Channel Compensation for Reciprocal TDD Massive MIMO-OFDM With IQ Imbalance

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***Abstract*—Massive multiple-input multiple-output (MIMO) orthogonal frequency division multiplexing (OFDM) systems are sensitive to in-phase and quadrature (IQ) imbalances, which destroys the reciprocity of time-division duplex (TDD) uplink/downlink channels with potentially large performance degradation. In this letter, we investigate the compensation of channel reciprocity for TDD massive MIMO-OFDM systems in the presence of IQ imbalances at both the transmitter and receiver. A frequency-domain compensation scheme is proposed, which uses compensation matrices to precode uplink and downlink transmissions. By disassembling the effective channel model into pairs of mirror subcarriers and utilizing block matrix inversion, the compensation matrices can be calculated with a complexity that is linear in the number of antennas at the base station and mobile station. Simulations verify the effectiveness of the proposed scheme.**

***Index Terms*—IQ imbalance, TDD, reciprocity of channel, massive MIMO, OFDM.**

I. Introduction

Owing to its extremely high spectral and energy efficiencies, massive multiple-input multiple-output (MIMO) has been widely considered an important enabling technique for the fifth-generation (5G) cellular communication [1], [2]. Since massive MIMO systems are equipped with a large number of base station (BS) antennas, it is impractical for the mobile station (MS) to estimate each downlink channel link and feedback the channel state information to the BS. For time-division duplex (TDD) systems with reciprocal uplink and downlink channels, uplink pilot transmission can be used to estimate the uplink channel. Then the BS can make use of the uplink channel estimation result for downlink signal processing. So the TDD mode is a popular choice for massive MIMO systems [3].

The effective communication channel consists of not only the physical channel but also the radio-frequency (RF) transceivers and antennas. Due to limitations of analog components, there are unavoidable phase and amplitude mismatches between the in-phase and quadrature branches in the RF transceivers, which are usually referred to as in-phase and quadrature (IQ) imbalances [4]. It has been found that IQ imbalances tend to be more severe in massive MIMO systems [5]. Such Tx/Rx mismatches would destroy the reciprocity between the effective uplink and downlink channels, leading to performance degradation of reciprocity-based systems [6], [7]. Thus the compensation of the effective channel is necessary in order to exploit the channel reciprocity for downlink transmission in TDD MIMO systems with IQ imbalances. The effect of transceiver non-reciprocity in TDD systems was studied and various reciprocity compensation schemes were investigated in [7]–[11]. In particular, [7] considered the problem for a narrowband TDD MIMO system and proposed two compensation methods. Compensation techniques in the presence of frequency offsets between the transmitter and receiver were proposed in [8].

We consider herein wideband massive MIMO systems. It is noted that the methods of [7] cannot be applied because IQ imbalances in MIMO-OFDM systems exhibit interferences among mirror subcarriers [4]. Because of such coupling among subcarriers, we cannot treat the wideband MIMO-OFDM system as a collection of narrowband MIMO systems and use methods in, e.g., [7], to deal with the IQ imbalance problem. This makes the compensation of channel reciprocity in MIMO-OFDM systems a more challenging task. To our best knowledge, the compensation of channel reciprocity for TDD MIMO-OFDM systems with IQ imbalances over frequency-selective channels has not been discussed in current literatures. This is the motivation of this letter.

In this letter, a frequency-domain compensation scheme is proposed to restore channel reciprocity for TDD massive MIMO-OFDM systems in the presence of frequency-independent IQ imbalances. The proposed scheme employs two compensation matrices to precode downlink and uplink transmissions, respectively. By disassembling the effective channel model into pairs of mirror subcarriers and utilizing block matrix inversion method, the proposed scheme has a linear computational complexity in the number of antennas at the BS and MS, making it suitable for massive MIMO applications. Simulations verify the effectiveness of the proposed scheme.

