Differential and Coherent Decorrelating Multiuser Receivers for Space-Time-Coded CDMA Systems

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***Abstract*—Recently, we proposed a differential space-code modulation (DSCM) scheme that integrates the strength of differential space-time coding and spreading to achieve interference suppression and resistance to time-varying channel fading in single-user environments. In this paper, we consider the problem of multiuser receiver design for code-division multiple-access (CDMA) systems that utilize DSCM for transmission. In particular, we propose two differential receivers for such systems. These differential receivers do not require the channel state information (CSI) for detection and, still, are resistant to multiuser interference (MUI) and time-varying channel fading. We also propose a coherent receiver that requires only the CSI of the desired user for detection. The coherent receiver yields improved performance over the differential receivers when reliable channel estimates are available (e.g., in slowly fading channels). The proposed differential/coherent receivers are decorrelative schemes that decouple the detection of different users. Both long and short spreading codes can be employed in these schemes. Numerical examples are presented to demonstrate the effectiveness of the proposed receivers.**

***Index Terms*—Code-division multiple-access, interference suppression, multiuser receiver, space-time coding, time-varying fading.**

I. Introduction

FUTURE wireless mobile systems are expected to provide broadband multimedia data services to mobile users under high mobility conditions (e.g., high-speed trains moving up to 500 km/h [1]). Reliable transmission over the wireless channel, however, is a challenging task due to the inherent difficulties (e.g., channel fading and radio interference) of the wireless transmission medium. Moreover, the detrimental effect of fading and interference exacerbates as the transmission rate and/or mobility increases. As such, there is an urgent need to develop interference and fading mitigation schemes that are suitable for high-speed transmission and high mobility environments.

Space-time coding, which makes use of multiple-antenna transmission and signal processing at the receiver, has been receiving significant interest recently. Space-time coding has been shown to be able to produce dramatic increases in transmission rate, due to its ability of fully exploiting the spatial and temporal diversity [2], [3]. A number of space-time coding schemes, including space-time trellis codes [4] and space-time block codes [5]–[8], have been proposed thus far. While space-time trellis coding achieves the maximum diversity gain and coding advantage, the trellis complexity (and thus the decoding complexity) increases exponentially with the transmission rate [4, Lemma 3.3.2]. Meanwhile, space-time block coding offers the maximum diversity gain based on linear processing at the receiver [5], [6]. Despite a loss in coding advantage, space-time block coding is still attractive, particularly in complexity-sensitive applications, since diversity gain is very effective in reducing the error probability at high signal-to-noise ratio (SNR) [3], [4].

Coherent decoding of space-time codes requires the channel state information (CSI) at the receiver, which may be estimated through either training-based or blind channel estimators. Channel estimation in space-time-coded systems, however, is more difficult than in single-antenna systems since the number of unknowns to be estimated increases proportionally to the number of transmit antennas. This translates to more required training data for training-based channel estimators or a longer convergence time for blind channel estimators [3]. To circumvent the difficulty of channel estimation, differential space-time coding schemes have been studied most recently in [9]–[13]. Among them, the *unitary space-time group codes* [9], [10], [13] are particularly suitable for low-complexity differential modulation and decoding. A complete characterization of all full-diversity constellations that form a group for all rates and numbers of transmit antennas is provided in [13]. Although similar to the scalar (single-antenna) differential modulation scheme, differential decoding of space-time codes incurs approximately a 3-dB penalty in SNR compared with coherent decoding [10]; the price is often worthwhile in fast-fading channels, where coherent decoding becomes practically infeasible [9]–[13].

Interference suppression in space-time-coded systems is also more challenging than in single-antenna systems [3]. For a system with K users where each is equipped with M transmit antennas, multiuser interference (MUI) is composed of $(K-1)M$ interfering signals, rather than $K-1$ interfering signals in a single-antenna system. Interference suppression for space-time block coded systems was considered in [2]. In particular, under the condition that the system is equipped with more receive antennas than the number of users, zero-forcing (ZF) and minimum mean squared error (MMSE) interference

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cancellation techniques were derived based on the availability of the CSI of both the desired and interfering users at the receiver. These techniques, however, were not designed for wireless mobile systems since the number of antennas that can be installed at the mobile is, in general, limited. Furthermore, it would also be difficult to apply them in (fast) time-varying channels due to the need for the CSI.

Recently, we proposed a *differential space-code modulation* (DSCM) scheme [14], [15], which enhances the *differential space-time modulation* (DSTM) scheme in [9] by adding a spreading component (see Section II for a brief review and comparison of DSTM and DSCM). DSCM was shown to attain a remarkable interference suppression ability. Similar to the ZF and MMSE schemes [2], however, DSCM utilizes only the spatial degrees of freedom. As a result, the number of interferers that can be effectively canceled is also limited by the number of receive antennas.

Although DSCM was considered only for single-user applications in [15], we show herein that it naturally extends to a multiuser code-division multiple-access (CDMA) scenario. We examine the problem of multiuser receiver design for CDMA systems that employ DSCM for transmission. To obviate the need for channel estimation, we propose two differential receivers that are resistant to both MUI and time-varying fading. Unlike previous schemes [2], [15], which rely on spatial degrees of freedom for interference cancellation, the proposed differential receivers exploit the spreading structure offered by DSCM to achieve effective MUI suppression, without requiring an excessive number of receive antennas. As shown in Section IV-A2, one of the proposed differential receivers bears a close relation to the single-user DSTM receiver [9] and, thus, can be thought of as a multiuser extension of DSTM. The proposed differential receivers can also be classified as *decorrelators* since both involve a decorrelating stage to remove the MUI (see Section IV for details). For single-transmit-antenna multiuser systems, noncoherent decorrelating receivers have been studied for a general class of nonorthogonal multipulse modulation schemes in Gaussian [16] and Rayleigh fading [17] channels; decorrelating receivers have also been investigated for systems with differential phase-shift keying (DPSK), which can be considered a special member of multipulse modulation (see [17]), and diversity combining [18], [19]. Our proposed receivers can be thought of the extensions of these single-transmit-antenna noncoherent/differential decorrelating receivers to multitransmit-antenna systems.

We also introduce a coherent decorrelating receiver for differentially space-time-coded CDMA systems. The coherent receiver requires only the CSI of the desired transmission. In slowly fading environments when channel estimates can be reliably derived at the receiver, the coherent receiver achieves improved performance over the differential receivers. Coherent decorrelating detection requiring only the CSI of the desired user has also been studied for single-transmit-antenna CDMA systems [20]. The proposed coherent receiver can be considered to be an extension of the coherent decorrelating scheme therein.

The rest of the paper is organized as follows. In Section II, we briefly review the DSTM [9] and DSCM [15] modulation schemes. The data model for CDMA systems with the DSCM scheme is presented in Section III. Section IV contains the derivations of the proposed differential and coherent receivers. Numerical results are presented in Section V. Finally, the paper is concluded in Section VI.

Notation: Vectors (matrices) are denoted by boldface lower (upper) case letters; all vectors are column vectors; superscripts $(\cdot)^*, (\cdot)^\mathsf{T}, (\cdot)^\mathsf{H}$ denote the complex conjugate, transpose, and conjugate transpose, respectively; $\mathbf{I}_N$ denotes the $N \times N$ identity matrix; $E\{\cdot\}$ denotes the statistical expectation; $\|\cdot\|_F$ denotes the matrix Frobenius norm [21]; $\mathrm{tr}\{\cdot\}$ denotes the trace of a matrix; $\otimes$ denotes the Kronecker product [21]; $\mathrm{vec}(\cdot)$ denotes the operation of stacking the columns of a matrix on top of one another [21]; $\Re[\cdot]$ takes the real-valued part of the argument.

