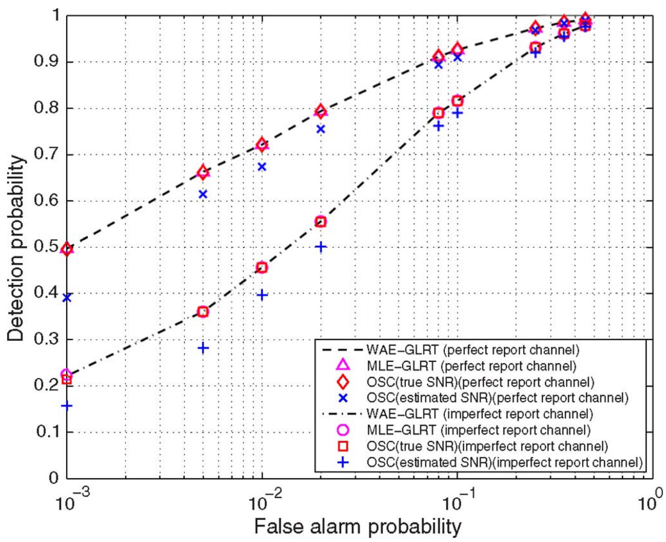

Fig. 11. Detection performance with noisy reporting channels when reporting channel SNR = 10 dB, observation SNR $\rho = 0$ dB, $K_m = 2$, and $D_m = 100$ m $\forall m$.

VI. Conclusion

A GLRT for cooperative spectrum sensing among multiple CUs has been proposed by utilizing the location information. The underlying estimation problem of the GLRT has been studied. We have first derived the MLE, which involves high-order polynomial rooting. To simplify implementation, a WAE has also been introduced. The estimation performance of both estimators has been analyzed. Moreover, the asymptotic performance of the proposed GLRT detector has been provided. Simulation results have verified that the proposed GLRT with either estimators yields competitive performance, compared to several conventional hard and soft decision based cooperative spectrum sensing. The WAE-GLRT is particularly appealing due to its computational efficiency, with all calculations involved in estimation and detection in closed form.

Appendix A
Statistics of u_m Under H_1 and H_0

The first and second moments of u_m are calculated here. Under H_1, u_m, as defined in (1), can be expressed as

$$u_m = \frac{\sigma^2 + L_m\alpha}{2K_m}\sum_{k=1}^{K_m}\left|\frac{\sqrt{2}x_m(k)}{\sqrt{\sigma^2+L_m\alpha}}\right|^2 = \frac{\sigma^2+L_m\alpha}{2K_m}v'_m$$

where v'_m is the summation of $2K_m$ normal random variables with zero mean and unit variance and, thus, follows a central chi-square distribution with $2K_m$ degrees of freedom. As a result, the pdf of u_m under H_1 can be expressed as

$$p_1(u_m|\alpha) = \frac{1}{2^{K_m}\Gamma(K_m)}u_m^{K_m-1}\left(\frac{2K_m}{\sigma^2+L_m\alpha}\right)^{K_m} \times \exp\left(-\frac{K_m u_m}{\sigma^2+L_m\alpha}\right). \quad (23)$$

The mean of u_m under H_1 is (note that $\mathbb{E}[v'_m] = 2K_m$)

$$\mathbb{E}[u_m] = \sigma^2 + L_m\alpha. \quad (24)$$

The second moment is (note that $\mathrm{Var}(v'_m) = 4K_m$)

$$\mathbb{E}\left[u_m^2\right] = \mathrm{Var}(u_m) + \mathbb{E}^2[u_m] = \frac{K_m+1}{K_m}(\sigma^2+L_m\alpha)^2. \quad (25)$$

Similarly, the energy under H_0 is

$$u_m = \frac{\sigma^2}{2K_m}\sum_{k=1}^{K_m}\left|\frac{\sqrt{2}w_m(k)}{\sigma}\right|^2 = \frac{\sigma^2}{2K_m}v'_m.$$