Notations: Superscripts $(\cdot)^*$, $(\cdot)^{\mathrm{T}}$, $(\cdot)^{\mathrm{H}}$ denote the conjugate, transpose and conjugate transpose operations, respectively. $\mathbf{F}_N$

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denotes the $N \times N$ unitary FFT matrix. $\mathbf{I}_N$ is the $N \times N$ identity matrix. $\mathbf{0}_{M\times N}$ is the $M \times N$ all zero matrix. $\mathbf{D}(\mathbf{x}) = \text{diag}(\mathbf{x})$ is a diagonal matrix with the vector $\mathbf{x}$ on its main diagonal. Operator $\otimes$, $\odot$ and $\oslash$ denote kronecker product, hadamard product and hadamard division, respectively.

II. Problem Formulation

We consider a massive MIMO-OFDM system, where the BS is equipped with L antennas and the MS is equipped with M antennas. For downlink transmission, the frequency-domain signal transmitted by the BS is denoted by $\mathbf{x} = [\mathbf{x}_1^{\mathrm{T}}, \ \mathbf{x}_2^{\mathrm{T}}, \ldots, \mathbf{x}_L^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{C}^{LN\times 1}$, where $\mathbf{x}_l = [x_{l,1}, \ x_{l,2}, \ldots, \ x_{l,N}]^{\mathrm{T}}$ is the OFDM symbol transmitted at the lth BS antenna and N is the number of OFDM subcarriers. An IFFT operation is performed at each Tx branch and cyclic prefix (CP) of length P is added to produce the time-domain transmitted signal $\bar{\mathbf{x}} = [\bar{\mathbf{x}}_1^{\mathrm{T}}, \ \bar{\mathbf{x}}_2^{\mathrm{T}}, \ldots, \ \bar{\mathbf{x}}_L^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{C}^{L(N+P)\times 1}$. Due to the IQ imbalance at the BS Tx, the equivalent transmitted signal is [5], [7]:

$$\bar{\mathbf{x}}_{\text{iqi}} = (\mathbf{D}(\mathbf{u}_{\text{bt}}) \otimes \mathbf{I}_{N+P})\bar{\mathbf{x}} + (\mathbf{D}(\mathbf{v}_{\text{bt}}) \otimes \mathbf{I}_{N+P})\bar{\mathbf{x}}^* \quad (1)$$

where the IQ imbalance parameters associated with BS transmission (denoted by subscript "bt") are $\mathbf{u}_{\text{bt}} = [\mu_{\text{bt},1}, \ \mu_{\text{bt},2}, \ldots, \ \mu_{\text{bt},L}]^{\mathrm{T}}$, $\mathbf{v}_{\text{bt}} = [\nu_{\text{bt},1}, \ \nu_{\text{bt},2}, \ldots, \ \nu_{\text{bt},L}]^{\mathrm{T}}$, $\mu_{\text{bt},l} = \cos(\theta_{\text{bt},l}/2) - j\alpha_{\text{bt},l}\sin(\theta_{\text{bt},l}/2)$ and $\nu_{\text{bt},l} = \alpha_{\text{bt},l}\cos(\theta_{\text{bt},l}/2) - j\sin(\theta_{\text{bt},l}/2)$, with $\theta_{\text{bt},l}$ and $\alpha_{\text{bt},l}$ denoting the phase and amplitude mismatches between the I and Q branches at the lth BS Tx, respectively. When stated in dB, the amplitude imbalance is computed as $10log(1+\alpha)$. We also define $\mathbf{U}_{\text{bt}} = \mathbf{D}(\mathbf{u}_{\text{bt}}) \otimes \mathbf{I}_N$ and $\mathbf{V}_{\text{bt}} = \mathbf{D}(\mathbf{v}_{\text{bt}}) \otimes \mathbf{I}_N$, which are both $LN \times LN$ diagonal IQ imbalance matrices composed of the IQ imbalance parameters at the BS Tx.