II. Review of DSTM and DSCM

Consider a wireless communication system that is equipped with M transmit antennas and N receive antennas. Assume a baseband transmission employing a unit-energy signal constellation $\mathcal{C}$. At the transmitter, the information symbols are encoded by a space-time encoder that outputs M parallel coded streams. Let $c_{ml} \in \mathcal{C}$ denote the constellation point selected by the encoder, which is transmitted from transmit antenna $m = 1, \ldots, M$ at time $l = 1, \ldots, L$. The *space-time code matrix* is defined as

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1L} \\ c_{21} & c_{22} & \cdots & c_{2L} \\ \vdots & \vdots & \ddots & \vdots \\ c_{M1} & c_{M2} & \cdots & c_{ML} \end{bmatrix} \tag{1}$$

where each column corresponds to the M symbols transmitted in parallel from the M transmit antennas. A space-time block code is a collection of $M \times L$ code matrices defined in the signal constellation $\mathcal{C}$.

In order to deal with rapidly time-varying Rayleigh environments, where the CSI are unknown and cannot be reliably estimated at the receiver, Marzetta and Hochwald [1], [22] suggested the use of code matrices with equal-energy, orthogonal rows, i.e.,

$$\mathbf{C}\mathbf{C}^\mathsf{H} = L\mathbf{I}_M \tag{2}$$

in order to approach the channel capacity. This leads to the so-called unitary space-time modulation [1].

Differential encoding can be introduced between two adjacent space-time code matrices so that the decoding at the receiver is independent of the underlying channel. Among various differential space-time coding schemes proposed so far [9]–[13], the *unitary space-time group codes* introduced in [9], [10], and [13] are particularly interesting for low-complexity differential modulation and decoding. Specifically, by assuming $M = L$, unitary space-time group codes are used to yield a simple DSTM scheme [9]. For example, the code

$$\mathcal{G} = \left\{ \pm \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \pm \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \right\}, \ \mathbf{D} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \tag{3}$$

is a unitary group code of dimension two over the binary phase-shift keying (BPSK) constellation [9]. Each pair of information bits in $\{00, 01, 10, 11\}$ is mapped into a space-time code in $\mathcal{G}$. To send the message $\mathbf{G}_t \in \mathcal{G}$ at time t, the transmitter sends $\mathbf{C}_t \in \mathcal{C}^{M\times L}$, where

$$\mathbf{C}_t = \mathbf{C}_{t-1}\mathbf{G}_t \tag{4}$$

and the transmitter begins with $\mathbf{C}_0 = \mathbf{D}$. Note the analogy between (4) and the standard scalar (single-antenna) DPSK [23]. For the above BPSK example, two transmit antennas are used to transmit each of the two elements of one column of $\mathbf{C}_t$ at a time. The unitary space-time group code matrices are orthogonal [9], i.e.,

$$\mathbf{C}_t\mathbf{C}_t^{\mathsf{H}} = \mathbf{C}_t^{\mathsf{H}}\mathbf{C}_t = M\mathbf{I}_M, \ \forall\, t \tag{5}$$

where we have assumed $M = L$.

The DSTM scheme [9] works well in time-varying channels, provided that the channel do not change significantly over two adjacent code matrices. DSTM itself, however, is not immune to unknown interference [15]. Recently, we introduced a new DSCM scheme that integrates the strength of unitary space-time group codes with spread spectrum to achieve interference suppression and fading resilience [14], [15]. The idea is to *vector-spread and combine* each column of a space-time block code matrix. Specifically, each column of $\mathbf{C}_t$ is spread temporally with a unique spreading code of unit energy, and the coded columns of $\mathbf{C}_t$ are transmitted *at the same time*. Let the output of the DSCM modulator corresponding to the tth block and jth chip be $\mathbf{x}_t(j)$, which has the form [15]

$$\mathbf{x}_t(j) = \mathbf{C}_t\mathbf{s}(j), \quad j = 1, \ldots, J \tag{6}$$

where $\mathbf{s}(j) \in \mathbb{C}^{J\times 1}, \quad j = 1, \ldots, J$ correspond to L spreading codes of length J used to spread $\mathbf{C}_t$. The L spreading codes $\{\mathbf{s}(j)\}$ need not be periodic, which implies that both short and long spreading codes can be used in this modulation scheme. We note that similar spreading schemes have been considered in [24] under the name of *space-time spreading*, as well as in [25, eq. (27)] for general space-time communications.

III. Data Model and Problem Formulation

We consider a synchronous K-user system operating in a frequency-nonselective, Rayleigh fading channel. The fading coefficients may either change slowly or vary rapidly. Extensions may be made to include frequency-selective channels by, for example, considering multicarrier CDMA systems [26]; however, such an extension is beyond the scope of the current paper. Under these assumptions, the baseband discrete-time signal received by the N receive antennas at the receiver can be expressed as (see [9] and [15])

$$\mathbf{y}_t(j) = \sum_{k=1}^{K}\sqrt{\rho_{k,t}}\mathbf{H}_{k,t}\mathbf{C}_{k,t}\mathbf{s}_k(j) + \mathbf{n}_t(j), \quad j = 1, \ldots, J \tag{7}$$

where $M\rho_{k,t}$ denotes user k's *average* received power per receive antenna during the transmission of the tth space-time code matrix $\mathbf{C}_{k,t} \in \mathbb{C}^{M\times L}$, $\mathbf{H}_{k,t} \in \mathbb{C}^{N\times M}$ is user k's channel matrix assumed (approximately) unchanged during the transmission of $\mathbf{C}_{k,t}$ (for differential modulation, we assume that $\mathbf{H}_{k,t}$ remains approximately unchanged during the transmission of two adjacent code matrices $\mathbf{C}_{k,t-1}$ and $\mathbf{C}_{k,t}$), $\{\mathbf{s}_k(j)\}_{j=1}^{J}$ denote the L spreading codes of length J assigned to user k, and $\mathbf{n}_t(j) \in \mathbb{C}^{N\times 1}$ is the channel noise corresponding to time t. Note that both t and j are time indexes: The former denotes the tth space-time block code, whereas the latter denotes the jth chip of the tth code matrix. We assume a Rayleigh fading channel model so that elements in $\mathbf{H}_{k,t}$ and $\mathbf{n}_t(j)$ are independent, identically-distributed (i.i.d.), complex Gaussian random variables with zero mean and unit variance [27]. The *average* SNR per receive antenna for user k during the transmission of $\mathbf{C}_{k,t}$ is thus $M\rho_{k,t}$. Note that the channel matrices $\{\mathbf{H}_{k,t}\}$ are different from one another for the uplink; for the downlink, $\mathbf{H}_{1,t} = \cdots = \mathbf{H}_{K,t}$.

Two transmission schemes are considered in this paper. The first one makes use of differential modulation whereby the space-time code matrices $\mathbf{C}_{k,t}$ are differentially encoded, as in (4). This transmission scheme is adopted when the CSI is not available, and differential detection is performed at the receiver. The second scheme does not need differential encoding, in which case, $\mathbf{C}_{k,t}$ can be any space-time block code introduced in, e.g., [6]. This scheme is applicable when the CSI is estimated and coherent detection is employed at the receiver.

The problem of interest herein is to determine the code matrices $\mathbf{C}_{k,t}, \quad k = 1, \ldots, K$ or, equivalently, the transmitted symbols for K users from the received data $\{\mathbf{y}_t(j)\}_{j=1}^{J}$.

