Differential Space-Time-Frequency Modulation Over Frequency-Selective Fading Channels

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Abstract—**We present herein a differential space-time-frequency (DSTF) modulation scheme for systems with two transmit antennas over frequency-selective fading channels. The proposed DSTF scheme employs a concatenation of a spectral encoder and a differential encoder/mapper, which are designed to yield the maximum spatio-spectral diversity and significant coding gain. To reduce the decoding complexity, the differential encoder is designed with a unitary structure that decouples the maximum likelihood (ML) detection in space and time; meanwhile, the spectral encoder utilizes a *linear constellation decimation (LCD)* coding scheme that encodes across a minimally required set of subchannels for full diversity and, hence, incurs the least decoding complexity among all full-diversity codes.**

Index Terms—**Differential modulation, frequency-selective fading, linear constellation decimation (LCD) codes, maximum spatio-spectral diversity, space-time coding.**

I. Introduction

DIFFERENTIAL space-time coding (DSTC), which circumvents the challenging task of multi-channel estimation in time-varying channels, has generated significant interest recently [1]–[3]. Current DSTC schemes are designed primarily for flat-fading channels. One possible wideband extension is to use DSTC with orthogonal frequency-division multiplexing (OFDM) on each subcarrier across the transmit antennas (e.g., [4]). Such an extension, however, does not exploit additional degrees of freedom offered by multipath propagation in wideband systems. It achieves only spatial diversity.

We present herein a novel *differential space-time-frequency (DSTF)* modulation scheme for systems with two transmit antennas in frequency-selective channels. The DSTF scheme employs a concatenation of a spectral encoder and a differential encoder that are designed to maximize the spatio-spectral diversity and coding gain. Our differential encoder can be thought of as a block extension of the scalar DSTC scheme in [1]; in particular, it reduces to the latter when the symbol block size (i.e., P defined in Section II) is one. The differential encoder provides full spatial diversity if working alone. To achieve full spectral diversity as well, we introduce a class of *linear constellation decimation (LCD)* codes that encode across a minimally necessary number of subchannels and, thus, incur the least decoding complexity among all full-diversity codes.

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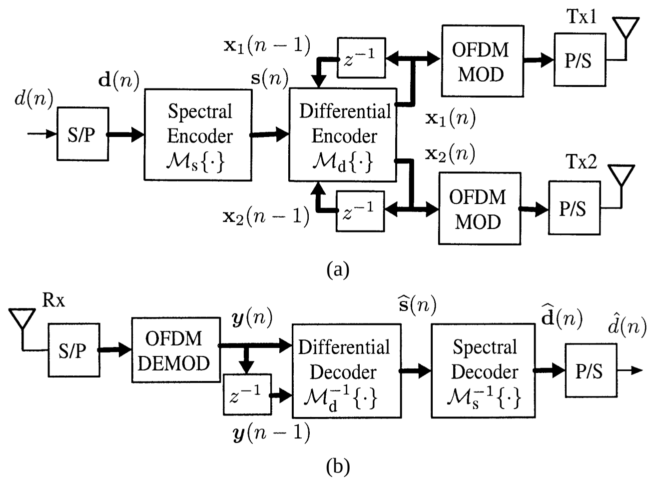

Fig. 1. A baseband DSTF system with two Tx's and one Rx. (a) Transmitter. (b) Receiver.

Notation: Vectors (matrices) are denoted by boldface lower (upper) case letters; superscripts $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^H$ denote the transpose, conjugate, and conjugate transpose, respectively; $\mathbf{I}_M$ is the $M \times M$ identity matrix; $\mathbf{0}$ (respectively, $\mathbf{1}$) is a vector with all zero (resp., one) elements; $\otimes$ denotes the Kronecker product; finally, $\mathrm{diag}\{\cdot\}$ denotes a diagonal matrix.