Its pdf is given by

$$p_0(u_m) = \frac{1}{2^{K_m}\Gamma(K_m)}u_m^{K_m-1}\left(\frac{2K_m}{\sigma^2}\right)^{K_m}\exp\left(-\frac{K_m u_m}{\sigma^2}\right). \quad (26)$$

Appendix B
Mean and Variance of the Maximum Likelihood Estimate of α

The ML estimate of α takes the following form:

$$\hat{\alpha} = \max\{0, \alpha^*\}$$

where α^* is given by (11). Let P_0 be the probability that $\hat{\alpha} = 0$, and the probability that $\hat{\alpha} = \alpha^*$ is $1 - P_0$. Then

$$\mathbb{E}(\hat{\alpha}) = 0 \cdot P_0 + (1-P_0)\mathbb{E}[\alpha^*] = (1-P_0)\mathbb{E}[\alpha^*] \quad (27)$$

and its variance is

$$\begin{aligned}\mathrm{Var}(\hat{\alpha}) &= \mathbb{E}\left[(0-\mathbb{E}(\hat{\alpha}))^2\right]P_0 + \mathbb{E}\left[(\alpha^*-\mathbb{E}(\hat{\alpha}))^2\right](1-P_0)\\ &= \left[\mathrm{Var}(\alpha^*) + P_0\mathbb{E}^2[\alpha^*]\right](1-P_0). \quad (28)\end{aligned}$$

The ML estimate $\hat{\alpha}$ is 0 when α^* is less than 0. From (11)

$$P_0 = Pr\left(\frac{u-\sigma^2}{L} < 0\right) = \int_0^{\sigma^2} P(u)\,du = \frac{\bar{\gamma}\left(K, \frac{K}{1+\rho}\right)}{\Gamma(K)}$$

where $\Gamma(.)$ is the Gamma function (5), and $\bar{\gamma}(.,.)$ is the lower incomplete Gamma function: $\bar{\gamma}(s,x) = \int_0^x t^{s-1}e^{-t}dt$.

Appendix C
Best Linear Unbiased Estimate of α

The BLUE of α in (13) can be expressed as

$$\tilde{\alpha} = \mathbf{a}^T\hat{\boldsymbol{\alpha}}$$

where $\mathbf{a} = [a_1, \ldots, a_m]^T$ is the weight vector, and $\hat{\boldsymbol{\alpha}} = [\hat{\alpha}_1, \ldots, \hat{\alpha}_m]^T$ is a vector containing the single-user ML estimates. Since $\hat{\alpha}_m$ are independent, the mean of $\tilde{\alpha}$ is

$$\mathbb{E}[\tilde{\alpha}] = \mathbf{a}^T\mathbb{E}[\hat{\boldsymbol{\alpha}}] = \mathbf{a}^T\mathbf{p}\alpha$$

where $\mathbf{p}$ is a column vector with the mth element given by $p_m = 1 - P_{0m}$. To ensure that $\tilde{\alpha}$ is unbiased, we need

$$\mathbf{a}^T\mathbf{p} = 1.$$

Moreover, the variance of $\tilde{\alpha}$ is given by

$$\mathrm{Var}(\tilde{\alpha}) = \mathbf{a}^T\mathbf{C}_{\hat{\boldsymbol{\alpha}}}\mathbf{a}$$

where $\mathbf{C}_{\hat{\boldsymbol{\alpha}}}$ is the covariance matrix of $\hat{\boldsymbol{\alpha}}$, which is diagonal, with the mth diagonal element given by [see (12)]

$$C_m = \left[\frac{\alpha^2}{K_m}\left(\frac{1}{\rho_m}+1\right)^2 + \alpha^2 P_{0m}\right](1-P_{0m}).$$

