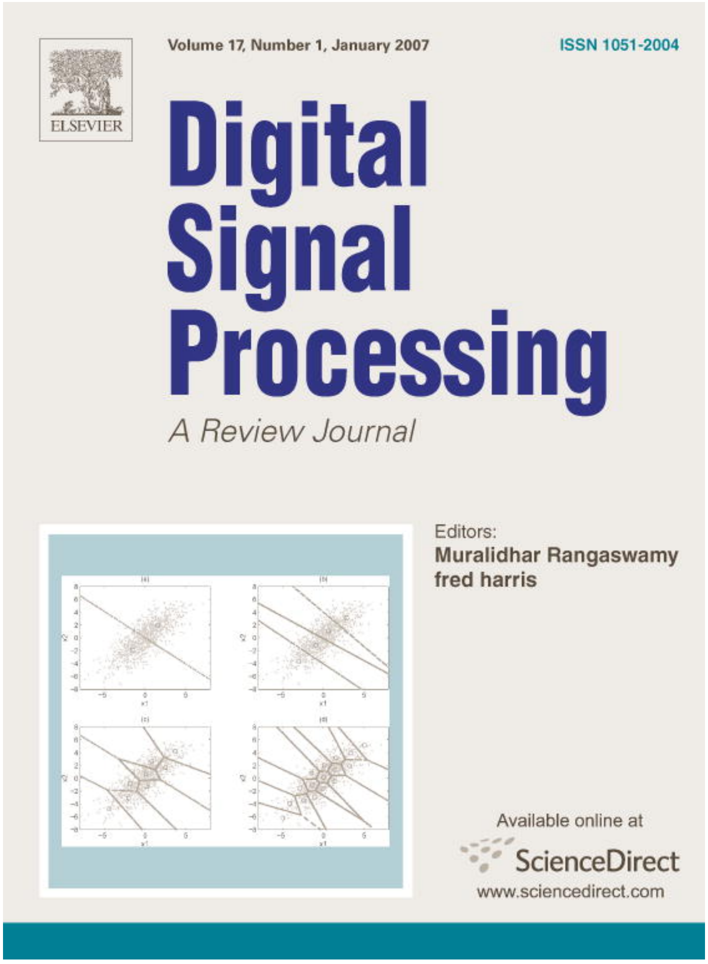
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Blind channel estimation and detection for space–time coded CDMA in ISI channels ☆

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Abstract

In order to accommodate high data-rate applications, there is a significant interest in extending space–time (ST) coding, originally proposed for *frequency-nonselective* channels assumed *known* to the receiver, to *unknown ISI* (inter-symbol interference) channels. In this paper, we consider the problem of blind channel identification and linear multiuser receiver design for ST-coded CDMA (code-division multiple-access) systems operating in frequency-selective fading channels. We investigate the identifiability conditions and present a subspace-based blind channel estimator for such systems. To assess the performance of the proposed and other potential estimators, we derive an unconditional Cramér–Rao bound (CRB) which, similarly to the proposed blind channel estimator, does not assume the knowledge of the transmitted information symbols. We discuss various linear multiuser detection schemes that can be used in conjunction with the proposed channel estimator for symbol demodulation in ST-coded CDMA systems. Numerical examples are presented to illustrate the performance of the proposed channel estimator and linear detectors in multipath Rayleigh fading channels.

Keywords: Space–time coding; Transmit diversity; Blind channel identification; Multiuser detection; Cramér–Rao bound

1. Introduction

Future wireless communication systems have to deal with increasing demands for high data-rate, high-quality data services. Space–time (ST) coding, which relies on multiple-antenna transmissions and appropriate signal processing at the receiver, is able to provide drastic increases in transmission rate, due to its ability to fully exploit spatial and temporal diversity [1]. Most previous studies on ST coding, however, were focused on the design of ST codes, assuming *frequency-nonselective* channels that are *perfectly known* to the receiver [2–4]. In practice, the channel state information (CSI) is unknown and has to be estimated. Channel estimation is of particular importance in future broadband wireless networks since high data-rate transmissions lead to severe frequency-selective channel fading, which necessitates the use of channel estimation/equalization techniques to combat significant inter-symbol interference (ISI).

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Channel estimation in ST-coded systems, however, is more challenging than in single-antenna systems since the number of unknown channel coefficients to be estimated increases proportionally to the number of transmit antennas. For training-based channel estimation schemes [5], this transforms to more required training data and, accordingly, decreases the channel throughput. Although the recently proposed differential ST coding schemes for frequency-nonselective channels (e.g., [6] and references therein) obviate the need for channel estimation and, therefore, are particularly attractive in fast fading environments when channel estimation becomes very difficult or even infeasible, differential decoding of ST codes incurs approximately a 3 dB penalty in SNR (signal-to-noise ratio) compared to coherent decoding which requires channel estimation. Hence, channel estimation is well motivated, especially in cases when the channel experiences relatively slow fading.

It should also be noted that interference in ST-coded systems is more severe than in single-antenna systems. For a system with K users where each is equipped with Q transmit antennas, MUI (multiuser interference) is composed of $(K-1)Q$ interfering signals, rather than $K-1$ interfering signals in a single-antenna system. To deal with the MUI more effectively, it is necessary to utilize/extend the multiuser detection schemes devised for conventional multiuser systems without ST coding [7] to ST-coded multiuser systems, such as the ST-coded CDMA (code-division multiple-access) systems considered in this paper.

The problem of interest to this work is blind channel estimation and multiuser receiver design for ST-coded CDMA systems. Channel estimation for single- and multiuser ST-coded systems in frequency-selective channels has been considered in, for example, [5,8–11]. Those studies all assume multicarrier modulation which converts a frequency-selective channel into a set of nonoverlapping frequency-nonselective sub-channels. In contrast, we consider in this paper single-carrier ST-coded CDMA systems. In Section 2, we formulate the data model for such systems, which is shown to coincide with an MIMO (multiple-input–multiple-output) system. In Section 3, the classical MIMO identification results (e.g., [12]) are utilized to determine the identifiability conditions for ST-coded CDMA systems. Also in Section 3, we present a subspace-based blind channel estimator to estimate the underlying frequency-selective channels by exploiting the unique signal structure inherent in ST-coded CDMA systems. The proposed subspace-based channel estimator is an extension of the acclaimed work on blind identification of SIMO (single-input–multiple-output) channels in [13], which has found extensive applications under various scenarios (e.g., [14–16]). To assess the performance of the proposed and other potential channel estimators for ST-coded CDMA systems, we derive in Section 4 an unconditional Cramér–Rao bound (CRB) which is not conditioned on the transmitted information symbols. It is meaningful to compare with this unconditional CRB since the proposed blind channel estimator either does not assume the knowledge of the transmitted symbols. In Section 5, we discuss several linear multiuser detectors that can be used in conjunction with the proposed channel estimation algorithm for symbol demodulation in ST-coded CDMA systems. Numerical examples are presented in Section 6 to illustrate the performance of the proposed channel estimator and linear receivers in multipath Rayleigh fading channels. Finally, the study is concluded in Section 7.

Notation. Vectors (matrices) are denoted by boldface lower (upper) case letters; all vectors are column vectors; superscripts $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$ denote the complex conjugate, transpose, and conjugate transpose, respectively; $\mathbf{I}_N$ denotes the $N \times N$ identity matrix; $\mathbf{0}$ denotes an all-zero vector/matrix; $E\{\cdot\}$ denotes the statistical expectation; $\|\cdot\|$ denotes the vector 2-norm [17]; $\otimes$ denotes the matrix Kronecker product [17]; diag$\{\cdot\}$ denotes a diagonal matrix; $\Re\{\cdot\}$ and $\Im\{\cdot\}$ takes the real and imaginary part of a complex quantity, respectively; $\lceil\cdot\rceil$ denotes the smallest integer no less than the argument; finally, given a sequence of $P \times K$ matrices $\{\mathbf{H}(m)\}_{m=0}^{M-1}$ (or, equivalently, a $P \times K$ polynomial matrix $\mathbf{H}_z(z) \triangleq \sum_{m=0}^{M-1} \mathbf{H}(m) z^{-m}$) [18], we define the *generalized Sylvester matrix* as

$$\mathcal{T}_N(\mathbf{H}) \triangleq \begin{bmatrix} \mathbf{H}(M-1) & \cdots & \mathbf{H}(0) & & \mathbf{0} \\ & \ddots & & \ddots & \\ \mathbf{0} & & \mathbf{H}(M-1) & \cdots & \mathbf{H}(0) \end{bmatrix} \in \mathbb{C}^{NP \times (M+N-1)K}. \tag{1}$$