After transmission over the frequency-selective radio channel and discarding the cyclic prefix, the received time-domain baseband signal at the MS is [4]:

$$\bar{\mathbf{y}} = \mathbf{C}\left(\begin{bmatrix}\mathbf{0}_{N\times P}, & \mathbf{I}_N\end{bmatrix} \otimes \mathbf{I}_L\right)\bar{\mathbf{x}}_{\text{iqi}} + \bar{\mathbf{n}} \quad (2)$$

where $\bar{\mathbf{y}} = [\bar{\mathbf{y}}_1^{\mathrm{T}}, \ \bar{\mathbf{y}}_2^{\mathrm{T}}, \ldots, \ \bar{\mathbf{y}}_M^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{C}^{MN\times 1}$. The radio channel matrix $\mathbf{C} \in \mathbb{C}^{MN\times LN}$ is a block matrix consisting of $M \times L$ sub-blocks. The (m, l)-th block $\mathbf{C}_{ml}$ is an $N \times N$ circulant channel matrix with the first column being $[h_{ml,1}, \ h_{ml,2}, \ldots, \ h_{ml,K}, \ 0, \ldots, \ 0]^{\mathrm{T}}$ and the first row as $[h_{ml,1}, \ 0, \ldots, \ 0, \ h_{ml,K}, \ldots, \ h_{ml,3}, \ h_{ml,2}]$, where $[h_{ml,1}, \ h_{ml,2}, \ldots, \ h_{ml,K}]$ is the radio channel impulse response between the lth BS antenna and the mth MS antenna, and K denotes the effective radio channel length. $\bar{\mathbf{n}}$ is an $MN \times 1$ additive white Gaussian noise (AWGN) vector at the MS.

Taking into account the IQ imbalance at the MS, the received time-domain signal is [4]–[6]:

$$\bar{\mathbf{y}}_{\text{iqi}} = \mathbf{U}_{\text{mr}}\bar{\mathbf{y}} + \mathbf{V}_{\text{mr}}\bar{\mathbf{y}}^* \quad (3)$$

where $\mathbf{U}_{\text{mr}}$ and $\mathbf{V}_{\text{mr}} \in \mathbb{C}^{MN\times MN}$ are diagonal matrices composed of IQ imbalance parameters associated with the MS reception (hence the subscript "mr"): $\mathbf{U}_{\text{mr}} = \mathbf{D}(\mathbf{u}_{\text{mr}}) \otimes \mathbf{I}_N$ and $\mathbf{V}_{\text{mr}} = \mathbf{D}(\mathbf{v}_{\text{mr}}) \otimes \mathbf{I}_N$, with $\mathbf{u}_{\text{mr}} = [\mu_{\text{mr},1}, \ \mu_{\text{mr},2}, \ldots, \ \mu_{\text{mr},M}]^{\mathrm{T}}$, $\mathbf{v}_{\text{mr}} = [\nu_{\text{mr},1}, \ \nu_{\text{mr},2}, \ldots, \ \nu_{\text{mr},M}]^{\mathrm{T}}$, $\mu_{\text{mr,m}} = \cos(\theta_{\text{mr,m}}/2) + j\alpha_{\text{mr,m}}\sin(\theta_{\text{mr,m}}/2)$ and $\nu_{\text{mr,m}} = \alpha_{\text{mr,m}}\cos(\theta_{\text{mr,m}}/2) - j\sin(\theta_{\text{mr,m}}/2)$.

By substituting (1), (2) into (3) and taking the FFT operation, we have the following frequency-domain input-output signal relationship of the downlink transmission [4]:

$$\mathbf{y}^{\text{d}} = \underbrace{\left(\mathbf{U}_{\text{mr}}\mathbf{H}\mathbf{U}_{\text{bt}} + \mathbf{V}_{\text{mr}}\mathbf{H}^{\#}\mathbf{V}_{\text{bt}}^*\right)}_{\mathbf{H}^{\text{d},1}}\mathbf{x}^{\text{d}} + \underbrace{\left(\mathbf{U}_{\text{mr}}\mathbf{H}\mathbf{V}_{\text{bt}} + \mathbf{V}_{\text{mr}}\mathbf{H}^{\#}\mathbf{U}_{\text{bt}}^*\right)}_{\mathbf{H}^{\text{d},2}}\mathbf{x}^{\text{d}\#} + \mathbf{w}^{\text{d}} \quad (4)$$