The BLUE of α is obtained by solving

$$\min_{\mathbf{a}} \ \mathbf{a}^T\mathbf{C}_{\hat{\boldsymbol{\alpha}}}\mathbf{a} \qquad \text{s.t.} \quad \mathbf{a}^T\mathbf{p} = 1$$

and the solution is

$$\mathbf{a}^* = \frac{\mathbf{C}_{\hat{\boldsymbol{\alpha}}}^{-1}\mathbf{p}}{\mathbf{p}^T\mathbf{C}_{\hat{\boldsymbol{\alpha}}}^{-1}\mathbf{p}}$$

where the mth element of $\mathbf{a}^*$ is given by

$$a_m^* = \frac{p_m C_m^{-1}}{\sum_{i=1}^{M} p_i^2 C_i^{-1}} = \frac{\left[\frac{1}{K_m}\left(\frac{1}{\rho_m}+1\right)^2 + P_{0m}\right]^{-1}}{\sum_{i=1}^{M}\left[\frac{1}{K_i}\left(\frac{1}{\rho_i}+1\right)^2 + P_{0i}\right]^{-1}(1-P_{0i})}.$$

Assuming that P_{0m} is negligible, the weights are approximated as

$$a_m^* \approx \frac{K_m\left(\frac{1}{\rho_m}+1\right)^{-2}}{\sum_{i=1}^{M} K_i\left(\frac{1}{\rho_i}+1\right)^{-2}}. \tag{29}$$

Appendix D
Cramer–Rao Bound of α

The second derivative of the log pdf is

$$\frac{\partial^2 \ln p_1(u_1,\ldots,u_M|\alpha)}{\partial\alpha^2} = \sum_{m=1}^{M}\frac{K_m L_m^2(\sigma^2+\alpha L_m - 2u_m)}{(\sigma^2+\alpha L_m)^3}.$$

The Fisher information (FI) is [cf. (24)]

$$I(\alpha) = -\mathbb{E}\left[\frac{\partial^2 \ln p_1(u_1,\ldots,u_M|\alpha)}{\partial\alpha^2}\right] = \sum_{m=1}^{M}\frac{K_m L_m^2}{(L_m\alpha+\sigma^2)^2}.$$

Thus, the CRB can be expressed as

$$\mathrm{var}(\hat{\alpha}) \geq \frac{1}{I(\alpha)} = \alpha^2\left[\sum_{m=1}^{M}\frac{K_m}{\left(\frac{1}{\rho_m}+1\right)^2}\right]^{-1}. \tag{30}$$

Appendix E
Asymptotic Performance of the Generalized Likelihood Ratio Test

The only unknown parameter is α under H_1. Therefore, the detection problem is a parametric testing problem, i.e.,

$$\begin{aligned} H_0: &\quad \theta_r = \theta_{r_0} \\ H_1: &\quad \theta_r = \theta_{r_1} \end{aligned}$$

where $\theta_{r_1} = \alpha$, and $\theta_{r_0} = 0$. From the aforementioned formulation, the asymptotic distribution of the GLRT statistic is [20]

$$T_{\mathrm{GLRT}} \overset{a}{\sim} \begin{cases} \chi_1^2, & \text{under } H_0 \\ \chi_1'^2(\lambda), & \text{under } H_1 \end{cases}$$

where the noncentrality parameter λ is given by

$$\lambda = (\theta_{r_1} - \theta_{r_0})^2 I(\theta_{r_0})$$

with $I(\theta)$ denoting the FI, with $\theta = \alpha = 0$. Thus

$$\lambda = \sum_{m=1}^{M}\frac{K_m L_m^2\alpha^2}{\sigma^4} = \sum_{m=1}^{M} K_m\rho_m^2.$$

References

[1] J. Mitola, III, "Cognitive radio for flexible mobile multimedia communications," in *Proc. IEEE Mobile Multimedia Commun. Conf.*, New York, Nov. 1999, pp. 3–10.

[2] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 2, pp. 201–220, Feb. 2005.

[3] S. Haykin, D. J. Thomson, and J. H. Reed, "Spectrum sensing for cognitive radio," *Proc. IEEE*, vol. 97, no. 5, pp. 849–877, May 2009.