2. Problem formulation

2.1. System model

Consider a synchronous (downlink) K-user CDMA system equipped with Q ($Q \geqslant 2$) transmit antennas ($T\times$'s) and L ($L \geqslant 1$) receive antennas ($R\times$'s). We consider only the downlink (i.e., base station to mobile) since multiple antennas are more often installed at the base station than at the mobile. We assume the Alamouti's ST coding scheme

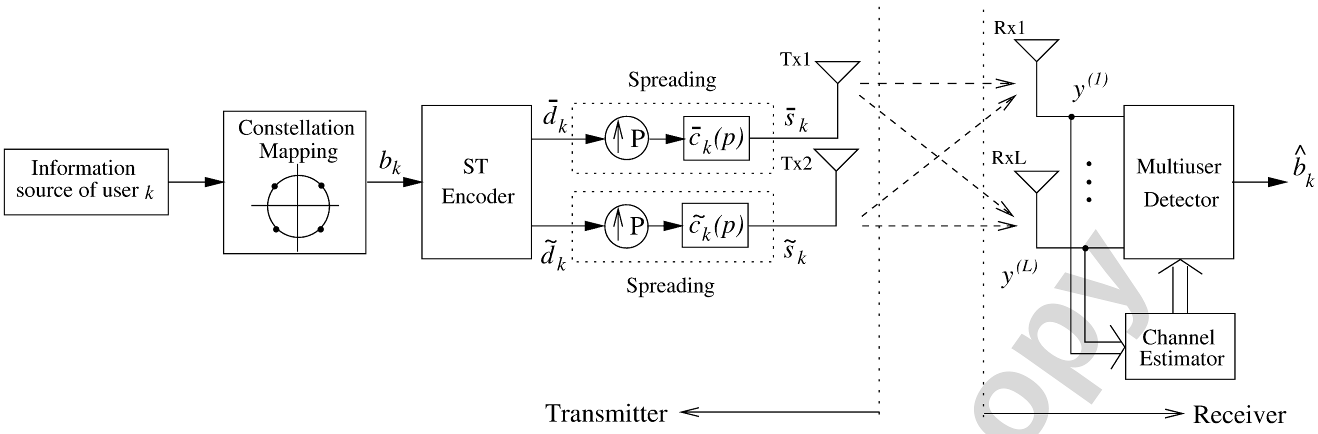


Fig. 1. Block diagram of a baseband ST-coded CDMA system shown for user k.

[3] which utilizes $Q = 2T\times$'s; extensions to other ST-block coding schemes are straightforward. Figure 1 depicts a diagram of the baseband ST-coded CDMA system, shown only for user k. At the transmitter, the ST encoder (specified in Section 2.2) maps the incoming symbol stream $\{b_k(n)\}$, drawn from some unit-energy constellation $\mathcal{B}$, into two ST-coded symbol streams: $\{d_k(n)\}$ and $\{\tilde{d}_k(n)\}$.[1] Next, the two ST-coded symbol streams are spread by two distinctive spreading codes $\{c_k(p)\}_{p=0}^{P-1}$ and $\{\tilde{c}_k(p)\}_{p=0}^{P-1}$, respectively, before being sent out by $T \times 1$ and $T \times 2$. Here $P = T/T_c$ is the processing gain with T and T_c being the symbol and chip duration, respectively. At the receiver, the channel estimator produces a (blind) channel estimate which is utilized by the multiuser detector for multiuser detection and ST decoding (not shown).

The baseband signals for user k transmitted from $T \times 1$ and $T \times 2$, respectively, are given by (in discrete form at the chip rate)[2]

$$\bar{s}_k(p) = \sum_{m=-\infty}^{\infty} \bar{d}_k(m)\bar{c}_k(p - mP), \qquad \tilde{s}_k(p) = \sum_{m=-\infty}^{\infty} \tilde{d}_k(m)\tilde{c}_k(p - mP). \tag{2}$$

Let $\bar{f}^{(l)}(t)$ [respectively, $\tilde{f}^{(l)}(t)$] be the impulse response of the overall channel including the physical wireless channel and the transmit/receive filters between $T \times 1$ (respectively $T \times 2$) and $R \times l$. Hereafter, $\bar{f}^{(l)}(t)$ and $\tilde{f}^{(l)}(t)$ are modeled as FIR (finite impulse response) filters [19] with a maximum length of W chips or, equivalently, $(M - 1)$ symbol periods, where

$$M \triangleq \lceil WT_c/T \rceil + 1 = \lceil W/P \rceil + 1. \tag{3}$$

Then, the received signal at $R \times l$, in discrete form at the chip rate, is given by

$$y^{(l)}(p) = \sum_{k=1}^{K} \sum_{q=0}^{W-1} \left[\bar{f}^{(l)}(q)\bar{s}_k(p - q) + \tilde{f}^{(l)}(q)\tilde{s}_k(p - q) \right] + v^{(l)}(p), \tag{4}$$

where $\bar{f}^{(l)}(q) \triangleq \bar{f}^{(l)}(t)|_{t=qT_c}$, $\tilde{f}^{(l)}(q) \triangleq \tilde{f}^{(l)}(t)|_{t=qT_c}$, $v^{(l)}(p)$ is the channel noise assumed to be spectrally white with zero-mean and variance σ_v^2, and it was assumed that a prior synchronization between the transmitter and receiver has been achieved (using one of the techniques discussed in, e.g., [20] and references therein). For brevity, in what follows we will mainly describe/define the quantities associated with $T \times 1$ with more details; similar definitions carry over to those associated with $T \times 2$.

Let $\bar{x}^{(l)}(p)$ denote the *noiseless* signals received at $R \times l$ due to transmissions from $T \times 1$ *only*, i.e.,

$$\bar{x}^{(l)}(p) = \sum_{k=1}^{K} \sum_{q=0}^{W-1} \bar{f}^{(l)}(q)\bar{s}_k(p - q). \tag{5}$$

[1] Throughout the paper, $\bar{(\cdot)}$ [respectively $\tilde{(\cdot)}$] is designated to quantities associated with the first (respectively second) transmit antenna.

[2] Throughout the paper, we use m and n for symbol indexes, p and q for chip indexes, k for user index, and l for $R\times$ index.

Substituting the expression of $\bar{s}_k(p)$ given in (2) into (5) yields

$$\bar{x}^{(l)}(p)=\sum_{k=1}^{K}\sum_{m=-\infty}^{\infty}\bar{d}_k(m)\sum_{q=0}^{W-1}\bar{f}^{(l)}(q)\bar{c}_k(p-q-mP)\triangleq\sum_{k=1}^{K}\sum_{m=-\infty}^{\infty}\bar{d}_k(m)\bar{h}_k^{(l)}(p-mP), \tag{6}$$

where $\bar{h}_k^{(l)}(p)$ is the *composite signature sequence* resulting from the convolution of the spreading code $\{\bar{c}_k(p)\}$ and the channel response $\{\bar{f}^{(l)}(p)\}$:

$$\bar{h}_k^{(l)}(p)\triangleq\sum_{q=0}^{W-1}\bar{f}^{(l)}(q)\bar{c}_k(p-q), \quad p=0,\dots,MP-1. \tag{7}$$

Let $\bar{x}_p^{(l)}(n)\triangleq\bar{x}^{(l)}(p+nP)$ denote the pth chip of the signal received during the nth symbol period at $R\times l$. It follows from (6) and (7) that $\bar{x}_p^{(l)}(n)$ is given by

$$\bar{x}_p^{(l)}(n)=\sum_{k=1}^{K}\sum_{m=-\infty}^{\infty}\bar{d}_k(m)\bar{h}_k^{(l)}(p-mP+nP)\triangleq\sum_{k=1}^{K}\sum_{m=-\infty}^{\infty}\bar{d}_k(m)\bar{h}_{p,k}^{(l)}(n-m)=\sum_{k=1}^{K}\sum_{m=0}^{M-1}\bar{h}_{p,k}^{(l)}(m)\bar{d}_k(n-m), \tag{8}$$

where

$$\bar{h}_{p,k}^{(l)}(n)\triangleq\bar{h}_k^{(l)}(p+nP), \quad p=0,\dots,P-1. \tag{9}$$