II. System Description

Fig. 1 depicts a baseband DSTF system with $N_t = 2$ transmit antennas (Tx) and $N_r = 1$ receive antenna (Rx). For space limitation, the extension to $N_t > 2$ will be considered elsewhere. At the transmitter, the information stream is serial-to-parallel (S/P) converted to $P \times 1$ vectors $\mathbf{d}(n)$, which are next spectrally encoded by $\mathcal{M}_s\{\cdot\}$ to form $P \times 1$ code vectors $\mathbf{s}(n)$. The coded symbols are, in general, drawn from a constellation of a larger size than that of the information symbols (cf. Section IV). Two adjacent coded vectors are differentially encoded by $\mathcal{M}_d\{\cdot\}$, which outputs a $2P \times 2$ DSTF code matrix: $\boldsymbol{\mathcal{X}}(n) \triangleq \begin{bmatrix} \mathbf{x}_1(2n-1) & \mathbf{x}_1(2n) \\ \mathbf{x}_2(2n-1) & \mathbf{x}_2(2n) \end{bmatrix}$, where $\mathbf{x}_i(t) \triangleq [x_i(t;0), \ldots, x_i(t;P-1)]^T$, $i = 1, 2$ and $t = 2n-1$, $2n$. Next, the $P \times 1$ vector $\mathbf{x}_i(t)$ is OFDM modulated on P subcarriers, parallel-to-serial (P/S) converted, and transmitted from $\mathrm{Tx}i$ during the tth OFDM symbol interval. At the receiver, the received data is S/P converted and OFDM demodulated to output $\mathbf{y}(n) \triangleq [y(n;0), \ldots, y(n;P-1)]^T$, where $y(n;p)$ denotes the sample corresponding to the pth subcarrier of the nth OFDM symbol. The differential decoder $\mathcal{M}_d^{-1}\{\cdot\}$ performs differential decoding, and finally, $\mathcal{M}_s^{-1}\{\cdot\}$ performs spectral decoding. The channel between $\mathrm{Tx}i$ and the Rx is modeled as an FIR filter with coefficients $\{h_i(l)\}_{l=0}^{L}$, where L denotes the channel order. The frequency response at the pth subchannel is $H_i(p) \triangleq \sum_{l=0}^{L} h_i(l)\exp(-j2\pi lp/P)$. Furthermore, we have

$$y(n;p) = \sum_{i=1}^{2} H_i(p)x_i(n;p) + w(n;p) \tag{1}$$

where $w(n;p)$ denotes the zero-mean complex white Gaussian noise with variance $N_0/2$ per dimension.

The problem of interest is to design $\mathcal{M}_d\{\cdot\}$ and $\mathcal{M}_s\{\cdot\}$ for wideband differential transmission that yields the maximum spatio-spectral diversity gain as well as significant coding gain.

III. Differential Encoding

We transmit the first DSTF code matrix as: $\boldsymbol{\mathcal{X}}(0) = \sqrt{E_s}\mathbf{I}_2 \otimes \mathbf{1}_{P\times 1}$. For subsequent transmission, we encode as follows:

$$\boldsymbol{\mathcal{X}}(n) = \boldsymbol{\mathcal{D}}_x(n-1)\boldsymbol{\mathcal{S}}(n), \quad n = 1, 2, \ldots \tag{2}$$