where the superscript "d" is included to signify downlink transmission, $\mathbf{y}^{\text{d}} = [\mathbf{y}_1^{\mathrm{T}}, \ \mathbf{y}_2^{\mathrm{T}}, \ldots, \mathbf{y}_M^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{C}^{MN\times 1}$, $\mathbf{x}^{\#} = [\mathbf{x}_1^{\#\mathrm{T}}, \ \mathbf{x}_2^{\#\mathrm{T}}, \ldots, \ \mathbf{x}_L^{\#\mathrm{T}}]^{\mathrm{T}}$, $\mathbf{x}_i^{\#} = \mathbf{F}_N\bar{\mathbf{x}}_i^*$, $\mathbf{H}$ and $\mathbf{H}^{\#} \in \mathbb{C}^{MN\times LN}$ are block matrices consisting of $M \times L$ diagonal sub-blocks, with the (m, l)-th block $\mathbf{H}_{ml} = \mathbf{F}_N\mathbf{C}_{ml}\mathbf{F}_N^{\mathrm{H}}$ and $\mathbf{H}_{ml}^{\#} = \mathbf{F}_N\mathbf{C}_{ml}^*\mathbf{F}_N^{\mathrm{H}}$, respectively. $\mathbf{w}^{\text{d}} = [\mathbf{w}_1^{\mathrm{T}}, \ \mathbf{w}_2^{\mathrm{T}}, \ldots, \mathbf{w}_M^{\mathrm{T}}]^{\mathrm{T}}$ is the frequency-domain noise which is also impacted by the MS Rx IQ imbalance.

Similarly, for uplink transmission, the frequency-domain input-output signal relationship is [4], [7], [12]:

$$\mathbf{y}^{\text{u}} = \underbrace{\left(\mathbf{U}_{\text{br}}\mathbf{H}^{\mathrm{T}}\mathbf{U}_{\text{mt}} + \mathbf{V}_{\text{br}}\left(\mathbf{H}^{\mathrm{T}}\right)^{\#}\mathbf{V}_{\text{mt}}^*\right)}_{\mathbf{H}^{\text{u},1}}\mathbf{x}^{\text{u}} + \underbrace{\left(\mathbf{U}_{\text{br}}\mathbf{H}^{\mathrm{T}}\mathbf{V}_{\text{mt}} + \mathbf{V}_{\text{br}}\left(\mathbf{H}^{\mathrm{T}}\right)^{\#}\mathbf{U}_{\text{mt}}^*\right)}_{\mathbf{H}^{\text{u},2}}\mathbf{x}^{\text{u}\#} + \mathbf{w}^{\text{u}} \quad (5)$$

where $\mathbf{y}^{\text{u}} \in \mathbb{C}^{LN\times 1}$ is the received uplink signal, $\mathbf{x}^{\text{u}} \in \mathbb{C}^{MN\times 1}$ is the transmitted uplink signal, $\mathbf{w}^{\text{u}} \in \mathbb{C}^{LN\times 1}$ is the noise at the BS Rx. $\mathbf{U}_{\text{br}}$ and $\mathbf{V}_{\text{br}} \in \mathbb{C}^{LN\times LN}$ are diagonal matrices composed of the IQ imbalance parameters associated with the BS reception (hence the subscript "br") , $\mathbf{U}_{\text{mt}}$ and $\mathbf{V}_{\text{mt}} \in \mathbb{C}^{MN\times MN}$ are diagonal matrices composed of IQ imbalance parameters associated with the MS transmission (hence the subscript "mt"). Finally, the superscript u denotes uplink transmission.

It can be seen from (4) and (5), in the presence of BS and MS Tx/Rx IQ imbalances, the IQ imbalance parameters are coupled with the radio channel during signal transmission. In TDD systems, although the downlink radio channel has an identical impulse response as the uplink radio channel in the same coherence interval, the impact of IQ imbalances causes the effective baseband channel to lose the channel reciprocity between uplink and downlink transmissions. The purpose of this letter is to address this problem and develop a new compensation technique to restore the reciprocity of the effective channel in the presence of IQ imbalances.