To express the input–output relation in compact matrix/vector form, we define

$$\begin{aligned}
\bar{\mathbf{d}}(n)&=\begin{bmatrix}\bar{d}_1(n), & \dots, & \bar{d}_K(n)\end{bmatrix}^T\in\mathbb{C}^{K\times 1},\\
\bar{\mathbf{H}}_p(m)&=\begin{bmatrix}\bar{h}_{p,1}^{(1)}(m) & \dots & \bar{h}_{p,K}^{(1)}(m)\\ & \vdots & \\ \bar{h}_{p,1}^{(L)}(m) & \dots & \bar{h}_{p,K}^{(L)}(m)\end{bmatrix}\in\mathbb{C}^{L\times K},\\
\bar{\mathbf{H}}(m)&=\begin{bmatrix}\bar{\mathbf{H}}_0^T(m), & \dots, & \bar{\mathbf{H}}_{P-1}^T(m)\end{bmatrix}^T\in\mathbb{C}^{LP\times K},\\
\bar{\mathbf{x}}(n)&=\begin{bmatrix}x_0^{(1)}(n), & \dots, & x_0^{(L)}(n), & \dots, & x_{P-1}^{(1)}(n), & \dots, & x_{P-1}^{(L)}(n)\end{bmatrix}^T\in\mathbb{C}^{LP\times 1}.
\end{aligned} \tag{10}$$

It follows from (5) and (8) that

$$\bar{\mathbf{x}}(n)=\sum_{m=0}^{M-1}\bar{\mathbf{H}}(m)\bar{\mathbf{d}}(n-m), \tag{11}$$

which corresponds to an MIMO system with K inputs and LP outputs.

Next, we collect samples received during N consecutive symbol periods, where N is the *smoothing factor* [21]. Specifically, let $\bar{\mathbf{x}}_N(n)=[\bar{\mathbf{x}}^T(n),\dots,\bar{\mathbf{x}}^T(n+N-1)]^T\in\mathbb{C}^{LNP\times 1}$, $\bar{\mathbf{d}}_N(n)=[\bar{\mathbf{d}}^T(n-M+1),\dots,\bar{\mathbf{d}}^T(n+N-1)]^T\in\mathbb{C}^{(M+N-1)K\times 1}$, and $\mathcal{T}_N(\bar{\mathbf{H}})\in\mathbb{C}^{LNP\times(M+N-1)K}$ be the generalized Sylvester matrix formed from $\{\bar{\mathbf{H}}(m)\}_{m=0}^{M-1}$ [cf. (1)]. Then, $\bar{\mathbf{x}}_N(n)$ can be expressed as

$$\bar{\mathbf{x}}_N(n)=\mathcal{T}_N(\bar{\mathbf{H}})\bar{\mathbf{d}}_N(n). \tag{12}$$

The *noiseless* signal due to the transmissions from $T\times 2$ *only* has a similar relation:

$$\tilde{\mathbf{x}}_N(n)=\mathcal{T}_N(\tilde{\mathbf{H}})\tilde{\mathbf{d}}_N(n), \tag{13}$$

where $\tilde{\mathbf{x}}_N(n)$, $\mathcal{T}_N(\tilde{\mathbf{H}})$, and $\tilde{\mathbf{d}}_N(n)$ are quantities associated with $T\times 2$ which are similarly defined to $\bar{\mathbf{x}}_N(n)$, $\mathcal{T}_N(\bar{\mathbf{H}})$ and $\bar{\mathbf{d}}_N(n)$, respectively.

Let $\mathbf{H}(m)=[\bar{\mathbf{H}}(m),\tilde{\mathbf{H}}(m)]$, and $\mathbf{d}(n)=[\bar{\mathbf{d}}^T(n),\tilde{\mathbf{d}}^T(n)]^T$. The *overall noisy* signal due to transmissions from *both* $T\times$'s is given by

$$\mathbf{y}_N(n)\triangleq\bar{\mathbf{x}}_N(n)+\tilde{\mathbf{x}}_N(n)+\mathbf{v}_N(n)=\mathcal{T}_N(\mathbf{H})\mathbf{d}_N(n)+\mathbf{v}_N(n), \quad n=0,\dots,N_y-1, \tag{14}$$

where $\mathcal{T}_N(\mathbf{H}) \in \mathbb{C}^{LNP\times 2(M+N-1)K}$ denotes the generalized Sylvester matrix formed from $\{\mathbf{H}(m)\}_{m=0}^{M-1}$ [cf. (1)], $\mathbf{d}_N(n) \triangleq [\mathbf{d}^T(n-M+1), \dots, \mathbf{d}^T(n+N-1)]^T \in \mathbb{C}^{2(M+N-1)K\times 1}$, and $\mathbf{v}_N(n)$ denotes the $LNP \times 1$ noise vector. Note that (14) corresponds to a noisy MIMO system with $2K$ inputs and LP outputs.

2.2. ST encoder

The ST encoder in Fig. 1 implements the Alamouti's ST coding scheme [3]. Specifically, for user k, the ST encoder takes two adjacent symbols $b_k(2n)$ and $b_k(2n+1)$ and outputs the following ST-coded matrix:

$$\boldsymbol{D}_k(n) \triangleq \begin{bmatrix} \bar{d}_k(2n) & \bar{d}_k(2n+1) \\ \tilde{d}_k(2n) & \tilde{d}_k(2n+1) \end{bmatrix},$$

where

$$\bar{d}_k(2n) = b_k(2n), \qquad \bar{d}_k(2n+1) = -b_k^*(2n+1), \qquad \tilde{d}_k(2n) = b_k(2n+1), \qquad \tilde{d}_k(2n+1) = b_k^*(2n). \tag{15}$$

The columns of $\boldsymbol{D}_k(n)$ are then transmitted one at a time over two successive symbol intervals, with the elements of each column sent from the two $T\times$'s simultaneously.

In the sequel, we assume that $\{b_k(n)\}$ are drawn from some unit-energy, circularly symmetric constellation (e.g., R-ary PSK with $R > 2$) such that $E\{|b_k(n)|^2\} = 1$ and $E\{b_k^2(n)\} = 0$; furthermore, we assume that $\{b_k(n)\}$ are independently and identically distributed (i.i.d.) for different n and k. Under these conditions, one can easily verify that $E\{\boldsymbol{d}_k(n)\boldsymbol{d}_k^H(n)\} = \mathbf{I}_4$, where

$$\boldsymbol{d}_k(n) \triangleq \begin{bmatrix} \bar{d}_k(2n), & \tilde{d}_k(2n), & \bar{d}_k(2n+1), & \tilde{d}_k(2n+1) \end{bmatrix}^T. \tag{16}$$

We further assume that N is chosen such that $M+N-1$ is even, which implies that the $\mathbf{d}_N(n)$ in (14) contains a total of $(M+N-1)/2$ full ST code matrices for each user. This observation, along with the previous assumptions, suggests that

$$\mathbf{R}_{\mathbf{d}_N} \triangleq E\{\mathbf{d}_N(n)\mathbf{d}_N^H(n)\} = \mathbf{I}_{2(M+N-1)}. \tag{17}$$

The problem of interest is to estimate the unknown channel coefficients $\{\bar{f}^{(l)}(p)\}_{p=0}^{W-1}$ and $\{\tilde{f}^{(l)}(p)\}_{p=0}^{W-1}$, for $l = 1, \dots, L$, that are embedded in the channel matrix $\mathcal{T}_N(\mathbf{H})$, and then to recover the information symbols $\{b_k(n)\}$ from the noisy observations $\{\mathbf{y}_N(n)\}$.

3. Subspace-based blind channel identification

3.1. Blind identification of MIMO FIR channels

The formulation of the ST-coded CDMA system model in Section 2.1 indicates that the estimation of the FIR channels $\{\bar{f}^{(l)}(p)\}$ and $\{\tilde{f}^{(l)}(p)\}$ is clearly related to the MIMO channel identification problem, a topic that has been extensively addressed in the literature (e.g., [15,22–24] and references therein). Most blind identification schemes relying only on the second-order statistics (SOS) of the received signal require the MIMO channel to satisfy certain *disparity conditions* such that the generalized Sylvester matrix $\mathcal{T}_N(\mathbf{H})$ has full column rank. This necessitates the choice of the smoothing factor N to make $\mathcal{T}_N(\mathbf{H})$ a tall matrix, i.e., $LNP \geqslant 2(M+N-1)K$. More specifically, we recall the following result (see, e.g., [12,23]).