IV. Receiver Design

Optimum coherent and noncoherent detection based on maximum likelihood (ML) processing can be readily derived from (7) (see also, e.g., [25] and [28]). These optimum detectors perform joint detection for all users and incur an exponential complexity in both the user number K and transmission rate. When differential modulation is utilized, optimum performance in general requires joint decoding of all transmitted code matrices (i.e., for all t) [9], [29]. The enormous complexity of such optimum receivers renders them mainly of theoretical interest. In what follows, we present several suboptimal decorrelating receivers that decouple the detection of different users. The complexity of these suboptimal receivers is exponential only in the transmission rate, similarly to single-user space-time receivers. The first two proposed receivers are customarily referred to as *differential receivers* since they exploit the inherent structure of differential space-time codes and do not require the CSI for detection. The third proposed receiver is a *coherent* decorrelating receiver that requires only the CSI of the desired user. As one can expect, considerable performance improvement can be obtained if the CSI is available and incorporated in detection.

In order to express (7) more compactly, we define

$$\mathbf{Y}_t \triangleq [\mathbf{y}_t(1), \ldots, \mathbf{y}_t(J)] \tag{8}$$

$$\mathbf{N}_t \triangleq [\mathbf{n}_t(1), \ldots, \mathbf{n}_t(J)] \tag{9}$$

$$\mathbf{S}_k \triangleq [\mathbf{s}_k(1), \ldots, \mathbf{s}_k(J)] \tag{10}$$

$$\mathbf{S} \triangleq \left[\mathbf{S}_1^{\top}, \ldots, \mathbf{S}_K^{\top}\right]^{\top} \tag{11}$$

$$\mathbf{A}_{k,t} \triangleq \sqrt{\rho_{k,t}}\mathbf{H}_{k,t}\mathbf{C}_{k,t} \tag{12}$$

$$\mathbf{A}_t \triangleq [\mathbf{A}_{1,t}, \ldots, \mathbf{A}_{K,t}]. \tag{13}$$

Then, (7) can be expressed as

$$\mathbf{Y}_t = \mathbf{A}_t\mathbf{S} + \mathbf{N}_t. \tag{14}$$

Since all information about the transmitted code matrices $\{\mathbf{C}_{k,t}\}_{k=1}^K$ is contained in $\mathbf{A}_t$, the key of the differential and coherent receivers introduced in the sequel is to obtain an initial estimate of $\mathbf{A}_t$, which is used for subsequent detection.

The *unstructured* ML estimate of $\mathbf{A}_t$, which is conditioned on $\{\mathbf{H}_{k,t}\}_{k=1}^K$ and $\{\mathbf{C}_{k,t}\}_{k=1}^K$, is given by

$$\hat{\mathbf{A}}_t = \mathbf{Y}_t\mathbf{S}^{\mathsf{H}}\left(\mathbf{S}\mathbf{S}^{\mathsf{H}}\right)^{-1} \tag{15}$$

where we have assumed that $\mathbf{S}\mathbf{S}^{\mathsf{H}}$ has full rank, and by "unstructured," we mean that the structure of $\mathbf{A}_t$, as specified in (12) and (13), is not enforced in the estimation. We note that the ML estimate $\hat{\mathbf{A}}_t$ is effectively the output of a *decorrelator* with the input being the received data $\mathbf{Y}_t$. The decorrelator cancels the MUI and decouples the detection of different users. Due to the block structure of $\mathbf{A}_t$, the ML estimate $\hat{\mathbf{A}}_{k,t}$ of $\mathbf{A}_{k,t}$ is easily obtained from $\hat{\mathbf{A}}_t$. Note that all information about the transmitted code matrix $\mathbf{C}_{k,t}$ for user k is contained in $\mathbf{A}_{k,t}$.

A. Differential Detection

As in the standard scalar DPSK modulation scheme where detection is performed based on the received signal over two adjacent symbol periods (e.g., [23]), our proposed differential receivers for space-time-coded CDMA systems make use of $\hat{\mathbf{A}}_{k,t-1}$ and $\hat{\mathbf{A}}_{k,t}$ to detect $\mathbf{C}_{k,t}$. To elaborate, we first need to determine the distribution of $\hat{\mathbf{A}}_{k,t-1}$ and $\hat{\mathbf{A}}_{k,t}$, which is stated in the following result.

Theorem 1: Let $\hat{\boldsymbol{\mathcal{A}}}_{k,t} \triangleq \left[\hat{\mathbf{A}}_{k,t-1}, \hat{\mathbf{A}}_{k,t}\right]$. Then, conditioned on $\mathbf{G}_{k,t}$, $\mathrm{vec}(\hat{\mathcal{A}}_{k,t})$ is a complex Gaussian random vector with zero mean and covariance matrix

$$\boldsymbol{\Sigma}_{k,t} = \rho_{k,t}(\boldsymbol{\mathcal{C}}_{k,t}^{\mathsf{H}}\boldsymbol{\mathcal{C}}_{k,t})^{\mathsf{T}} \otimes \mathbf{I}_N + \mathbf{I}_2 \otimes \boldsymbol{\Phi}_k \tag{16}$$

where

$$\boldsymbol{\mathcal{C}}_{k,t} \triangleq [\mathbf{C}_{k,t-1}, \mathbf{C}_{k,t}] \tag{17}$$

and $\boldsymbol{\Phi}_k \in \mathbb{C}^{LN\times LN}$ is the kth diagonal block of $\boldsymbol{\Phi}$:

$$\boldsymbol{\Phi} \triangleq \left[(\mathbf{S}\mathbf{S}^{\mathsf{H}})^{-1}\right]^{\mathsf{T}} \otimes \mathbf{I}_N. \tag{18}$$

Proof: According to (14) and (15), we have

$$\begin{aligned}\hat{\mathbf{A}}_t =&(\mathbf{A}_t\mathbf{S} + \mathbf{N}_t)\mathbf{S}^{\mathsf{H}}(\mathbf{S}\mathbf{S}^{\mathsf{H}})^{-1}\\ =&\mathbf{A}_t + \mathbf{N}_t\mathbf{S}^{\mathsf{H}}(\mathbf{S}\mathbf{S}^{\mathsf{H}})^{-1} = \mathbf{A}_t + \mathbf{E}_t\end{aligned} \tag{19}$$

where $\mathbf{E}_t \triangleq \mathbf{N}_t\mathbf{S}^{\mathsf{H}}(\mathbf{S}\mathbf{S}^{\mathsf{H}})^{-1}$. We decompose $\hat{\mathbf{A}}_t$ and $\mathbf{E}_t$ into $N \times L$ blocks, i.e., let $\hat{\mathbf{A}}_t \triangleq \left[\hat{\mathbf{A}}_{1,t}, \ldots, \hat{\mathbf{A}}_{K,t}\right]$ and $\mathbf{E}_t \triangleq [\mathbf{E}_{1,t}, \ldots, \mathbf{E}_{K,t}]$. Let $\mathbf{A}_{t-1}$ and $\mathbf{E}_{t-1}$ corresponding to time $t-1$ be similarly decomposed. Then, we have

$$\mathrm{vec}(\hat{\boldsymbol{\mathcal{A}}}_{k,t}) = \mathrm{vec}(\boldsymbol{\mathcal{A}}_{k,t}) + \mathrm{vec}(\boldsymbol{\mathcal{E}}_{k,t}) \tag{20}$$

where $\boldsymbol{\mathcal{A}}_{k,t} \triangleq [\mathbf{A}_{k,t-1}, \mathbf{A}_{k,t}]$ and $\boldsymbol{\mathcal{E}}_{k,t} \triangleq [\mathbf{E}_{k,t-1}, \mathbf{E}_{k,t}]$. We first write $\mathrm{vec}(\mathbf{E}_t)$ as

$$\mathrm{vec}(\mathbf{E}_t) = \left\{\left[\mathbf{S}^{\mathsf{H}}(\mathbf{S}\mathbf{S}^{\mathsf{H}})^{-1}\right]^{\mathsf{T}} \otimes \mathbf{I}_N\right\}\mathrm{vec}(\mathbf{N}_t) \tag{21}$$