III. Compensation of Effective Channel Reciprocity

Referring to (4), it can be shown that the kth element of $\mathbf{x}^{\#}$ is the conjugate of the k'th element of $\mathbf{x}$, where $k' = N-k+2$ [4]. Thus k and k' can be viewed as a pair of mirror subcarriers, among which there are inter-subcarrier interferences caused by IQ imbalances. The downlink system model in (4) can be re-written pairwise as in the following equation to reduce processing dimension:

$$\begin{bmatrix}\mathbf{y}_k^{\text{d}} \\ \mathbf{y}_{k'}^{\text{d}*}\end{bmatrix} = \begin{bmatrix}\mathbf{H}_k^{\text{d},1} & \mathbf{H}_k^{\text{d},2} \\ \mathbf{H}_{k'}^{\text{d},2*} & \mathbf{H}_{k'}^{\text{d},1*}\end{bmatrix}\begin{bmatrix}\mathbf{x}_k^{\text{d}} \\ \mathbf{x}_{k'}^{\text{d}*}\end{bmatrix} + \begin{bmatrix}\mathbf{w}_k^{\text{d}} \\ \mathbf{w}_{k'}^{\text{d}*}\end{bmatrix} \quad (6)$$

where $\mathbf{y}_k^{\text{d}} = [y_{1,k}, \ y_{2,k}, \ldots, \ y_{M,k}]^{\mathrm{T}}$, $\mathbf{x}_k^{\text{d}} = [x_{1,k}, \ x_{2,k}, \ldots, \ x_{L,k}]^{\mathrm{T}}$, $\mathbf{H}_k^{\text{d},1} \in \mathbb{C}^{M\times L}$ with its mth row constructed as $[\mathbf{H}^{d,1}((m-1)N+k,k),\ \mathbf{H}^{d,1}((m-1)N+k,N+k),\ldots,\ \mathbf{H}^{d,1}((m-1)N+k,(L-1)N+k)]$, $\mathbf{H}_k^{d,2} \in \mathbb{C}^{M\times L}$, whose mth row is $[\mathbf{H}^{d,2}((m-1)N+k,k),\ \mathbf{H}^{d,2}((m-1)N+k,N+k),\ldots,\ \mathbf{H}^{d,2}((m-1)N+k,(L-1)N+k)]$, $\mathbf{w}_k^d = [w_{1,k},\ w_{2,k},\ldots,\ w_{M,k}]^T$. By separating the Rx IQ imbalance parameters, the radio channel and the Tx IQ imbalance parameters, the above equation can be decomposed as

$$\begin{bmatrix}\mathbf{y}_k^d\\ \mathbf{y}_{k'}^{d*}\end{bmatrix} = \underbrace{\begin{bmatrix}\mathbf{D}(\mathbf{u}_{mr}) & \mathbf{D}(\mathbf{v}_{mr})\\ \mathbf{D}(\mathbf{v}_{mr}^*) & \mathbf{D}(\mathbf{u}_{mr}^*)\end{bmatrix}}_{\mathbf{G}_{mr}} \underbrace{\begin{bmatrix}\mathbf{H}_k & \mathbf{0}_{M\times L}\\ \mathbf{0}_{M\times L} & \mathbf{H}_{k'}^*\end{bmatrix}}_{\tilde{\mathbf{H}}_k^d} \times \underbrace{\begin{bmatrix}\mathbf{D}(\mathbf{u}_{bt}) & \mathbf{D}(\mathbf{v}_{bt})\\ \mathbf{D}(\mathbf{v}_{bt}^*) & \mathbf{D}(\mathbf{u}_{bt}^*)\end{bmatrix}}_{\mathbf{G}_{bt}} \begin{bmatrix}\mathbf{x}_k^d\\ \mathbf{x}_{k'}^{d*}\end{bmatrix} + \begin{bmatrix}\mathbf{w}_k^d\\ \mathbf{w}_{k'}^{d*}\end{bmatrix} \tag{7}$$

where $\mathbf{H}_k \in \mathbb{C}^{M\times L}$ with its mth row constructed as $[\mathbf{H}_{m1}(k,k),\ \mathbf{H}_{m2}(k,k),\ldots,\ \mathbf{H}_{mL}(k,k)]$, $m \in [1,2,\ldots,M]$. For simplicity, the downlink channel corresponding to the subcarrier pair of k and k' is denoted as

$$\mathbf{J}_k^d = \mathbf{G}_{mr}\tilde{\mathbf{H}}_k^d\mathbf{G}_{bt} \tag{8}$$