Proposition 1. *Define the $LP \times 2K$ transfer function $\mathbf{H}_z(z) \triangleq \sum_{m=0}^{M-1} \mathbf{H}(m)z^{-m}$. The generalized Sylvester matrix $\mathcal{T}_N(\mathbf{H})$ has full column rank if and only if the following conditions are satisfied*:

(A1) $\mathbf{H}_z(z)$ *has full column rank for all z* [*i.e.,* $\mathbf{H}_z(z)$ *is* irreducible].
(A2) $\mathbf{H}(M-1)$ *has full column rank* [*i.e.,* $\mathbf{H}_z(z)$ *is* column reduced].
(A3) *The degrees of the columns of* $\mathbf{H}_z(z)$ *are identical.*
(A4) $N \geqslant \max_{1\leqslant j\leqslant LP-2K} \mu_j^{\perp} - 1$, *where* $\{\mu_j^{\perp}\}_{j=1}^{LP-2K}$ *denote the* dual Kronecker indices [18] *of the rational subspace spanned by the columns of* $\mathbf{H}_z(z)$.

(A3) is usually satisfied in a downlink scenario where all users share identical channels. The case of different column degrees of $\mathbf{H}_z(z)$ (e.g., uplink) is addressed in [15]. (A4) reiterates the need for a large enough smoothing factor N to build the necessary "embedding dimension." Although $\{\mu_j^{\perp}\}_{j=1}^{LP-2K}$ depend on the unknown channel, general criteria on how to choose N are provided in [12].

The subspace-based blind channel identification method, originally proposed for the SIMO channel identification [13], can be used to identify an MIMO FIR filter $\mathbf{H}_z(z)$ up to a constant nonsingular matrix, providing that certain conditions are satisfied. More specifically, under the conditions stated in Proposition 1 and, furthermore, that (A4) is augmented to

(A4′) $N \geqslant \max_{1\leqslant j\leqslant LP-2K} \mu_j^{\perp}$,

if there exists an $LP \times 2K$ polynomial matrix $\hat{\mathbf{H}}_z(z) \triangleq \sum_{m=0}^{M-1} \hat{\mathbf{H}}(m)z^{-m}$ which satisfies $\operatorname{rank}\{\hat{\mathbf{H}}_z(z)\} = 2K$ and $\mathbf{\Pi}_{\mathbf{H}}^{\perp}\mathcal{T}(\hat{\mathbf{H}}) = \mathbf{0}$, where $\mathbf{\Pi}_{\mathbf{H}}^{\perp}$ denotes the orthogonal projector on the left null space of $\mathcal{T}_N(\mathbf{H})$, then [12]

$$\hat{\mathbf{H}}_z(z) = \mathbf{H}_z(z)\mathbf{\Omega}, \tag{18}$$

where $\mathbf{\Omega} \in \mathbb{C}^{2K\times 2K}$ is a constant, nonsingular matrix.

3.2. Blind channel identification for ST-coded CDMA systems

We note that the MIMO FIR channel $\mathcal{T}_N(\mathbf{H})$ is *partly known* due to the presence of spreading [cf. (7)]. The knowledge of the spreading codes can be utilized to produce channel estimates with much reduced ambiguity, as shown in [25] for conventional CDMA systems (without ST coding). In the sequel, we derive a subspace-based blind channel estimator which yields estimates of $\{\bar{f}^{(l)}(p)\}$ and $\{\tilde{f}^{(l)}(p)\}$ to within only a scalar ambiguity. The remaining scalar ambiguity can be easily removed by differentially encoding/decoding the information symbols or using a few pilot symbols.

The covariance matrix of $\mathbf{y}_N(n)$ is given by [see (14) and (17)]

$$\mathbf{R}_{\mathbf{y}_N} = E\{\mathbf{y}_N(n)\mathbf{y}_N^H(n)\} = \mathcal{T}_N(\mathbf{H})\mathcal{T}_N^H(\mathbf{H}) + \sigma_v^2\mathbf{I}_{LNP}.$$

Assuming that the conditions in Proposition 1 are satisfied, $\mathcal{T}_N(\mathbf{H})$ has full column rank. Hence, the eigendecomposition of $\mathbf{R}_{\mathbf{y}_N}$ can be expressed as

$$\mathbf{R}_{\mathbf{y}_N} = \mathbf{E}_{\mathrm{s}}\mathbf{\Lambda}_{\mathrm{s}}\mathbf{E}_{\mathrm{s}}^H + \sigma_v^2\mathbf{E}_{\mathrm{n}}\mathbf{E}_{\mathrm{n}}^H, \tag{19}$$

where $\mathbf{\Lambda}_{\mathrm{s}} = \operatorname{diag}\{\lambda_1, \ldots, \lambda_{2(M+N-1)K}\}$ contains the $2(M+N-1)K$ largest eigenvalues, i.e., *signal eigenvalues*, $\mathbf{E}_{\mathrm{s}} \in \mathbb{C}^{LNP\times 2(M+N-1)K}$ contains the $2(M+N-1)K$ *signal eigenvectors* which span the *signal subspace*, i.e., $\operatorname{span}\{\mathbf{E}_{\mathrm{s}}\} = \operatorname{span}\{\mathcal{T}_N(\mathbf{H})\}$, and $\mathbf{E}_{\mathrm{n}} \in \mathbb{C}^{LNP\times J}$ contains the J *noise eigenvectors* which span the *noise subspace*, with

$$J \triangleq LNP - 2(M+N-1)K.$$

The signal and noise subspaces are mutually orthogonal. Hence, $\mathbf{E}_{\mathrm{n}}^H\mathcal{T}_N(\mathbf{H}) = \mathbf{0}$ or, equivalently,

$$\mathbf{E}_{\mathrm{n}}^H\mathcal{T}_N(\bar{\mathbf{H}}) = \mathbf{0}, \qquad \mathbf{E}_{\mathrm{n}}^H\mathcal{T}_N(\tilde{\mathbf{H}}) = \mathbf{0}, \tag{20}$$

where $\mathcal{T}_N(\bar{\mathbf{H}})$ and $\mathcal{T}_N(\tilde{\mathbf{H}})$ are $LNP \times (M+N-1)K$ generalized Sylvester matrices defined as in (12) and (13), respectively.

Let

$$\begin{aligned}
\bar{\boldsymbol{H}} &= \left[\bar{\mathbf{H}}^T(0), \quad \ldots, \quad \bar{\mathbf{H}}^T(M-1)\right]^T \triangleq \left[\bar{\mathbf{h}}_1, \quad \ldots, \quad \bar{\mathbf{h}}_K\right] \in \mathbb{C}^{LMP\times K}, \\
\tilde{\boldsymbol{H}} &= \left[\tilde{\mathbf{H}}^T(0), \quad \ldots, \quad \tilde{\mathbf{H}}^T(M-1)\right]^T \triangleq \left[\tilde{\mathbf{h}}_1, \quad \ldots, \quad \tilde{\mathbf{h}}_K\right] \in \mathbb{C}^{LMP\times K}.
\end{aligned} \tag{21}$$

Decompose $\mathbf{E}_{\mathrm{n}}$ into $\mathbf{E}_{\mathrm{n}}^H \triangleq [\mathbf{G}_0, \ \ldots, \ \mathbf{G}_{N-1}]$, where $\mathbf{G}_n \in \mathbb{C}^{J\times LP}$ for $n = 0, \ldots, N-1$, and form

$$\boldsymbol{\mathcal{G}}^H = \begin{bmatrix} \mathbf{G}_{N-1} & & \mathbf{0} \\ \vdots & \ddots & \\ \mathbf{G}_0 & & \mathbf{G}_{N-1} \\ & \ddots & \vdots \\ \mathbf{0} & & \mathbf{G}_0 \end{bmatrix} = \mathcal{T}_M^H(\mathbf{G}^H) \in \mathbb{C}^{J(M+N-1)\times LMP}. \tag{22}$$