where we have used the identity

$$\mathrm{vec}(\mathbf{X}\mathbf{Y}\mathbf{Z}) = (\mathbf{Z}^{\mathsf{T}} \otimes \mathbf{X})\mathrm{vec}(\mathbf{Y}) \tag{22}$$

for arbitrary matrices $\mathbf{X}$, $\mathbf{Y}$, and $\mathbf{Z}$ of appropriate sizes [30]. Since $\mathrm{vec}(\mathbf{N}_t)$ has a Gaussian distribution with zero mean and covariance matrix $\mathbf{I}_{JN}$, $\mathrm{vec}(\mathbf{E}_t)$ is also Gaussian with zero mean and covariance matrix

$$\begin{aligned}E\left\{\mathrm{vec}(\mathbf{E}_t)\mathrm{vec}^{\mathsf{H}}(\mathbf{E}_t)\right\} &= \left\{\left[\mathbf{S}^{\mathsf{H}}(\mathbf{S}\mathbf{S}^{\mathsf{H}})^{-1}\right]^{\mathsf{T}} \otimes \mathbf{I}_N\right\}\\ &\quad\times\left\{\left[\mathbf{S}^{\mathsf{H}}(\mathbf{S}\mathbf{S}^{\mathsf{H}})^{-1}\right]^{\mathsf{T}} \otimes \mathbf{I}_N\right\}^{\mathsf{H}}\\ &= \left[(\mathbf{S}\mathbf{S}^{\mathsf{H}})^{-1}\right]^{\mathsf{T}} \otimes \mathbf{I}_N = \boldsymbol{\Phi}.\end{aligned} \tag{23}$$

Accordingly, $\mathrm{vec}(\mathbf{E}_{k,t})$ has a Gaussian distribution with zero mean and covariance matrix $\boldsymbol{\Phi}_k$, where $\boldsymbol{\Phi}_k$ is a $LN \times LN$ matrix that corresponds to the kth diagonal block of $\boldsymbol{\Phi}$. It is trivial to show that $\mathrm{vec}(\mathbf{E}_{k,t-1})$ and $\mathrm{vec}(\mathbf{E}_{k,t})$ are i.i.d. Therefore, $\mathrm{vec}(\boldsymbol{\mathcal{E}}_{k,t})$ has a Gaussian distribution with zero mean and covariance matrix $\mathbf{I}_2 \otimes \boldsymbol{\Phi}_k$.

Next, we examine $\mathrm{vec}(\boldsymbol{\mathcal{A}}_{k,t})$. Note that $\boldsymbol{\mathcal{A}}_{k,t} = \sqrt{\rho_{k,t}}\mathbf{H}_{k,t}\boldsymbol{\mathcal{C}}_{k,t}$ [cf. (12). Using (22) again, we have

$$\mathrm{vec}(\boldsymbol{\mathcal{A}}_{k,t}) = \sqrt{\rho_{k,t}}(\boldsymbol{\mathcal{C}}_{k,t}^{\mathsf{T}} \otimes \mathbf{I}_N)\mathrm{vec}(\mathbf{H}_{k,t}). \tag{24}$$

The fact that $\mathrm{vec}(\mathbf{H}_{k,t})$ is Gaussian with zero mean and covariance matrix $\mathbf{I}_{MN}$ implies that $\mathrm{vec}(\boldsymbol{\mathcal{A}}_{k,t})$ is also Gaussian with zero mean and covariance matrix

$$\begin{aligned}E\left\{\mathrm{vec}(\boldsymbol{\mathcal{A}}_{k,t})\mathrm{vec}^{\mathsf{H}}(\boldsymbol{\mathcal{A}}_{k,t})\right\} =&\rho_{k,t}(\boldsymbol{\mathcal{C}}_{k,t}^{\mathsf{T}} \otimes \mathbf{I}_N)(\boldsymbol{\mathcal{C}}_{k,t}^{\mathsf{T}} \otimes \mathbf{I}_N)^{\mathsf{H}}\\ =&\rho_{k,t}(\boldsymbol{\mathcal{C}}_{k,t}^{\mathsf{H}}\boldsymbol{\mathcal{C}}_{k,t})^{\mathsf{T}} \otimes \mathbf{I}_N.\end{aligned} \tag{25}$$

Hence, $\mathrm{vec}(\hat{\boldsymbol{\mathcal{A}}}_{k,t})$ is the sum of two zero-mean Gaussian random vectors that are independent of each other (since the channel coefficients and the noise samples are independent). It follows that $\mathrm{vec}(\hat{\boldsymbol{\mathcal{A}}}_{k,t})$ also has a Gaussian distribution with zero mean and covariance matrix given by (16), and the proof is complete. ■

1) Differential Receiver A: With the distribution of $\mathrm{vec}(\hat{\boldsymbol{\mathcal{A}}}_{k,t})$ at hand, the ML detector based on $\mathrm{vec}(\hat{\boldsymbol{\mathcal{A}}}_{k,t})$ is given by (e.g., [31])

$$\begin{aligned}\hat{\mathbf{G}}_{k,t} =&\arg\max_{\mathbf{G}_{k,t}\in\mathcal{G}}\Big\{-\ln|\boldsymbol{\Sigma}_{k,t}|\\ &\qquad - \mathrm{tr}\left\{\boldsymbol{\Sigma}_{k,t}^{-1}\mathrm{vec}(\hat{\boldsymbol{\mathcal{A}}}_{k,t})\mathrm{vec}^{\mathsf{H}}(\hat{\boldsymbol{\mathcal{A}}}_{k,t})\right\}\Big\}\\ k =&1, \ldots, K.\end{aligned} \tag{26}$$

The above receiver requires an estimate of the received signal power $\rho_{k,t}$ [see (16)], which can be obtained as follows. Note that

$$\begin{aligned}\mathrm{tr}\{\boldsymbol{\mathcal{A}}_{k,t}\boldsymbol{\mathcal{A}}_{k,t}^{\mathsf{H}}\} =&\rho_{k,t}\ \mathrm{tr}\{\mathbf{H}_{k,t}\boldsymbol{\mathcal{C}}_{k,t}\boldsymbol{\mathcal{C}}_{k,t}^{\mathsf{H}}\mathbf{H}_{k,t}^{\mathsf{H}}\}\\ =&2M\rho_{k,t}\ \mathrm{tr}\{\mathbf{H}_{k,t}\mathbf{H}_{k,t}^{\mathsf{H}}\}\\ =&2M\rho_{k,t}||\mathbf{H}_{k,t}||_F^2\end{aligned} \tag{27}$$

where we have used the fact that [see (17)]

$$\boldsymbol{\mathcal{C}}_{k,t}\boldsymbol{\mathcal{C}}_{k,t}^{\mathsf{H}} = [\mathbf{C}_{k,t-1},\ \mathbf{C}_{k,t}]\begin{bmatrix}\mathbf{C}_{k,t-1}^{\mathsf{H}}\\ \mathbf{C}_{k,t}^{\mathsf{H}}\end{bmatrix} = \mathbf{C}_{k,t-1}\mathbf{C}_{k,t-1}^{\mathsf{H}} + \mathbf{C}_{k,t}\mathbf{C}_{k,t}^{\mathsf{H}} = 2M\mathbf{I}_M \quad (28)$$

due to the unitary structure of the code matrices [cf. (5)]. Taking the expectation of (27), while noting the fading coefficients in $\mathbf{H}_{k,t}$ are i.i.d., yields

$$E\left\{\mathrm{tr}\{\boldsymbol{\mathcal{A}}_{k,t}\boldsymbol{\mathcal{A}}_{k,t}^{\mathsf{H}}\}\right\} = 2M^2N\rho_{k,t}. \quad (29)$$

It follows that an estimate of $\rho_{k,t}$ is given by

$$\hat{\rho}_{k,t} = \frac{1}{2M^2N}\mathrm{tr}\{\hat{\boldsymbol{\mathcal{A}}}_{k,t}\hat{\boldsymbol{\mathcal{A}}}_{k,t}^{H}\}. \quad (30)$$

To summarize, Differential Receiver A consists of the following steps.