Following a similar decomposition on (5), the uplink channel associated with subcarriers k and k' can be written as

$$\mathbf{J}_k^u = \mathbf{G}_{br}\tilde{\mathbf{H}}_k^u\mathbf{G}_{mt} \tag{9}$$

where

$$\mathbf{G}_{br} = \begin{bmatrix}\mathbf{D}(\mathbf{u}_{br}) & \mathbf{D}(\mathbf{v}_{br})\\ \mathbf{D}(\mathbf{v}_{br}^*) & \mathbf{D}(\mathbf{u}_{br}^*)\end{bmatrix},\quad \tilde{\mathbf{H}}_k^u = \begin{bmatrix}\mathbf{H}_k^T & \mathbf{0}_{L\times M}\\ \mathbf{0}_{L\times M} & \left(\mathbf{H}_{k'}^T\right)^*\end{bmatrix}$$

and

$$\mathbf{G}_{mt} = \begin{bmatrix}\mathbf{D}(\mathbf{u}_{mt}) & \mathbf{D}(\mathbf{v}_{mt})\\ \mathbf{D}(\mathbf{v}_{mt}^*) & \mathbf{D}(\mathbf{u}_{mt}^*)\end{bmatrix}.$$

We have the following relationship

$$\tilde{\mathbf{H}}_k^u = \begin{bmatrix}\mathbf{H}_k^T & \mathbf{0}_{L\times M}\\ \mathbf{0}_{L\times M} & \left(\mathbf{H}_{k'}^T\right)^*\end{bmatrix} = \begin{bmatrix}\mathbf{H}_k & \mathbf{0}_{M\times L}\\ \mathbf{0}_{M\times L} & \mathbf{H}_{k'}^*\end{bmatrix}^T = \left(\tilde{\mathbf{H}}_k^d\right)^T \tag{10}$$

which is a direct result of the reciprocity between the uplink and downlink radio channels. However, the cascade of the Rx IQ imbalance, radio channel and Tx IQ imbalance causes the effective channel to lose channel reciprocity. To address this issue, we propose to use two compensation matrices $\mathbf{R}_k^u$ and $\mathbf{R}_k^d$ to restore the reciprocity of the effective channel, which satisfy:

$$\mathbf{J}_k^d\mathbf{R}_k^d = \left(\mathbf{J}_k^u\mathbf{R}_k^u\right)^T \tag{11}$$

Effectively, these compensation matrices function as precoding matrices that are applied to the communication signals prior to transmission. The explicit expressions of these compensation matrices can be found by substituting (8) – (10) into (11), yielding

$$\mathbf{R}_k^d = \mathbf{G}_{bt}^{-1}\mathbf{G}_{br}^T \tag{12}$$

$$\mathbf{R}_k^u = \mathbf{G}_{mt}^{-1}\mathbf{G}_{mr}^T \tag{13}$$

In order to obtain the downlink compensation matrix $\mathbf{R}_k^d$, we need to calculate the inverse of $\mathbf{G}_{bt} \in \mathbb{C}^{2L\times 2L}$, where L denotes the number of antennas at the BS. In massive MIMO systems, L is typically very large, which would lead to high computational complexity. However, as shown in (7), $\mathbf{G}_{bt}$ is a block matrix formed by four diagonal matrices. By using the block matrix inversion lemma in [15], $\mathbf{G}_{bt}^{-1}$ can be computed efficiently as

$$\mathbf{G}_{bt}^{-1} = \begin{bmatrix}\Sigma_1^{-1} & -\Sigma_1^{-1}\mathbf{D}(\mathbf{v}_{bt}\oslash\mathbf{u}_{bt}^*)\\ -\mathbf{D}(\mathbf{v}_{bt}^*\oslash\mathbf{u}_{bt}^*)\Sigma_1^{-1} & \Sigma_2\end{bmatrix} \tag{14}$$