where $\boldsymbol{\mathcal{D}}_x(n-1) \triangleq \begin{bmatrix} \mathbf{D}_{x_1}(2(n-1)-1) & \mathbf{D}_{x_1}(2(n-1)) \\ \mathbf{D}_{x_2}(2(n-1)-1) & \mathbf{D}_{x_2}(2(n-1)) \end{bmatrix}$ and $\boldsymbol{\mathcal{S}}(n) \triangleq (1/\sqrt{2})\begin{bmatrix} \mathbf{s}(2n-1) & -\mathbf{s}^*(2n) \\ \mathbf{s}(2n) & \mathbf{s}^*(2n-1) \end{bmatrix}$, with $\mathbf{D}_{x_i}(t) \triangleq \mathrm{diag}\{\mathbf{x}_i(t)\}$. Assuming that $\mathbf{s}(t)$ are drawn from a constant-modulus, unit-energy constellation $\mathcal{A}_s$ (e.g., PSK), it can be readily verified that $\boldsymbol{\mathcal{D}}_x(n)$, similarly defined as $\boldsymbol{\mathcal{D}}_x(n-1)$, is unitary: $\boldsymbol{\mathcal{D}}_x(n)\boldsymbol{\mathcal{D}}_x^H(n) = E_s\mathbf{I}_{2P}$. Rewrite (1) in vector/matrix form: $\mathbf{y}(t) = \sum_{i=1}^{2}\mathbf{H}_i\mathbf{x}_i(t) + \mathbf{w}(t)$, $t = 2n-1, 2n$, where $\mathbf{H}_i \triangleq \mathrm{diag}\{H_i(0), \ldots, H_i(P-1)\}$ and $\mathbf{w}(t)$ denotes the $P \times 1$ noise vector. Let $\boldsymbol{y}(n) \triangleq [\mathbf{y}^T(2n-1), \mathbf{y}^H(2n)]^T$, $\boldsymbol{s}(n) \triangleq [\mathbf{s}^T(2n-1), \mathbf{s}^T(2n)]^T$, and $\boldsymbol{\mathcal{D}}_y(n-1) \triangleq \begin{bmatrix} \mathbf{D}_y(2(n-1)-1) & \mathbf{D}_y(2(n-1)) \\ \mathbf{D}_y^*(2(n-1)) & -\mathbf{D}_y^*(2(n-1)-1) \end{bmatrix}$, where $\mathbf{D}_y(t) \triangleq \mathrm{diag}\{\mathbf{y}(t)\}$. Using (2), we can readily show that

$$\boldsymbol{y}(n) = 2^{-1/2}\boldsymbol{\mathcal{D}}_y(n-1)\boldsymbol{s}(n) + \boldsymbol{v}(n) \tag{3}$$

where $\boldsymbol{v}(n)$ are $2P \times 1$ vectors formed by independent Gaussian entries with zero-mean and variance N_0 per dimension. Equation (3) is the *fundamental differential receiver equation*.

Due to the unitary structure of the DSTF codes, the maximum likelihood (ML) detection of the space-time multiplexed code vectors $\mathbf{s}(2n-1)$ and $\mathbf{s}(2n)$ is decoupled. To see this, let $\boldsymbol{\Omega}_y(n-1) \triangleq \sum_{t=2n-3}^{2n-2}\mathbf{D}_y^H(t)\mathbf{D}_y(t)$ and $\widetilde{\boldsymbol{\mathcal{D}}}_y(n-1) \triangleq \boldsymbol{\mathcal{D}}_y(n-1)[\mathbf{I}_2 \otimes \boldsymbol{\Omega}_y^{-1/2}(n-1)]$. Note that $\widetilde{\boldsymbol{\mathcal{D}}}_y(n-1)$ is unitary. Let

$$\begin{aligned} \boldsymbol{z}(n) &\triangleq \widetilde{\boldsymbol{\mathcal{D}}}_y^H(n-1)\boldsymbol{y}(n) \\ &= 2^{-1/2}\left[\mathbf{I}_2 \otimes \boldsymbol{\Omega}_y^{1/2}(n-1)\right]\boldsymbol{s}(n) + \widetilde{\boldsymbol{\mathcal{D}}}_y^H(n-1)\boldsymbol{v}(n). \end{aligned} \tag{4}$$

Due to the block diagonal structure of matrix $\mathbf{I}_2 \otimes \boldsymbol{\Omega}_y^{1/2}(n-1)$, (4) reduces to the following two independent equations:

$$\mathbf{z}(t) = 2^{-1/2}\boldsymbol{\Omega}_y^{1/2}(n-1)\mathbf{s}(t) + \boldsymbol{\nu}(t), \quad t = 2n-1, 2n \tag{5}$$

where $\mathbf{z}(2n-1)$ and $\mathbf{z}(2n)$ are the first and second halves of $\boldsymbol{z}(n)$, whereas $\boldsymbol{\nu}(2n-1)$ and $\boldsymbol{\nu}(2n)$ are similarly formed from $\widetilde{\boldsymbol{\mathcal{D}}}_y(n-1)\boldsymbol{v}(n)$. Hence, the ML detection of $\mathbf{s}(2n-1)$ and $\mathbf{s}(2n)$ is independent.