Since convolution is commutative, one can easily verify that (20) is equivalent to

$$\boldsymbol{\mathcal{G}}^H \bar{\boldsymbol{H}} = \mathbf{0}, \qquad \boldsymbol{\mathcal{G}}^H \tilde{\boldsymbol{H}} = \mathbf{0}. \tag{23}$$

Let

$$\bar{\mathbf{f}}(p) = \left[\bar{f}^{(1)}(p), \quad \ldots, \quad \bar{f}^{(L)}(p)\right]^T \in \mathbb{C}^{L\times 1}, \qquad \bar{\mathbf{f}} = \left[\bar{\mathbf{f}}^T(0), \quad \ldots, \quad \bar{\mathbf{f}}^T(W-1)\right]^T \in \mathbb{C}^{LW\times 1},$$

and $\tilde{\mathbf{f}}$ is similarly formed from $\{\tilde{f}^{(l)}(p)\}$. We note that $\bar{\boldsymbol{H}}$ and $\tilde{\boldsymbol{H}}$ are parameterized by $\bar{\mathbf{f}}$ and $\tilde{\mathbf{f}}$, respectively. In particular, we have [see (7)]

$$\bar{\mathbf{h}}_k = \bar{\boldsymbol{\mathcal{C}}}_k \bar{\mathbf{f}}, \qquad \tilde{\mathbf{h}}_k = \tilde{\boldsymbol{\mathcal{C}}}_k \tilde{\mathbf{f}}, \tag{24}$$

where

$$\bar{\boldsymbol{\mathcal{C}}}_k = \bar{\mathbf{C}}_k \otimes \mathbf{I}_L, \qquad \tilde{\boldsymbol{\mathcal{C}}}_k = \tilde{\mathbf{C}}_k \otimes \mathbf{I}_L, \tag{25}$$

$$\bar{\mathbf{C}}_k = \begin{bmatrix} \bar{c}_k(0) & & 0 \\ \vdots & \ddots & \vdots \\ \bar{c}_k(P-1) & & \bar{c}_k(0) \\ \vdots & \ddots & \vdots \\ 0 & & \bar{c}_k(P-1) \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix} \in \mathbb{C}^{MP\times W}, \tag{26}$$

and $\tilde{\mathbf{C}}_k$ is similarly formed from the spreading code $\{\tilde{c}_k(p)\}_{p=0}^{P-1}$.

Substituting (24) into (23) yields

$$\boldsymbol{\mathcal{G}}^H \bar{\boldsymbol{\mathcal{C}}}_k \bar{\mathbf{f}} = \mathbf{0}, \qquad \boldsymbol{\mathcal{G}}^H \tilde{\boldsymbol{\mathcal{C}}}_k \tilde{\mathbf{f}} = \mathbf{0}, \quad k = 1, \ldots, K. \tag{27}$$

Hence, $\bar{\mathbf{f}}$ and $\tilde{\mathbf{f}}$ are within the null space of $\boldsymbol{\mathcal{G}}^H \bar{\boldsymbol{\mathcal{C}}}_k$ and $\boldsymbol{\mathcal{G}}^H \tilde{\boldsymbol{\mathcal{C}}}_k$, respectively. If the dimension of the two null spaces is one, then any vector within $\{\boldsymbol{\mathcal{G}}^H \bar{\boldsymbol{\mathcal{C}}}_k\}$ (respectively $\{\boldsymbol{\mathcal{G}}^H \tilde{\boldsymbol{\mathcal{C}}}_k\}$) is proportional to $\bar{\mathbf{f}}$ (respectively $\tilde{\mathbf{f}}$). Specifically, we have the following result.

Proposition 2. *In addition to* (A1)–(A3) *and* (A4′)*, assume*

(A5) *The two matrices* $[\bar{\mathbf{h}}_1, \ \ldots, \ \bar{\mathbf{h}}_{k-1}, \ \bar{\boldsymbol{\mathcal{C}}}_k, \ \bar{\mathbf{h}}_{k+1}, \ \ldots, \ \bar{\mathbf{h}}_K, \ \tilde{\boldsymbol{H}}]$ *and* $[\tilde{\mathbf{h}}_1, \ \ldots, \ \tilde{\mathbf{h}}_{k-1}, \ \tilde{\boldsymbol{\mathcal{C}}}_k, \ \tilde{\mathbf{h}}_{k+1}, \ \ldots, \ \tilde{\mathbf{h}}_K, \ \bar{\boldsymbol{H}}]$ *have full column rank.*

If there exist $\hat{\bar{\mathbf{f}}}$ *and* $\hat{\tilde{\mathbf{f}}}$ *such that* $\boldsymbol{\mathcal{G}}^H \bar{\boldsymbol{\mathcal{C}}}_k \hat{\bar{\mathbf{f}}} = \mathbf{0}$ *and* $\boldsymbol{\mathcal{G}}^H \tilde{\boldsymbol{\mathcal{C}}}_k \hat{\tilde{\mathbf{f}}} = \mathbf{0}$*, then* $\hat{\bar{\mathbf{f}}} \propto \bar{\mathbf{f}}$ *and* $\hat{\tilde{\mathbf{f}}} \propto \tilde{\mathbf{f}}$.

Proof. Let $\hat{\bar{\boldsymbol{H}}}$ and $\hat{\tilde{\boldsymbol{H}}}$ be formed from $\hat{\bar{\mathbf{f}}}$ and $\hat{\tilde{\mathbf{f}}}$, respectively, by using (24) and (21). According to (18), we have

$$[\,\hat{\bar{\boldsymbol{H}}}, \quad \hat{\tilde{\boldsymbol{H}}}\,] = [\,\bar{\boldsymbol{H}}, \quad \tilde{\boldsymbol{H}}\,] \begin{bmatrix} \bar{\boldsymbol{\Omega}} \\ \tilde{\boldsymbol{\Omega}} \end{bmatrix}, \tag{28}$$

where $\bar{\boldsymbol{\Omega}} = [\bar{\boldsymbol{\omega}}_1, \ \ldots, \bar{\boldsymbol{\omega}}_{2K}]$ and $\tilde{\boldsymbol{\Omega}} = [\tilde{\boldsymbol{\omega}}_1, \ \ldots, \tilde{\boldsymbol{\omega}}_{2K}]$ are some $K \times 2K$ constant matrices. Equivalently, we can express (28) as

$$\hat{\bar{\mathbf{h}}}_k = \bar{\boldsymbol{H}} \bar{\boldsymbol{\omega}}_k + \tilde{\boldsymbol{H}} \tilde{\boldsymbol{\omega}}_k, \qquad \hat{\tilde{\mathbf{h}}}_k = \bar{\boldsymbol{H}} \bar{\boldsymbol{\omega}}_{K+k} + \tilde{\boldsymbol{H}} \tilde{\boldsymbol{\omega}}_{K+k}. \tag{29}$$

Let $\bar{\omega}_{j,k}$ (respectively $\tilde{\omega}_{j,k}$) denote the (j,k)th element of $\bar{\boldsymbol{\Omega}}$ (respectively $\tilde{\boldsymbol{\Omega}}$). We rewrite (29) as

$$\bar{\mathcal{C}}_k(\hat{\bar{\mathbf{f}}} - \bar{\omega}_{k,k}\bar{\mathbf{f}}) = \hat{\bar{\mathbf{h}}}_k - \bar{\omega}_{k,k}\bar{\mathbf{h}}_k = \sum_{j\neq k}\bar{\omega}_{j,k}\bar{\mathbf{h}}_j + \tilde{\boldsymbol{H}}\tilde{\boldsymbol{\omega}}_k,$$

$$\tilde{\mathcal{C}}_k(\hat{\tilde{\mathbf{f}}} - \tilde{\omega}_{k,K+k}\tilde{\mathbf{f}}) = \hat{\tilde{\mathbf{h}}}_k - \tilde{\omega}_{k,K+k}\tilde{\mathbf{h}}_k = \bar{\boldsymbol{H}}\bar{\boldsymbol{\omega}}_{K+k} + \sum_{j\neq k}\tilde{\omega}_{j,K+k}\tilde{\mathbf{h}}_j,$$

which, in conjunction with (A5), suggest that

$$\hat{\bar{\mathbf{f}}} = \omega_{k,k}\bar{\mathbf{f}}, \qquad \hat{\tilde{\mathbf{f}}} = \omega_{k,K+k}\tilde{\mathbf{f}},$$

and $\boldsymbol{\Omega} \triangleq [\bar{\boldsymbol{\Omega}}^T,\ \tilde{\boldsymbol{\Omega}}^T]^T$ is a strictly diagonal matrix. This concludes the proof. □

As pointed out in [23], the dependence on (A2) (i.e., $\mathbf{H}_z(z)$ is column reduced) may not be well adjusted in CDMA systems since the last $LMP - L(P+W-1)$ rows of $\bar{\boldsymbol{H}}$ and $\tilde{\boldsymbol{H}}$ are zeros [cf. (7), (21), and (26)]. Nevertheless, it was shown that (A2) is not necessary to ensure blind identification in CDMA systems if the spreading codes are properly designed; see [23] for details.