[4] A. Sahai and D. Cabric, "Spectrum sensing: fundamental limits and practical challenges," in *Proc. IEEE Int. Symp. New Frontiers DySPAN*, Baltimore, MD, Nov. 2005.

[5] H. Urkowitz, "Energy detection of unknown deterministic signals," *Proc. IEEE*, vol. 55, no. 4, pp. 523–531, Apr. 1967.

[6] V. I. Kostylev, "Energy detection of a signal with random amplitude," in *Proc. IEEE ICC*, Apr. 2002, vol. 3, pp. 1606–1610.

[7] F. F. Digham, M. S. Alouini, and M. K. Simon, "On the energy detection of unknown signals over fading channels," *IEEE Trans. Commun.*, vol. 55, no. 1, pp. 21–24, Jan. 2007.

[8] P. D. Sutton, K. E. Nolan, and L. E. Doyle, "Cyclostationary signatures in practical cognitive radio applications," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 1, pp. 13–24, Jan. 2008.

[9] D. Cabric, S. M. Mishra, and R. W. Brodersen, "Implementation issues in spectrum sensing for cognitive radios," in *Proc. 38th Asilomar Conf. Syst. Comput.*, Pacific Grove, CA, Nov. 7–11, 2004, pp. 772–776.

[10] T. Yucek and H. Arslan, "A survey of spectrum sensing algorithms for cognitive radio applications," *IEEE Commun. Surveys Tuts.*, vol. 11, no. 1, pp. 116–130, First Quarter, 2009.

[11] I. F. Akyildiz, W. Y. Lee, M. C. Vuran, and S. Mohanty, "Next generation/dynamic spectrum access/cognitive radio wireless networks: A survey," *Comput. Netw. J.*, vol. 50, no. 13, pp. 2127–2159, Sep. 2006.

[12] Z. Quan, S. Cui, and A. H. Sayed, "Optimal linear cooperation for spectrum sensing in cognitive radio networks," *IEEE J. Sel. Areas Commun.*, vol. 2, no. 1, pp. 28–40, Feb. 2008.

[13] A. Ghasemi and E. S. Sousa, "Collaborative spectrum sensing for opportunistic access in fading environments," in *Proc. IEEE Int. Symp. New Frontiers Dyn. Spectrum Access Netw.*, Nov. 2005, pp. 131–136.

[14] J. Ma and Y. Li, "Soft combination and detection for cooperative spectrum sensing in cognitive radio networks," in *Proc. IEEE GLOBECOM*, Washington, DC, Nov. 26–30, 2007, pp. 3139–3143.

[15] L. C. Wang and A. Chen, "Effects of location awareness on concurrent transmissions for cognitive ad hoc networks overlaying infrastructure-based systems," *IEEE Trans. Mobile Comput.*, vol. 8, no. 5, pp. 577–589, May 2009.

[16] A. J. Viterbi, *CDMA: Principles of Spread Spectrum Communication*. Reading, MA: Addison-Wesley, 1995.

[17] A. Goldsmith, *Wireless Communications*. Cambridge, U.K.: Cambridge Univ. Press, 2006.

[18] P. Wang, J. Fang, N. Han, and H. Li, "Multi-antenna assisted spectrum sensing for cognitive radio," *IEEE Trans. Veh. Technol.*, vol. 59, no. 4, pp. 1791–1800, May 2010, Special Issue Achievements and the Road Ahead: The First Decade of Cognitive Radio.

[19] I. S. Gradshteyn, I. M. Ryzhik, and A. Jeffrey, *Table of Integrals, Series, and Products*. San Diego, CA: Academic, 1994.

[20] S. M. Kay, *Fundamentals of Statistical Signal Processing: Detection Theory*. Upper Saddle River, NJ: Prentice-Hall, 1998.

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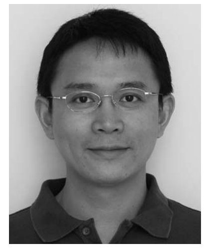

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