1) Compute $\hat{\mathbf{A}}_{t-1}$ and $\hat{\mathbf{A}}_t$ by (15), and obtain $\hat{\boldsymbol{\mathcal{A}}}_{k,t} = \left[\hat{\mathbf{A}}_{k,t-1},\ \hat{\mathbf{A}}_{k,t}\right]$ by identifying the kth block of $\hat{\mathbf{A}}_{t-1}$ and $\hat{\mathbf{A}}_t$, respectively.
2) Compute $\hat{\rho}_{k,t}$ by (30).
3) Compute $\mathbf{\Sigma}_{k,t}$ by (16), using $\hat{\rho}_{k,t}$ in place of $\rho_{k,t}$, for each member of the information code matrix set $\mathcal{G}$; choose $\hat{\mathbf{G}}_{k,t}$ as the one that maximizes (26).

2) Differential Receiver B: Differential Receiver A has to compute the $2LN \times 2LN$ covariance matrix $\mathbf{\Sigma}_{k,t}$ as well as its determinant and inverse for every t and every space-time code matrix in $\mathcal{G}$, which may be computationally involved especially for high transmission rate with a large value of L and/or N. When orthogonal spreading codes are used, we can see

$$\mathbf{\Phi}_k = \mathbf{I}_{LN}. \quad (31)$$

In this case, Differential Receiver A can be significantly simplified, as shown in the following result.

Theorem 2: When orthogonal spreading codes are used, the ML detector based on $\hat{\boldsymbol{\mathcal{A}}}_{k,t}$ is given by

$$\hat{\mathbf{G}}_{k,t} = \arg\max_{\mathbf{G}_{k,t}\in\mathcal{G}} \Re\left[\mathrm{tr}\left\{\mathbf{G}_{k,t}\hat{\mathbf{A}}_{k,t}^{\mathsf{H}}\hat{\mathbf{A}}_{k,t-1}\right\}\right] \quad k = 1,\ldots,K. \quad (32)$$

Proof: First, we write

$$\mathbf{\Sigma}_{k,t} = \mathbf{I}_{2LN} + \rho_{k,t}(\boldsymbol{\mathcal{C}}_{k,t}^{\mathsf{H}}\boldsymbol{\mathcal{C}}_{k,t})^{\mathsf{T}} \otimes \mathbf{I}_N = \mathbf{I}_{2LN} + \mathbf{\Gamma}_{k,t}\mathbf{\Gamma}_{k,t}^{\mathsf{H}} \quad (33)$$

where

$$\mathbf{\Gamma}_{k,t} \triangleq \sqrt{\rho_{k,t}}(\boldsymbol{\mathcal{C}}_{k,t}^{\mathsf{T}} \otimes \mathbf{I}_N). \quad (34)$$

Observe that

$$\begin{aligned}\left|\mathbf{\Sigma}_{k,t}\right| &= \left|\mathbf{I}_{2LN} + \mathbf{\Gamma}_{k,t}\mathbf{\Gamma}_{k,t}^{\mathsf{H}}\right| = \left|\mathbf{I}_{MN} + \mathbf{\Gamma}_{k,t}^{\mathsf{H}}\mathbf{\Gamma}_{k,t}\right| \\ &= \left|\mathbf{I}_{MN} + \rho_{k,t}(\boldsymbol{\mathcal{C}}_{k,t}\boldsymbol{\mathcal{C}}_{k,t}^{\mathsf{H}})^{\mathsf{T}} \otimes \mathbf{I}_N\right| \\ &= \left|\mathbf{I}_{MN} + 2M\rho_{k,t}\mathbf{I}_{MN}\right| = (1 + 2M\rho_{k,t})^{MN} \end{aligned} \quad (35)$$

where in the second equality, we used the fact that $|\mathbf{I} + \mathbf{XY}| = |\mathbf{I} + \mathbf{YX}|$ for arbitrary matrices $\mathbf{X}$ and $\mathbf{Y}$ of appropriate sizes, and in the fourth equality, we used (28). As such, $|\mathbf{\Sigma}_{k,t}|$ is independent of the information code matrix $\mathbf{G}_{k,t}$ and can be dropped from (26).

Applying the matrix inversion lemma (e.g., [21]), we have

$$\begin{aligned}\mathbf{\Sigma}_{k,t}^{-1} &= (\mathbf{I}_{2LN} + \mathbf{\Gamma}_{k,t}\mathbf{\Gamma}_{k,t}^{\mathsf{H}})^{-1} \\ &= \mathbf{I}_{2LN} - \mathbf{\Gamma}_{k,t}(\mathbf{I}_{MN} + \mathbf{\Gamma}_{k,t}^{\mathsf{H}}\mathbf{\Gamma}_{k,t})^{-1}\mathbf{\Gamma}_{k,t}^{\mathsf{H}} \\ &= \mathbf{I}_{2LN} - \mathbf{\Gamma}_{k,t}(\mathbf{I}_{MN} + 2M\rho_{k,t}\mathbf{I}_{MN})^{-1}\mathbf{\Gamma}_{k,t}^{\mathsf{H}} \\ &= \mathbf{I}_{2LN} - \frac{1}{1 + 2M\rho_{k,t}}\mathbf{\Gamma}_{k,t}\mathbf{\Gamma}_{k,t}^{\mathsf{H}}. \end{aligned} \quad (36)$$

Substituting (36) into (26), while dropping all constant terms (including $\ln|\mathbf{\Sigma}_{k,t}|$), we have

$$\begin{aligned}\hat{\mathbf{G}}_{k,t} &= \arg\max_{\mathbf{G}_{k,t}\in\mathcal{G}} \mathrm{tr}\left\{\mathbf{\Gamma}_{k,t}\mathbf{\Gamma}_{k,t}^{\mathsf{H}}\mathrm{vec}(\hat{\boldsymbol{\mathcal{A}}}_{k,t})\mathrm{vec}^{\mathsf{H}}(\hat{\boldsymbol{\mathcal{A}}}_{k,t})\right\} \\ &= \arg\max_{\mathbf{G}_{k,t}\in\mathcal{G}} \mathrm{tr}\left\{\left[(\boldsymbol{\mathcal{C}}_{k,t}^{\mathsf{H}}\boldsymbol{\mathcal{C}}_{k,t})^{\mathsf{T}} \otimes \mathbf{I}_N\right] \times\mathrm{vec}(\hat{\boldsymbol{\mathcal{A}}}_{k,t})\mathrm{vec}^{\mathsf{H}}(\hat{\boldsymbol{\mathcal{A}}}_{k,t})\right\} \end{aligned} \quad (37)$$

where in the second equality, we used $\mathbf{\Gamma}_{k,t} \triangleq \sqrt{\rho_{k,t}}(\boldsymbol{\mathcal{C}}_{k,t}^{\mathsf{T}} \otimes \mathbf{I}_N)$. Applying (22) again yields

$$\left[(\boldsymbol{\mathcal{C}}_{k,t}^{\mathsf{H}}\boldsymbol{\mathcal{C}}_{k,t})^{\mathsf{T}} \otimes \mathbf{I}_N\right]\mathrm{vec}(\hat{\boldsymbol{\mathcal{A}}}_{k,t}) = \mathrm{vec}(\hat{\boldsymbol{\mathcal{A}}}_{k,t}\boldsymbol{\mathcal{C}}_{k,t}^{\mathsf{H}}\boldsymbol{\mathcal{C}}_{k,t}). \quad (38)$$