IV. Spectral Encoding

We assume (correlated) Rayleigh fading channels: **A1)** $\mathbf{h}_i \triangleq [h_i(0), \ldots, h_i(L)]^T$ are zero-mean complex Gaussian with nonsingular covariance matrix $\mathbf{R}_h \triangleq E\{\mathbf{h}\mathbf{h}^H\}$, where $\mathbf{h} \triangleq [\mathbf{h}_1^T, \mathbf{h}_2^T]^T$. To minimize decoding complexity, we consider *minimum-length full-diversity* codes that encode across a minimum number of subchannels for full diversity. The coded symbols have to be transmitted in well separated subchannels by *subcarrier interleaving (SI)* [5]. Let $\mathcal{I} \triangleq \{0, 1, \ldots, P-1\}$ collect the indices of all subcarriers. Briefly stated, SI is a partition of $\mathcal{I}$ into M nonoverlapping subsets $\mathcal{I}^{(m)} \triangleq \{p_{m,0}, \ldots, p_{m,Q_m-1}\}$, where Q_m is the number of subcarriers in the mth subset. For channels satisfying **A1)**, we need $Q_m \geq L+1$ to achieve the maximum spectral diversity [5]. We choose the minimum $Q_m = L+1$ so that the decoding complexity is minimized. Among other alternatives, the following SI scheme is conceptually simple [5]:

$$\mathcal{I}^{(m)} = \{m, M+m, \ldots, LM+m\} \tag{6}$$

where $M \triangleq P/(L+1)$, and P is assumed a multiple of $L+1$.

The input–output relation, when SI is utilized, for the mth subcarrier subset is given by [cf. (5)]

$$\mathbf{z}^{(m)}(t) = 2^{-1/2}\left[\boldsymbol{\Omega}_y^{(m)}(n-1)\right]^{1/2}\mathbf{s}^{(m)}(t) + \boldsymbol{\nu}^{(m)}(t) \tag{7}$$

where $\mathbf{z}^{(m)}(t) \in \mathbb{C}^{(L+1)\times 1}$, $\boldsymbol{\Omega}_y^{(m)}(n-1) \in \mathbb{C}^{(L+1)\times(L+1)}$, $\mathbf{s}^{(m)}(t) \in \mathcal{A}_s^{(L+1)\times 1}$, and $\boldsymbol{\nu}^{(m)}(t) \in \mathbb{C}^{(L+1)\times 1}$ are the counterparts of the corresponding quantities in (5). The probability of erroneously choosing $\mathbf{s}_2^{(m)}(t)$ as $\mathbf{s}_1^{(m)}(t)$ by the ML detector is upper-bounded by (dropping indices m and t for brevity) [6]:

$$P(\mathbf{s}_1 \to \mathbf{s}_2) \leq \left[\frac{E_s}{(8N_0)}\right]^{-r_e}\left[\det(\mathbf{R}_h)\prod_{l=1}^{r_e}\lambda_l\right]^{-1} \tag{8}$$