where $\Sigma_1 \triangleq \mathbf{D}(\mathbf{u}_{bt}) - \mathbf{D}(\mathbf{v}_{bt}\odot\mathbf{v}_{bt}^*\oslash\mathbf{u}_{bt}^*)$ and $\Sigma_2 \triangleq \mathbf{D}(\mathbf{u}_{bt}^*)^{-1} + \mathbf{D}(\mathbf{v}_{bt}^*\oslash\mathbf{u}_{bt}^*)\Sigma_1^{-1}\mathbf{D}(\mathbf{v}_{bt}\oslash\mathbf{u}_{bt}^*)$. By substituting $\mathbf{u}_{bt}$ and $\mathbf{v}_{bt}$ into (14), we have

$$\mathbf{G}_{bt}^{-1} = \begin{bmatrix}\mathbf{D}(\mathbf{a}) & \mathbf{D}(\mathbf{b})\\ \mathbf{D}(\mathbf{b}^*) & \mathbf{D}(\mathbf{a}^*)\end{bmatrix} \tag{15}$$

where $\mathbf{a}$ and $\mathbf{b}$ are $L\times 1$ vectors, with the lth element given by

$$a_l = \mu_{bt,l}^*/\left(\|\mu_{bt,l}\|^2 - \|\nu_{bt,l}\|^2\right)$$
$$b_l = -\nu_{bt,l}/\left(\|\mu_{bt,l}\|^2 - \|\nu_{bt,l}\|^2\right)$$

Finally, using (15) in (12), we obtain the compensation matrix for downlink transmission as:

$$\mathbf{R}_k^d = \begin{bmatrix}\mathbf{D}(\mathbf{c}) & \mathbf{D}(\mathbf{d})\\ \mathbf{D}(\mathbf{d}^*) & \mathbf{D}(\mathbf{c}^*)\end{bmatrix} \tag{16}$$

where $\mathbf{c}$ and $\mathbf{d}$ are $L\times 1$ vectors, with the lth element given by

$$c_l = \left(\mu_{bt,l}^*\mu_{br,l} - \nu_{bt,l}\nu_{br,l}\right)/\left(\|\mu_{bt,l}\|^2 - \|\nu_{bt,l}\|^2\right)$$
$$d_l = \left(\mu_{bt,l}^*\nu_{br,l}^* - \nu_{bt,l}\mu_{br,l}^*\right)/\left(\|\mu_{bt,l}\|^2 - \|\nu_{bt,l}\|^2\right)$$

Likewise, the compensation matrix for uplink transmission can be simplified as:

$$\mathbf{R}_k^u = \begin{bmatrix}\mathbf{D}(\mathbf{e}) & \mathbf{D}(\mathbf{f})\\ \mathbf{D}(\mathbf{f}^*) & \mathbf{D}(\mathbf{e}^*)\end{bmatrix} \tag{17}$$

where $\mathbf{e}$ and $\mathbf{f}$ are $M\times 1$ vectors, with the mth element given by

$$e_m = \left(\mu_{mt,m}^*\mu_{mr,m} - \nu_{mt,m}\nu_{mr,m}\right)/\left(\|\mu_{mt,m}\|^2 - \|\nu_{mt,m}\|^2\right)$$
$$f_m = \left(\mu_{mt,m}^*\nu_{mr,m}^* - \nu_{mt,m}\mu_{mr,m}^*\right)/\left(\|\mu_{mt,m}\|^2 - \|\nu_{mt,m}\|^2\right)$$

It can be seen from (16) and (17), the compensation matrices are only dependent on IQ imbalance parameters, which can be estimated by a variety of IQ imbalance parameter estimators in, e.g., [4], [13], and [14]. These estimators offer separate estimates of the IQ imbalance parameters and the radio channel. Since the IQ imbalance parameters associated with any transmitter cannot be estimated locally, they are estimated at the receiver side and then fed back to the transmitter. For example, the IQ imbalance parameters of the BS Tx are estimated and fed back by the MS, and visa versa. It has been found that the IQ imbalance parameters are relatively static compared with channel variations [6], [7]. As such, even for fast fading channels, the IQ imbalance parameters as well as the compensation matrices need to be re-calibrated only occasionally.