In practice, $\mathbf{R}_{\mathbf{y}_N}$ and, therefore, $\mathcal{G}$ are unknown. Denote by $\hat{\mathcal{G}}$ the estimate of $\mathcal{G}$ obtained from the eigendecomposition of, e.g., the *sample covariance matrix*:

$$\hat{\mathbf{R}}_{\mathbf{y}_N} = \frac{1}{N_y}\sum_{n=0}^{N_y-1}\mathbf{y}_N(n)\mathbf{y}_N^H(n). \tag{30}$$

Note that other estimators of $\mathbf{R}_{\mathbf{y}_N}$ and $\mathcal{G}$, including adaptive schemes, are possible; but explorations of such alternatives are beyond the scope of the current paper. Due to estimation and roundoff errors, (27) in general does not hold exactly. Instead, we seek solutions in the least-squares (LS) sense. That is,

$$\hat{\bar{\mathbf{f}}} = \arg\min_{\mathbf{f}\in\mathbf{C}^{LW\times 1}} \mathbf{f}^H\bar{\mathcal{C}}_k^H\hat{\mathcal{G}}^H\hat{\mathcal{G}}\bar{\mathcal{C}}_k\mathbf{f}, \tag{31}$$

$$\hat{\tilde{\mathbf{f}}} = \arg\min_{\mathbf{f}\in\mathbf{C}^{LW\times 1}} \mathbf{f}^H\tilde{\mathcal{C}}_k^H\hat{\mathcal{G}}^H\hat{\mathcal{G}}\tilde{\mathcal{C}}_k\mathbf{f}. \tag{32}$$

Under the standard constraint $\|\mathbf{f}\| = 1$ (to avoid trivial solutions), $\hat{\bar{\mathbf{f}}}$ and $\hat{\tilde{\mathbf{f}}}$ are the eigenvectors corresponding to the smallest eigenvalue of $\bar{\mathcal{C}}_k^H\hat{\mathcal{G}}^H\hat{\mathcal{G}}\bar{\mathcal{C}}_k$ and $\tilde{\mathcal{C}}_k^H\hat{\mathcal{G}}^H\hat{\mathcal{G}}\tilde{\mathcal{C}}_k$, respectively.

In the event that the receiver has the knowledge of some other users' spreading codes, say users within $\mathcal{K} \subseteq [1, K]$, the knowledge can be used to improve channel estimation, by using the following modified criteria:

$$\hat{\bar{\mathbf{f}}} = \arg\min_{\mathbf{f}\in\mathbf{C}^{LW\times 1}} \mathbf{f}^H\left(\sum_{k\in\mathcal{K}}\bar{\mathcal{C}}_k^H\hat{\mathcal{G}}^H\hat{\mathcal{G}}\bar{\mathcal{C}}_k\right)\mathbf{f}, \tag{33}$$

$$\hat{\tilde{\mathbf{f}}} = \arg\min_{\mathbf{f}\in\mathbf{C}^{LW\times 1}} \mathbf{f}^H\left(\sum_{k\in\mathcal{K}}\tilde{\mathcal{C}}_k^H\hat{\mathcal{G}}^H\hat{\mathcal{G}}\tilde{\mathcal{C}}_k\right)\mathbf{f}, \tag{34}$$

the solutions to which, under again the unit-norm constraints, are the eigenvectors corresponding to the smallest eigenvalue of $(\sum_{k\in\mathcal{K}}\bar{\mathcal{C}}_k^H\hat{\mathcal{G}}^H\hat{\mathcal{G}}\bar{\mathcal{C}}_k)$ and $(\sum_{k\in\mathcal{K}}\tilde{\mathcal{C}}_k^H\hat{\mathcal{G}}^H\hat{\mathcal{G}}\tilde{\mathcal{C}}_k)$, respectively.

To summarize, the proposed subspace-based blind channel identification algorithm for ST-coded CDMA systems consists of the following steps:

Step 1. Estimate the covariance matrix $\mathbf{R}_{\mathbf{y}_N}$ by using, e.g., the sample covariance matrix $\hat{\mathbf{R}}_{\mathbf{y}_N}$ in (30).

Step 2. Obtain the noise eigenvectors $\mathbf{E}_\mathrm{n}$ by computing the eigendecomposition of $\hat{\mathbf{R}}_{\mathbf{y}_N}$ [cf. (19)], and form $\hat{\mathcal{G}}$ by (22).

Step 3. Compute the channel estimates $\hat{\bar{\mathbf{f}}}$ and $\hat{\tilde{\mathbf{f}}}$ by (31) and (32) if only spreading codes for user k are known, or by (33) and (34) if additional spreading codes are known.

4. Cramér–Rao bound

In this section, we derive the Cramér–Rao bound (CRB), the lower bound on the variance of any unbiased estimators, for the ST-coded CDMA channel estimation problem. Specifically, we derive an *unconditional CRB* which is not conditioned on the information symbols (see, e.g., [26]). It is meaningful to compare the proposed blind channel estimator with the unconditional CRB since the former does not assume the knowledge of the information symbols. Conditional CRBs (i.e., CRBs which are conditioned on the information symbols) for various blind channel identification problems have been investigated in the literature; see, e.g., [27–29] and references therein.

Assume that the observation time consists of a total of N_s symbol intervals. We collect all samples obtained within this observation time and define $\mathbf{y} \triangleq [\mathbf{y}^T(0),\ \ldots,\ \mathbf{y}^T(N_s-1)]^T \in \mathbb{C}^{LN_sP\times 1}$, where $\mathbf{y}(n)$ consists of $LP \times 1$ noisy samples obtained within one symbol interval [cf. (10) and (14)]: $\mathbf{y}(n) = \bar{\mathbf{x}}(n) + \tilde{\mathbf{x}}(n) + \mathbf{v}(n)$, $n = 0, \ldots, N_s - 1$, and $\mathbf{d} \triangleq [\mathbf{d}^T(-M+1),\ \ldots, \mathbf{d}^T(N_s-1)]^T \in \mathbb{C}^{2K(M+N_s-1)\times 1}$ contains all the information symbols that contribute to $\mathbf{y}$. Then, we have [see (14)]

$$\mathbf{y} = \mathcal{T}_{N_s}(\mathbf{H})\mathbf{d} + \mathbf{v}, \tag{35}$$

where $\mathcal{T}_{N_s}(\mathbf{H}) \in \mathbb{C}^{LN_sP\times 2(M+N_s-1)K}$ denotes the generalized Sylvester matrix formed from $\{\mathbf{H}(m)\}_{m=0}^{M-1}$ [cf. (1)] and $\mathbf{v} \in \mathbb{C}^{LN_sP\times 1}$ contains the overall noise samples.

The derivation of the CRB requires the knowledge of the exact distribution of $\mathbf{y}$, which is in general difficult to obtain. In the sequel, we assume that $\mathbf{y}$ follows a Gaussian distribution with zero-mean and covariance matrix:

$$\mathbf{R}_{\mathbf{y}} = \mathcal{T}_{N_s}(\mathbf{H})\mathcal{T}_{N_s}^H(\mathbf{H}) + \sigma_v^2\mathbf{I}_{LN_sP} = \mathcal{T}_{N_s}(\bar{\mathbf{H}})\mathcal{T}_{N_s}^H(\bar{\mathbf{H}}) + \mathcal{T}_{N_s}(\tilde{\mathbf{H}})\mathcal{T}_{N_s}^H(\tilde{\mathbf{H}}) + \sigma_v^2\mathbf{I}_{LN_sP}, \tag{36}$$

where we used the fact that $E\{\mathbf{d}\mathbf{d}^H\} = \mathbf{I}_{2K(M+N_s-1)}$ (see Section 2.2) and the second equality of (36) was due to $\mathbf{H}(m) = [\bar{\mathbf{H}}(m),\ \tilde{\mathbf{H}}(m)]$ for $m = 0, \ldots, M-1$. Comparing with the CRB based on the Gaussian assumption is well motivated not only because it is easily computable, but also because the Gaussian CRB is the lower bound for the covariance matrices of a large class of estimation methods, regardless of the data distribution [30, p. 293].