where $r_e \triangleq \mathrm{rank}(\boldsymbol{\Phi}_e) \leq 2(L+1)$, $\boldsymbol{\Phi}_e \triangleq 2^{-1}\mathbf{I}_2 \otimes \left(\boldsymbol{\mathcal{F}}_m^H\mathbf{D}_e^*\mathbf{D}_e\boldsymbol{\mathcal{F}}_m\right)$, and $\{\lambda_l\}_{l=1}^{r_e}$ are the r_e nonzero eigenvalues of $\boldsymbol{\Phi}_e$, with $\mathbf{D}_e \triangleq \mathrm{diag}(\mathbf{e})$, $\mathbf{e} \triangleq \mathbf{s}_1 - \mathbf{s}_2$, and $\boldsymbol{\mathcal{F}}_m \in \mathbb{C}^{(L+1)\times(L+1)}$ formed by rows $m, m+M, \ldots, m+LM$ of the P-point FFT matrix $\boldsymbol{\mathcal{F}} \in \mathbb{C}^{P\times(L+1)}$: $[\boldsymbol{\mathcal{F}}]_{p,q} \triangleq \exp(-j2\pi(p-1)(q-1)/P)$. Following [6], $G_d \triangleq \min_{\forall\, \mathbf{e}\neq\mathbf{0}} r_e$ is called the *diversity advantage*, while $G_c \triangleq \min_{\forall\, \mathbf{e}\neq\mathbf{0}}\left[\det(\mathbf{R}_h)\prod_{l=1}^{r_e}\lambda_l\right]^{1/r_e}$ is the *coding advantage* over an uncoded system. We summarize the optimum G_d and G_c for the DSTF system as follows:

Theorem 1: Under condition **A1)** and (6), the maximum diversity advantage of the DSTF system is $G_{d,\max} = 2(L+1)$, which is achieved iff the code $\mathbf{s}$ has a uniform Hamming distance of $L+1$. Any maximum-diversity achieving code has a coding advantage given by $G_{c,\max} = 2^{-1}(L+1)[\delta_{\min}^4\det(\mathbf{R}_h)]^{1/[2(L+1)]}$, where $\delta_{\min}$ denotes the *minimum product distance* of the code: $\delta_{\min} = \min_{\forall\, \mathbf{e}\neq\mathbf{0}}|\det(\mathbf{D}_e)|$.

Proof: Note that $\boldsymbol{\mathcal{F}}_m$ is orthogonal with $\boldsymbol{\mathcal{F}}_m^H\boldsymbol{\mathcal{F}}_m = (L+1)\mathbf{I}_{L+1}$. Hence, $\mathrm{rank}(\boldsymbol{\Phi}_e) = 2\,\mathrm{rank}(\mathbf{D}_e^*\mathbf{D}_e) \leq 2(L+1)$. The inequality becomes an equality iff $\mathbf{e}$ has no zero element over all error events, which occurs when the code $\mathbf{s}$ has a uniform Hamming distance of $L+1$. Hence, the maximum diversity order is $2(L+1)$. Note that the minimum diversity order is 2 since $\min_{\forall\, \mathbf{e}\neq\mathbf{0}}\mathrm{rank}(\boldsymbol{\Phi}_e) = 2$. Now, assume the maximum diversity. We have $\det(\boldsymbol{\Phi}_e) = 2^{-2(L+1)}\left[\det\left(\boldsymbol{\mathcal{F}}_m^H\mathbf{D}_e^*\mathbf{D}_e\boldsymbol{\mathcal{F}}_m\right)\right]^2 = 2^{-2(L+1)}\left[\det(\mathbf{D}_e^*\mathbf{D}_e)\det\left(\boldsymbol{\mathcal{F}}_m^H\boldsymbol{\mathcal{F}}_m\right)\right]^2 \triangleq [2^{-1}(L+1)]^{2(L+1)}|\det(\mathbf{D}_e)|^4$, which leads to the coding gain $G_{c,\max}$ as in Theorem 1 for all full-diversity codes. ∎

For notational brevity, we drop the subcarrier subset index m. To achieve a code rate of R bps/Hz, we need a codebook with

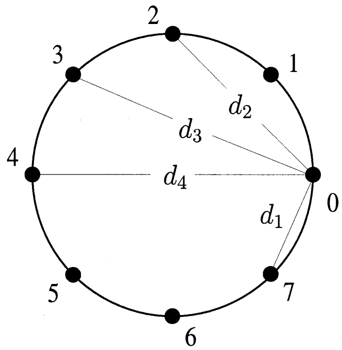

Fig. 2. 8-PSK constellation with unit symbol energy.