It follows that

$$\begin{aligned}&\mathrm{tr}\left\{\left[(\boldsymbol{\mathcal{C}}_{k,t}^{\mathsf{H}}\boldsymbol{\mathcal{C}}_{k,t})^{\mathsf{T}} \otimes \mathbf{I}_N\right]\mathrm{vec}(\hat{\boldsymbol{\mathcal{A}}}_{k,t})\mathrm{vec}^{\mathsf{H}}(\hat{\boldsymbol{\mathcal{A}}}_{k,t})\right\} \\ &= \mathrm{tr}\left\{\mathrm{vec}(\hat{\boldsymbol{\mathcal{A}}}_{k,t}\boldsymbol{\mathcal{C}}_{k,t}^{\mathsf{H}}\boldsymbol{\mathcal{C}}_{k,t})\mathrm{vec}^{\mathsf{H}}(\hat{\boldsymbol{\mathcal{A}}}_{k,t})\right\} \\ &= \mathrm{vec}^{\mathsf{H}}(\hat{\boldsymbol{\mathcal{A}}}_{k,t})\mathrm{vec}(\hat{\boldsymbol{\mathcal{A}}}_{k,t}\boldsymbol{\mathcal{C}}_{k,t}^{\mathsf{H}}\boldsymbol{\mathcal{C}}_{k,t}) \\ &= \mathrm{tr}\left\{\hat{\boldsymbol{\mathcal{A}}}_{k,t}^{\mathsf{H}}\hat{\boldsymbol{\mathcal{A}}}_{k,t}\boldsymbol{\mathcal{C}}_{k,t}^{\mathsf{H}}\boldsymbol{\mathcal{C}}_{k,t}\right\} \end{aligned} \quad (39)$$

where, in the last equality, we used the fact that $\mathrm{tr}\{\mathbf{X}^{\mathsf{H}}\mathbf{Y}) = \mathrm{vec}^{\mathsf{H}}(\mathbf{X})\mathrm{vec}(\mathbf{Y})$ for arbitrary matrices $\mathbf{X}$ and $\mathbf{Y}$ of appropriate sizes. Next, we observe that

$$\hat{\boldsymbol{\mathcal{A}}}_{k,t}^{\mathsf{H}}\hat{\boldsymbol{\mathcal{A}}}_{k,t} = \begin{bmatrix}\hat{\mathbf{A}}_{k,t-1}^{\mathsf{H}}\hat{\mathbf{A}}_{k,t-1} & \hat{\mathbf{A}}_{k,t-1}^{\mathsf{H}}\hat{\mathbf{A}}_{k,t} \\ \hat{\mathbf{A}}_{k,t}^{\mathsf{H}}\hat{\mathbf{A}}_{k,t-1} & \hat{\mathbf{A}}_{k,t}^{\mathsf{H}}\hat{\mathbf{A}}_{k,t}\end{bmatrix} \quad (40)$$

$$\boldsymbol{\mathcal{C}}_{k,t}^{\mathsf{H}}\boldsymbol{\mathcal{C}}_{k,t} = \begin{bmatrix}\mathbf{C}_{k,t-1}^{\mathsf{H}}\\ \mathbf{C}_{k,t}^{\mathsf{H}}\end{bmatrix}[\mathbf{C}_{k,t-1}, \mathbf{C}_{k,t}] = M\begin{bmatrix}\mathbf{I}_L & \mathbf{G}_{k,t} \\ \mathbf{G}_{k,t}^{\mathsf{H}} & \mathbf{I}_L\end{bmatrix}. \quad (41)$$

Using (40) and (41) in (39), we obtain

$$\begin{aligned}&\mathrm{tr}\left\{\hat{\boldsymbol{\mathcal{A}}}_{k,t}^{\mathsf{H}}\hat{\boldsymbol{\mathcal{A}}}_{k,t}\boldsymbol{\mathcal{C}}_{k,t}^{\mathsf{H}}\boldsymbol{\mathcal{C}}_{k,t}\right\} \\ &= M\,\mathrm{tr}\left\{\hat{\mathbf{A}}_{k,t-1}^{\mathsf{H}}\hat{\mathbf{A}}_{k,t-1} + \hat{\mathbf{A}}_{k,t-1}^{\mathsf{H}}\hat{\mathbf{A}}_{k,t}\mathbf{G}_{k,t}^{\mathsf{H}} + \hat{\mathbf{A}}_{k,t}^{\mathsf{H}}\hat{\mathbf{A}}_{k,t-1}\mathbf{G}_{k,t} + \hat{\mathbf{A}}_{k,t}^{\mathsf{H}}\hat{\mathbf{A}}_{k,t}\right\}. \end{aligned} \quad (42)$$

Dropping the terms in (42), which are independent of $\mathbf{G}_{k,t}$, while using the identity $\mathbf{X} + \mathbf{X}^{\mathsf{H}} = 2\Re[\mathbf{X}]$, one immediately sees that (37) reduces to (32). ■

The receiver (32) is identical to (26), or Differential Receiver A, for orthogonal spreading codes. When nonorthogonal spreading codes are used, the two receivers are generally different, and the former is referred to as Differential Receiver B in that case. It makes sense to consider Differential Receiver B for nonorthogonal spreading codes not only because it is computationally more efficient but also because spreading codes in real systems are typically *near-orthogonal*, in which case, Differential Receiver B yields a performance very close to that of Differential Receiver A; see Section V for numerical comparison.

We summarize Differential Receiver B as follows.

1) Compute $\hat{\mathbf{A}}_{t-1}$ and $\hat{\mathbf{A}}_t$ by (15), and obtain $\hat{\mathbf{A}}_{k,t-1}$ and $\hat{\mathbf{A}}_{k,t}$ as the kth block of $\hat{\mathbf{A}}_{t-1}$ and $\hat{\mathbf{A}}_t$, respectively.
2) Evaluate (32) for each member of the information code matrix set $\mathcal{G}$, and choose $\hat{\mathbf{G}}_{k,t}$ as the one that maximizes (32).

Remark 1: Note that Differential Receiver B does not need an estimate of the received signal power $\rho_{k,t}$, which is, however, required by Differential Receiver A.

Remark 2: A close relationship exists between the proposed *multiuser* Differential Receiver B and the *single-user* DSTM receiver introduced in [9]. Specifically, [9, eq. (16)] is produced as follows for easy reference (using our notation):

$$\hat{\mathbf{G}}_{k,t} = \arg \max_{\mathbf{G}_{k,t} \in \mathcal{G}} \Re \left[\mathrm{tr}\{\mathbf{G}_{k,t} \mathbf{Y}_t^{\mathsf{H}} \mathbf{Y}_{t-1}\} \right]. \tag{43}$$

A comparison of (32) and (43) indicates that $\hat{\mathbf{A}}_{k,t}$ and $\hat{\mathbf{A}}_{k,t-1}$, which correspond to the output of the decorrelator $\mathbf{S}^{\mathsf{H}}(\mathbf{S}\mathbf{S}^{\mathsf{H}})^{-1}$ when the input is the raw data $\mathbf{Y}_t$ and $\mathbf{Y}_{t-1}$, respectively [see (15)], play the same role as the raw data $\mathbf{Y}_{t-1}$ and $\mathbf{Y}_t$ in the single-user DSTM receiver. Hence, *the multiuser Differential Receiver B may be interpreted as consisting of a two-step process: with the first step of decorrelating by* $\mathbf{S}^{\mathsf{H}}(\mathbf{S}\mathbf{S}^{\mathsf{H}})^{-1}$, *which removes the MUI, followed by the single-user DSTM receiver, which detects the information code matrix* $\mathbf{G}_{k,t}$.