Let $\mathbf{f} \triangleq [\bar{\mathbf{f}}^T,\ \tilde{\mathbf{f}}^T]^T \in \mathbb{C}^{2LW\times 1}$ and $\boldsymbol{\theta} \triangleq [\Re^T\{\mathbf{f}\},\ \Im^T\{\mathbf{f}\}]^T \in \mathbb{R}^{4LW\times 1}$. For a Gaussian process with zero-mean and covariance matrix $\mathbf{R}_{\mathbf{y}}$, the (r,s)th element of the Fisher information matrix (FIM) is given by (e.g., [30, Appendix B])

$$[\mathbf{J}(\boldsymbol{\theta})]_{s,t} = \operatorname{tr}\left\{\frac{\partial \mathbf{R}_{\mathbf{y}}}{\partial \theta_s}\mathbf{R}_{\mathbf{y}}^{-1}\frac{\partial \mathbf{R}_{\mathbf{y}}}{\partial \theta_t}\mathbf{R}_{\mathbf{y}}^{-1}\right\}, \quad s, t = 1, \ldots, 4LW, \tag{37}$$

where θ_s denotes the sth element of $\boldsymbol{\theta}$. By (36), we have

$$\frac{\partial \mathbf{R}_{\mathbf{y}}}{\partial \theta_s} = \frac{\partial \mathcal{T}_{N_s}(\bar{\mathbf{H}})}{\partial \theta_s}\mathcal{T}_{N_s}^H(\bar{\mathbf{H}}) + \mathcal{T}_{N_s}(\bar{\mathbf{H}})\left(\frac{\partial \mathcal{T}_{N_s}(\bar{\mathbf{H}})}{\partial \theta_s}\right)^H + \frac{\partial \mathcal{T}_{N_s}(\tilde{\mathbf{H}})}{\partial \theta_s}\mathcal{T}_{N_s}^H(\tilde{\mathbf{H}}) + \mathcal{T}_{N_s}(\tilde{\mathbf{H}})\left(\frac{\partial \mathcal{T}_{N_s}(\tilde{\mathbf{H}})}{\partial \theta_s}\right)^H. \tag{38}$$

To facilitate the evaluation of $\partial \mathcal{T}_{N_s}(\bar{\mathbf{H}})/\partial \theta_s$ and $\partial \mathcal{T}_{N_s}(\tilde{\mathbf{H}})/\partial \theta_s$, we decompose the spreading code matrices in (25) as follows:

$$\bar{\boldsymbol{C}}_k \triangleq \begin{bmatrix}\bar{\boldsymbol{C}}_{k,0}^T, & \ldots, & \bar{\boldsymbol{C}}_{k,M-1}^T\end{bmatrix}^T, \qquad \tilde{\boldsymbol{C}}_k \triangleq \begin{bmatrix}\tilde{\boldsymbol{C}}_{k,0}^T, & \ldots, & \tilde{\boldsymbol{C}}_{k,M-1}^T\end{bmatrix}^T, \tag{39}$$

where $\bar{\boldsymbol{C}}_{k,m}$ and $\tilde{\boldsymbol{C}}_{k,m}$, for $m = 0, \ldots, M-1$, are $LP \times LW$ matrices. This decomposition, along with (21) and (24)–(26), allows the following expressions of the channel matrices:

$$\bar{\mathbf{H}}(m) = \begin{bmatrix}\bar{\boldsymbol{C}}_{1,m}\bar{\mathbf{f}}, & \ldots, & \bar{\boldsymbol{C}}_{K,m}\bar{\mathbf{f}}\end{bmatrix}, \qquad \tilde{\mathbf{H}}(m) = \begin{bmatrix}\tilde{\boldsymbol{C}}_{1,m}\tilde{\mathbf{f}}, & \ldots, & \tilde{\boldsymbol{C}}_{K,m}\tilde{\mathbf{f}}\end{bmatrix}, \quad m = 0, \ldots, M-1.$$

Hence,

$$\begin{aligned}
&\frac{\partial \bar{\mathbf{H}}(m)}{\partial \Re\{\bar{f}_s\}} = \begin{bmatrix}\bar{\boldsymbol{C}}_{1,m}(:,s), & \ldots, & \bar{\boldsymbol{C}}_{K,m}(:,s)\end{bmatrix} \triangleq \bar{\boldsymbol{\Phi}}_{s,m},\\
&\frac{\partial \tilde{\mathbf{H}}(m)}{\partial \Re\{\tilde{f}_s\}} = \begin{bmatrix}\tilde{\boldsymbol{C}}_{1,m}(:,s), & \ldots, & \tilde{\boldsymbol{C}}_{K,m}(:,s)\end{bmatrix} \triangleq \tilde{\boldsymbol{\Phi}}_{s,m},\\
&\frac{\partial \bar{\mathbf{H}}(m)}{\partial \Im\{\bar{f}_s\}} = j\bar{\boldsymbol{\Phi}}_{s,m}, \qquad \frac{\partial \tilde{\mathbf{H}}(m)}{\partial \Im\{\tilde{f}_s\}} = j\tilde{\boldsymbol{\Phi}}_{s,m},\\
&\frac{\partial \bar{\mathbf{H}}(m)}{\partial \Re\{\tilde{f}_s\}} = \frac{\partial \bar{\mathbf{H}}(m)}{\partial \Im\{\tilde{f}_s\}} = \frac{\partial \tilde{\mathbf{H}}(m)}{\partial \Re\{\bar{f}_s\}} = \frac{\partial \tilde{\mathbf{H}}(m)}{\partial \Im\{\bar{f}_s\}} = \mathbf{0},
\end{aligned} \tag{40}$$

where the Matlab notation $\mathbf{A}(:, s)$ is used to denote the sth column of any matrix $\mathbf{A}$, and $\bar{f}_s$ (respectively $\tilde{f}_s$) denotes the sth element of $\bar{\mathbf{f}}$ (respectively $\tilde{\mathbf{f}}$). It follows that

$$\begin{aligned}
&\frac{\partial \mathcal{T}_{N_s}(\bar{\mathbf{H}})}{\partial \Re\{\bar{f}_s\}} = \mathcal{T}_{N_s}(\bar{\boldsymbol{\Phi}}_s), \qquad \frac{\partial \mathcal{T}_{N_s}(\tilde{\mathbf{H}})}{\partial \Re\{\tilde{f}_s\}} = \mathcal{T}_{N_s}(\tilde{\boldsymbol{\Phi}}_s),\\
&\frac{\partial \mathcal{T}_{N_s}(\bar{\mathbf{H}})}{\partial \Im\{\bar{f}_s\}} = j\mathcal{T}_{N_s}(\bar{\boldsymbol{\Phi}}_s), \qquad \frac{\partial \mathcal{T}_{N_s}(\tilde{\mathbf{H}})}{\partial \Re\{\tilde{f}_s\}} = j\mathcal{T}_{N_s}(\tilde{\boldsymbol{\Phi}}_s),\\
&\frac{\partial \mathcal{T}_{N_s}(\bar{\mathbf{H}})}{\partial \Re\{\tilde{f}_s\}} = \frac{\partial \mathcal{T}_{N_s}(\bar{\mathbf{H}})}{\partial \Im\{\tilde{f}_s\}} = \frac{\partial \mathcal{T}_{N_s}(\tilde{\mathbf{H}})}{\partial \Re\{\bar{f}_s\}} = \frac{\partial \mathcal{T}_{N_s}(\tilde{\mathbf{H}})}{\partial \Im\{\bar{f}_s\}} = \mathbf{0},
\end{aligned} \tag{41}$$

where $\mathcal{T}_{N_s}(\bar{\boldsymbol{\Phi}}_s)$ [respectively $\mathcal{T}_{N_s}(\tilde{\boldsymbol{\Phi}}_s)$] is the $LN_sP \times (M + N_s - 1)K$ generalized Sylvester matrix formed from $\{\bar{\boldsymbol{\Phi}}(s, m)\}_{m=0}^{M-1}$ [respectively $\{\tilde{\boldsymbol{\Phi}}(s, m)\}_{m=0}^{M-1}$]. Using (41) in (38), which is subsequently substituted into (37), we can compute the $4LW \times 4LW$ FIM $\mathbf{J}(\boldsymbol{\theta})$ entry by entry.