$N_c \triangleq 2^{R(L+1)}$ distinct codewords of length $L+1$ (i.e., the minimum code length for full diversity), with coded symbols drawn from an M_c-PSK constellation $\mathcal{A}_s$. Let $\mathbf{s}_i \triangleq [s_{i,0}, \ldots, s_{i,L}]^T$ be the ith codeword, and $\boldsymbol{\mathcal{B}}_s \triangleq [\mathbf{s}_0, \ldots, \mathbf{s}_{N_c-1}]_{(L+1)\times N_c}$ be the codebook. To ensure that $\boldsymbol{\mathcal{B}}_s$ has a uniform Hamming distance of $L+1$, it can be shown that M_c must be no less than N_c. We choose $M_c = N_c$ to minimize the decoding complexity. Let us label the constellation points in $\mathcal{A}_s$ as $0, 1, \ldots, M_c - 1$ (e.g., the 8-PSK shown in Fig. 2) and form the sequence $\vec{c} \triangleq [0, 1, \ldots, M_c - 1]$. The uniform Hamming distance requirement mandates that each row of $\boldsymbol{\mathcal{B}}_s$ be a permutation of $\vec{c}$; any code formed by permutations also has a uniform Hamming distance of $L+1$. It is easy to see that there are a total of $(N_c!)^L$ such permutation codes, all achieving the full diversity!

To facilitate code construction, we introduce the idea of *constellation decimation* that effectively imposes a linear structure on the code. The linear structure makes the analysis of distance property and search for good codes significantly easier. Specifically, let $\vec{c}[k]$ be the kth element of $\vec{c}$. Denote by $\vec{c}_q \triangleq \{\vec{c}_q, \vec{c}_q 1, \ldots, \vec{c}_q[M_c - 1]\}$ the *qth decimation* of $\vec{c}$, $q = 1, 2, \ldots, M_c$, where $\vec{c}_q[k] \triangleq \vec{c}[qk \pmod{M_c}]$, $k = 0, 1, \ldots, M_c - 1$. Note that q and M_c have to be relatively prime so that the decimated sequence will be a permutation of $\vec{c}$.

A *linear constellation decimation (LCD)* code $\boldsymbol{\mathcal{B}}_s$ is an $(L+1) \times M_c$ matrix, each row obtained by a proper decimation of $\vec{c}$. We use the notation $\boldsymbol{\mathcal{B}}_s = \langle q_0, q_1, \ldots, q_L \rangle$ to signify that $\boldsymbol{\mathcal{B}}_s$ is obtained by using decimation factor q_j for the jth row of $\boldsymbol{\mathcal{B}}_s$. Two LCD codes are listed below for $L = 2$ (i.e., 3-ray channel), $\mathcal{A}_s = $ 8-PSK as shown in Fig. 2, and rate $R = 1$ bps/Hz:

$$\boldsymbol{\mathcal{B}}_s^{\langle 1,1,1 \rangle} = \begin{bmatrix} 0, & 1, & 2, & 3, & 4, & 5, & 6, & 7 \\ 0, & 1, & 2, & 3, & 4, & 5, & 6, & 7 \\ 0, & 1, & 2, & 3, & 4, & 5, & 6, & 7 \end{bmatrix}$$

$$\boldsymbol{\mathcal{B}}_s^{\langle 1,3,5 \rangle} = \begin{bmatrix} 0, & 1, & 2, & 3, & 4, & 5, & 6, & 7 \\ 0, & 3, & 6, & 1, & 4, & 7, & 2, & 5 \\ 0, & 5, & 2, & 7, & 4, & 1, & 6, & 3 \end{bmatrix}. \quad (9)$$