Remark 3: The unstructured estimate $\hat{\mathbf{A}}_t$ in (15) is a consistent (for high SNR) estimate of $\mathbf{A}_t$. When the SNR $\to \infty$, we have $\hat{\mathbf{A}}_t \to \mathbf{A}_t$, as one can see from (19). As a result, the proposed differential receivers are also consistent, and their error probability decreases as the SNR increases.

Remark 4: The proposed differential receivers introduced here can also be considered as extensions of the noncoherent decorrelating receivers in previous works [16]–[19] for the single transmit antenna case.

B. Coherent Detection

When the CSI is available at the receiver, by a procedure similar to the proof of Theorem 1, one can show that the conditional distribution of $\mathrm{vec}(\hat{\mathbf{A}}_{k,t})$ is complex Gaussian with mean $\mathrm{vec}(\mathbf{A}_{k,t})$ and covariance matrix $\mathbf{\Phi}_k$, where $\mathbf{\Phi}_k \in \mathbb{C}^{LN \times LN}$ is the kth diagonal block of $\mathbf{\Phi}$ given by (18). Maximizing the distribution of $\mathrm{vec}(\hat{\mathbf{A}}_{k,t})$ with respect to the coding matrix $\mathbf{C}_{k,t}$ yields the following coherent receiver:

$$\begin{aligned}\hat{\mathbf{C}}_{k,t} =& \arg \min_{\mathbf{C}_{k,t} \in \mathcal{C}} \left\{ \mathrm{vec}^{\mathsf{H}} \left(\hat{\mathbf{A}}_{k,t} - \sqrt{\rho_{k,t}} \mathbf{H}_{k,t} \mathbf{C}_{k,t} \right) \mathbf{\Phi}_k^{-1} \right. \\ & \left. \times \mathrm{vec} \left(\hat{\mathbf{A}}_{k,t} - \sqrt{\rho_{k,t}} \mathbf{H}_{k,t} \mathbf{C}_{k,t} \right) \right\} \\ k =& 1, \ldots, K. \end{aligned} \tag{44}$$

When orthogonal spreading codes are used, the receiver (44) in reduces to

$$\begin{aligned}\hat{\mathbf{C}}_{k,t} =& \arg \min_{\mathbf{C}_{k,t} \in \mathcal{C}} \left\| \hat{\mathbf{A}}_{k,t} - \sqrt{\rho_{k,t}} \mathbf{H}_{k,t} \mathbf{C}_{k,t} \right\|_F^2 \\ k =& 1, \ldots, K. \end{aligned} \tag{45}$$

Finally, we remark that the above coherent receivers may be thought of as extensions of prior works (e.g., [20]) on decorrelating coherent receivers for the case of one transmit antenna.

V. Simulations

We consider a $K = 10$ synchronous CDMA system equipped with $M = 2$ transmit antennas and $N = 1$ or 2 receive antennas. For differential modulation/detection, we utilize the unitary differential space-time code given by (3) with BPSK constellation; for coherent transmission/detection, the Alamouti's space-time code [5] with BPSK constellation is employed. We simulate a near–far environment, where the power level of all (nine) interfering users is 10 dB higher than that of the first (desired) user. Nonorthogonal Gold codes $(J = 63)$ are used for spreading. We consider independent Rayleigh fading channels, i.e., the fading coefficients are generated as i.i.d. complex Gaussian random variables with zero mean and unit variance.

We first consider differential modulation/detection. In particular, we evaluate the bit error rate (BER) of the following receivers:

- conventional decorrelating differential BPSK (DBPSK) receiver (see, e.g., [16], [17]) with Gold codes and $M = 1$ transmit antenna;
- Differential Receiver A (DRA) with Gold codes;
- Differential Receiver B (DRB) with Gold codes;
- DBPSK receiver in single-user environments (SU-DBPSK) with MN diversity channels [23, p. 783], which effectively serves as a lower bound on our multiuser differential system.

The BERs of these receivers are shown in Fig. 1(a) for $N = 1$ receive antenna and Fig. 1(b) for $N = 2$ receive antennas. We first note that DRA and DRB with nonorthogonal Gold codes yield nearly identical BERs; hence, the computationally more efficient DRB should be preferred if complexity is a concern. It is seen that the multitransmit-antenna based, space-time-coded DRA and DRB offer a substantial performance gain over the conventional single-transmit-antenna-based DBPSK system. For example, at BER $= 10^{-3}$, an improvement of approximately 10 dB (respectively, 5 dB) is obtained by DRA/DRB with $N = 1$ (resp. $N = 2$) receive antenna(s). This clearly motivates the use of multiantenna transmission and space-time coding in fading channels. We also note that the BERs of DRA and DRB are very close to that of the SU-DBPSK, with a difference less than 1 dB. Finally, a comparison of Fig. 1(a) and (b) indicates that a performance gain of close to 7 dB at BER $= 10^{-3}$ is achieved with $N = 2$ over $N = 1$ receive antenna(s). Hence, whenever feasible, receive diversity should be pursued.

We next consider coherent transmission/detection. The following receivers are examined:

- conventional decorrelating coherent BPSK (CBPSK) receiver (see, e.g, [20]) with Gold codes and $M = 1$ transmit antenna;
- proposed coherent receiver (CR) with Gold codes;
- CBPSK receiver in single-user environments (SU-CBPSK) with MN diversity channels [23, p.

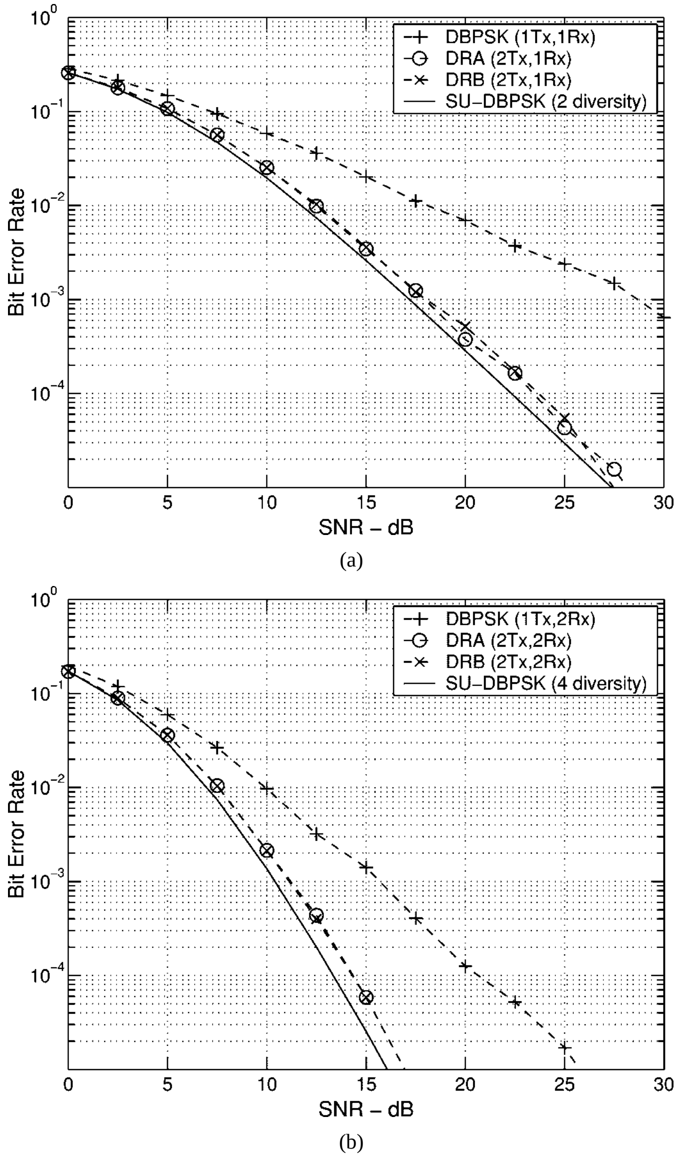

Fig. 1. BERs of conventional and proposed differential receivers in Rayleigh fading channels. (a) $N = 1$ receive antenna. (b) $N = 2$ receive antennas.