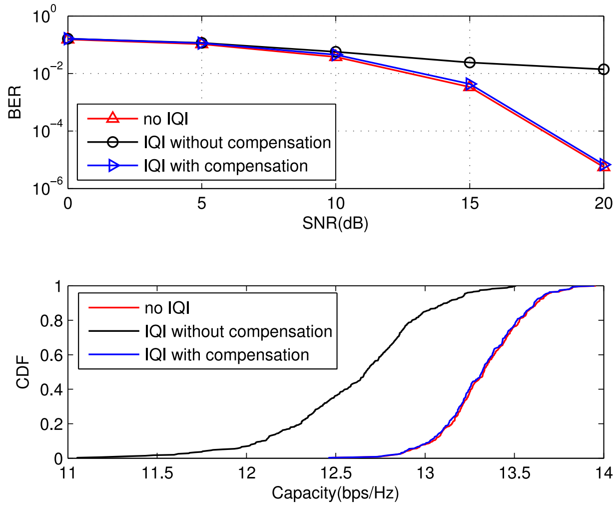

Fig. 1. (a) BER performance with 16QAM; (b) CDFs of the downlink capacity with SNR = 20 dB, when $L = 32$ and $M = 2$.

IV. Simulations and Discussion

In this section, simulation results are presented. We consider a TDD MIMO-OFDM system with 32 BS antennas, 2 MS antennas and 64 OFDM subcarriers. The carrier frequency is 2GHz, signal bandwidth is 5MHz, and signal constellation is 16 QAM. The mismatches in amplitude $\{\alpha_{mt}, \alpha_{mr}, \alpha_{bt}, \alpha_{br}\}$ and phase $\{\theta_{mt}, \theta_{mr}, \theta_{bt}, \theta_{br}\}$ at the different antenna branches of the MS and BS are randomly and uniformly distributed in the range of [0, 2] dB and [0°, 6°], respectively. Typical urban channel with maximum path delay of 2μs is employed.

Fig. 1 shows the bit error rate (BER) performance and cumulative distribution functions (CDFs) of the obtained downlink capacity. The curve of "no IQI" refers to a system with ideal IQ branches and ideal knowledge of the radio channel. The curve of "IQI without compensation" refers to a system with IQ imbalances but no compensation of channel reciprocity. The curve of "IQI with compensation" refers to a system with IQ imbalances and our proposed compensation scheme. The channel estimation scheme in [16] is adopted to estimate the uplink channel for the curve of "IQI with compensation". As can be seen from the simulation results, the proposed scheme can improve the BER performances and capacity significantly, and provide very close performances to the ideal case of no IQ imbalances.

The computational complexity of the compensation scheme is discussed in the following. Given the simple expression of c_l and d_l, the calculation of $\mathbf{R}_k^{\mathrm{d}}$ in (16) requires only $O(L)$ complex multiplications instead of $O(L^3)$ for full-size matrix inversion and multiplication. Since IQ imbalance parameters are frequency-independent, $\mathbf{R}_k^{\mathrm{d}}$ is constant for all OFDM subcarrier pairs and the complexity associated with calculating $\mathbf{R}^{\mathrm{d}}$ is $O(L)$, which is linear in the number of BS antennas L in the massive MIMO-OFDM system. Meanwhile, the complexity associated with calculating $\mathbf{R}^{\mathrm{u}}$ for all OFDM subcarrier pairs is only $O(M)$, where M is the number of antennas at the MS. Considering the relatively static character of IQ imbalance parameters, the additional complexity due to the calculation of the compensation matrices is small.

V. Conclusion

In this letter, a low-complexity compensation scheme was proposed to compensate the reciprocity of TDD massive MIMO-OFDM channels caused by frequency-independent IQ imbalances. It was shown that the proposed scheme is able to achieve the BER and capacity performance of the TDD MIMO-OFDM system close to that of the ideal scenario in the absence of IQ imbalances. The compensation of channel reciprocity for TDD MIMO-OFDM systems with frequency-dependent IQ imbalances will be discussed in our future work.

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