$\boldsymbol{\mathcal{B}}_s^{\langle 1,1,1 \rangle}$ is seen to coincide with a repetition code. It is easy to verify that both codes have a uniform Hamming distance $L + 1 = 3$. The minimum product distances are $\delta_{\min}^{\langle 1,1,1 \rangle} = d_1^3$ and $\delta_{\min}^{\langle 1,3,5 \rangle} = d_3 d_1^2$ (cf. Fig. 2), respectively. By Theorem 1, $\boldsymbol{\mathcal{B}}_s^{\langle 1,3,5 \rangle}$ achieves a coding gain of $10\log_{10}\left(\delta_{\min}^{\langle 1,3,5 \rangle}/\delta_{\min}^{\langle 1,1,1 \rangle}\right)^{2/(L+1)} \approx$ 2.55 dB relative to the repetition code. In fact, $\boldsymbol{\mathcal{B}}_s^{\langle 1,3,5 \rangle}$ can be shown (by a quick computer search) to be the optimum LCD code with the largest product distance. Due to space limitation, construction of optimum LCD codes for other values of R and L will be reported elsewhere.

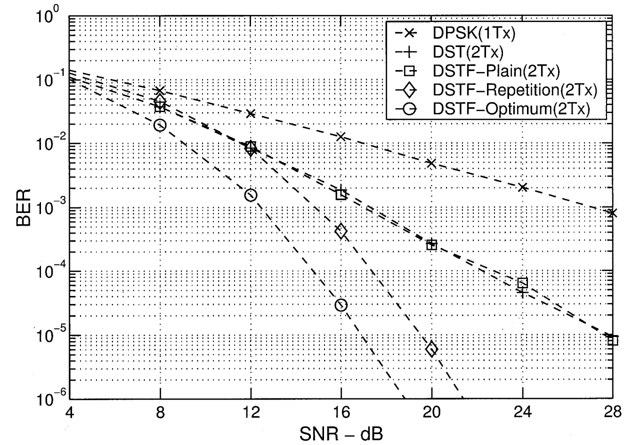

Fig. 3. BER versus SNR in 3-ray Rayleigh fading channels.

V. Simulation Results

Consider an OFDM system with $P = 48$ subcarriers and $R = 1$ bps/Hz. The transmitter has one or two Tx's, but the receiver has only one Rx. The channel coefficients are assumed complex Gaussian with zero-mean and variance $N_0 = 1/(L+1)$, where $L = 2$ (i.e., 3-ray Rayleigh channels). Fig. 3 depicts the BER versus SNR (defined as E_s/N_0) of the following transmission schemes. **1) DPSK (1Tx):** Differential OFDM with differential BPSK applied on each subcarrier, which yields no diversity and serves as a benchmark for other diversity systems. **2) DST (2Tx):** Differential space-time coded OFDM with the unitary DSTC [3] applied on each subcarrier. The constellation used by DST is QPSK. **3) DSTF-Plain (2Tx):** The proposed DSTF scheme without the spectral encoder $\mathcal{M}_s\{\cdot\}$ (thus the word *plain*), in order to show the additional gain obtained by $\mathcal{M}_s\{\cdot\}$. The information symbols are BPSK. **4) DSTF-Repetition (2Tx):** DSTF using the repetition code $\boldsymbol{\mathcal{B}}_s^{\langle 1,1,1 \rangle}$ in (9) with 8-PSK for spectral coding. **5) DSTF-Optimum (2Tx):** DSTF using the optimum LCD code $\boldsymbol{\mathcal{B}}_s^{\langle 1,3,5 \rangle}$ with 8-PSK in (9) for spectral coding. Fig. 3 indicates that both DST and DSTF-Plain achieve a diversity order of 2, since the BER-SNR slope of these two schemes is approximately 2. This is the spatial diversity. An inspection of the BER-SNR slope reveals that both DSTF-Repetition and DSTF-Optimum achieve a diversity order of 6 at high SNR, which is the maximum spatio-spectral diversity order offered by the system. It is also observed that DSTF-Optimum yields an additional coding gain of about 2.5 dB over DSTF-Repetition, which agrees with the calculation in Section IV.

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