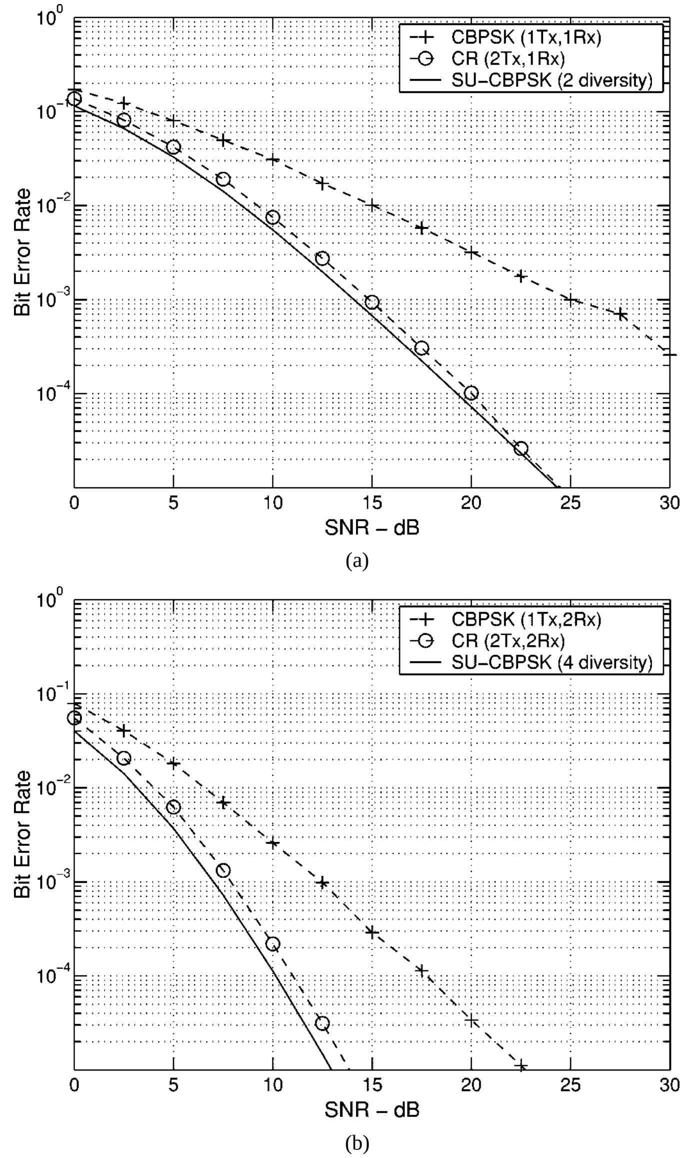

Fig. 2. BERs of conventional and proposed coherent receivers in Rayleigh fading channels. (a) $N = 1$ receive antenna. (b) $N = 2$ receive antennas.

781], which serves as a lower bound on our multiuser coherent system.

Fig. 2(a) and (b) depict the BERs of the above coherent receivers for $N = 1$ and $N = 2$ receive antennas, respectively. The multi-transmit-antenna-based, space-time-coded CR yields a significantly improved performance over the single-transmit-antenna based CBPSK system, with a gain of about 10 dB (resp. 5 dB) for $N = 1$ (resp. $N = 2$) at BER $= 10^{-3}$, similarly to the differential receivers. We also note that the BER of CR is very close to that of SU-CBPSK. As for receive diversity, CR with $N = 2$ outperforms CR with $N = 1$ by approximately 7 dB at BER $= 10^{-3}$.

Finally, we compare the differential and coherent receivers. Fig. 3(a) and (b) depict the BERs of DBPSK, CBPSK, DRA, and CR with Gold codes, $N = 1$ and, respectively, $N = 2$ receive antennas. It is well known that coherent detection outperforms differential detection by approximately 3 dB in single-user systems (e.g., [23]). This is also observed in the multiuser single-transmit-antenna systems (i.e., CBPSK and DBPSK) as well as our multiuser multitransmit-antenna systems (i.e., CR and DRA). At the cost of increased complexity, the performance loss of the differential receivers can be reduced by including multiple code matrices for joint detection, similarly to the use of multiple symbols for detecting scalar DPSK [29].

VI. Conclusions

We have presented two differential receivers and one coherent receiver for space-time-coded CDMA systems that utilize the DSCM transmission. The proposed differential/coherent receivers are MUI and (time-varying) fading resistant. These multitransmit-antenna based, space-time-coded transmission/reception schemes have been shown to significantly outperform the conventional single-transmit-antenna-based schemes. The proposed coherent receiver requires only the CSI of the desired transmission and can be used to yield about 3-dB performance gain over the proposed differential receivers

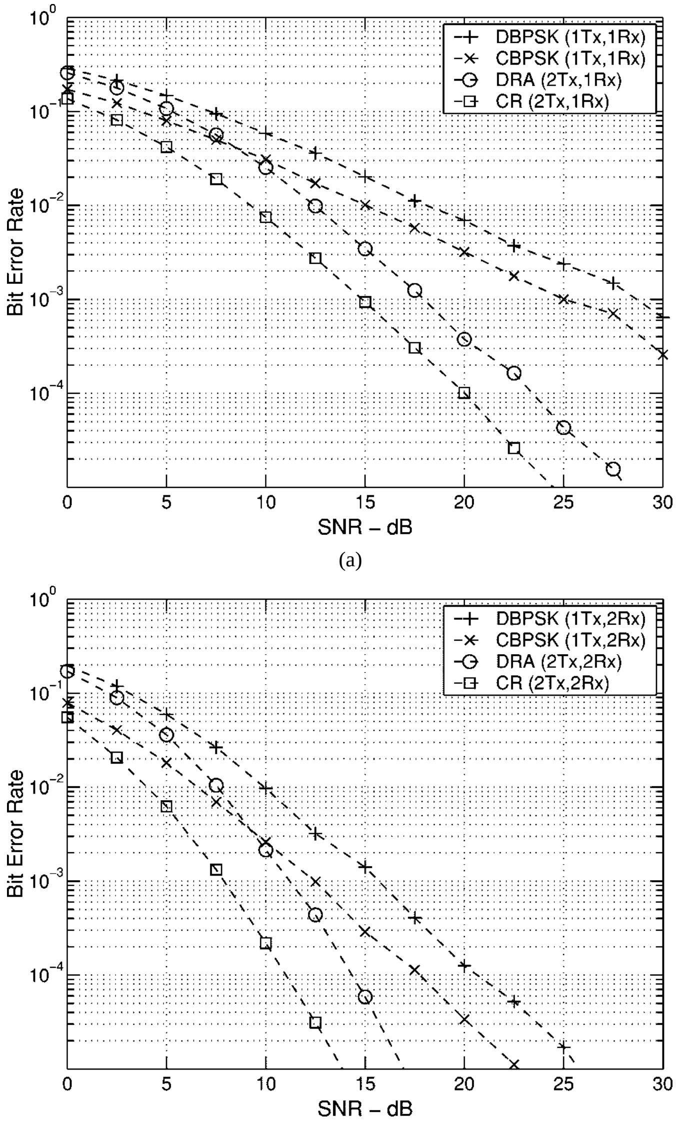

Fig. 3. Comparison of differential and coherent receivers in Rayleigh fading channels. (a) $N = 1$ receive antenna. (b) $N = 2$ receive antennas.

when reliable channel estimates can be obtained at the receiver (e.g., in slowly fading channels). Future research directions include extending the receivers to multicarrier CDMA systems to handle frequency-selective multipath fading and to utilize multiple code matrices for joint detection.

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