The FIR $\mathbf{J}(\boldsymbol{\theta})$, however, is singular due to the scalar ambiguity inherent in all blind channel identification problems [27–29]. To eliminate the ambiguity, various constraints can be enforced to regularize the estimation problem, e.g., the unit-norm constraint in Section 3 plus a constraint on the phase of one element of $\bar{\mathbf{f}}$ and $\tilde{\mathbf{f}}$, respectively; see [31]. Suppose we have a set of $N_u < 4LW$ constraints on the parameter $\boldsymbol{\theta}$ defined through

$$\mathbf{u}(\boldsymbol{\theta}) = \mathbf{0}. \tag{42}$$

Define the $4LW \times N_u$ gradient matrix of the constraints:

$$\mathbf{U}(\boldsymbol{\theta}) = \frac{\partial \mathbf{u}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}^T}.$$

The gradient matrix is assumed to have full row rank for any $\boldsymbol{\theta}$ satisfying the constraints (42) (i.e., the constraints are not redundant). Let $\mathbf{V} \in \mathbb{R}^{4LW \times (4LW - N_u)}$ whose columns form a basis for the null space of $\mathbf{U}(\boldsymbol{\theta})$, i.e., $\mathbf{U}(\boldsymbol{\theta})\mathbf{V} = \mathbf{0}$. Then, the constrained CRB is given by [28,31]

$$\mathrm{CRB}\big(\boldsymbol{\theta}; \mathbf{u}(\boldsymbol{\theta}) = \mathbf{0}\big) = \mathbf{V}\big[\mathbf{V}^T\mathbf{J}(\boldsymbol{\theta})\mathbf{V}\big]^{-1}\mathbf{V}^T. \tag{43}$$

5. Linear detection and ST decoding

Once estimates of the channel vectors $\bar{\mathbf{f}}$ and $\tilde{\mathbf{f}}$ are available, various detection techniques can be employed for user detection. Although the nonlinear ML (maximum likelihood) detector yields the optimum performance, it has an exponential complexity in K and, thus, is of only theoretical interest [7]. In this section, we consider linear detection schemes, including the single-user MF (matched-filter) detector and the multiuser ZF (zero-forcing) and MMSE (minimum mean-squared error) detectors, for demodulation in ST-coded CDMA systems. We also discuss how to perform ST decoding to fully recover the information symbols.

5.1. MF detector

Assume that user k is of interest. Decompose the composite channel vectors $\bar{\mathbf{h}}_k$ and $\tilde{\mathbf{h}}_k$ as [cf. (21)]: $\bar{\mathbf{h}}_k \triangleq [\bar{\mathbf{h}}_1^T(0), \ \ldots, \ \bar{\mathbf{h}}_K^T(M-1)]^T$ and $\tilde{\mathbf{h}}_k \triangleq [\tilde{\mathbf{h}}_1^T(0), \ \ldots, \ \tilde{\mathbf{h}}_K^T(M-1)]^T$. Let $\mathbf{H}_k(m) \triangleq [\bar{\mathbf{h}}_k(m), \ \tilde{\mathbf{h}}_k(m)] \in \mathbb{C}^{LP \times 2}$, $m = 0, \ldots, M-1$. We separate the contribution of user k from that of other users to the receiver signal $\mathbf{y}_N(n)$ and rewrite (14) as

$$\mathbf{y}_N(n) = \mathcal{T}_N(\mathbf{H}_k)\mathbf{d}_{N,k}(n) + \mathrm{MUI} + \mathbf{v}_N(n),$$

where $\mathcal{T}_N(\mathbf{H}_k) \in \mathbb{C}^{LNP \times 2(M+N-1)}$ is the generalized Sylvester matrix formed from $\{\mathbf{H}_k(m)\}_{m=0}^{M-1}$ [cf. (1)], the MUI (multiuser interference) term lumps signals from all users other than user k, and

$$\mathbf{d}_{N,k}(n) = \big[\mathbf{d}_k^T(n - M + 1), \quad \ldots, \quad \mathbf{d}_k^T(n + N - 1)\big]^T \in \mathbb{C}^{2(M+N-1)\times 1}, \tag{44}$$

$$\mathbf{d}_k(n) = \big[\bar{d}_k(n), \quad \tilde{d}_k(n)\big]^T. \tag{45}$$

The MF detector "matches" only to the composite channel of user k, while treating the MUI as white Gaussian noise:

$$\mathcal{W}_{\mathrm{MF}_k} = \mathcal{T}_N^{\dagger H}(\mathbf{H}_k). \tag{46}$$

The MF detector outputs an estimate of $\mathbf{d}_{N,k}(n)$ as $\hat{\mathbf{d}}_{N,k}(n) = \mathcal{W}_{\mathrm{MF}_k}^H \mathbf{y}_N(n)$.

5.2. ZF detector

The ZF detector is given by (e.g., [7])

$$\mathcal{W}_{\mathrm{ZF}} = \mathcal{T}_N^{\dagger H}(\mathbf{H}), \tag{47}$$

where $\mathcal{T}_N(\mathbf{H})$ is defined in (14). The ZF detector $\mathcal{W}_{\mathrm{ZF}}$ satisfies the ZF constraint: $\mathcal{W}_{\mathrm{ZF}}^H \mathcal{T}_N(\mathbf{H}) = \mathbf{I}_{2K(M+N-1)}$, by which it removes completely the MUI and ISI at the expense of increasing the additive noise level [7]. We note that $\mathcal{W}_{\mathrm{ZF}}$ requires the spreading codes of *all* users. It demodulates *all* users simultaneously and outputs an estimate of $\mathbf{d}_N(n)$ as $\hat{\mathbf{d}}_N(n) = \mathcal{W}_{\mathrm{ZF}}^H \mathbf{y}_N(n) \in \mathbb{C}^{2K(M+N-1)\times 1}$.

5.3. MMSE detector

The MMSE detector minimizes the mean-squared error criterion:

$$\mathcal{W}_{\mathrm{MMSE}_k} = \arg \min_{\mathcal{W} \in \mathbb{C}^{LNP\times 2(M+N-1)}} E\left\{\left\|\mathcal{W}^H \mathbf{y}_N(n) - \mathbf{d}_{N,k}(n)\right\|^2\right\},$$

the solution to which is well-known (e.g., [7])

$$\mathcal{W}_{\mathrm{MMSE}_k} = \mathbf{R}_{\mathbf{y}_N}^{-1} E\left\{\mathbf{y}_N(n)\mathbf{d}_{N,k}^H(n)\right\} = \mathbf{R}_{\mathbf{y}}^{-1} \mathcal{T}_N(\mathbf{H}_k). \tag{48}$$

In practice, $\mathbf{R}_{\mathbf{y}_N}$ can be replaced by $\hat{\mathbf{R}}_{\mathbf{y}_N}$ in (30) or some other recursively computed covariance matrix estimate. Hence, unlike the ZF detector, the spreading codes of other users are not required by the MMSE detector. The MMSE detector outputs an estimate of $\mathbf{d}_{N,k}(n)$ as $\hat{\mathbf{d}}_{N,k}(n) = \mathcal{W}_{\mathrm{MMSE}_k}^H \mathbf{y}_N(n)$.

5.4. ST decoding

Form $\hat{\mathbf{d}}_k(n) = [\hat{\bar{d}}_k(n),\ \hat{\tilde{d}}_k(n)]^T$ from the detector output $\hat{\mathbf{d}}_{N,k}(n)$ [see (44)–(45)]. Assume proper alignment/synchronization at the receiver such that two adjacent blocks of $\{\hat{\mathbf{d}}_k(n)\}$ form an estimate of $\boldsymbol{d}_k(n)$ [see (16)]

$$\hat{\boldsymbol{d}}_k(n) \triangleq \left[\hat{\mathbf{d}}_k^T(2n),\quad \hat{\mathbf{d}}_k^T(2n+1)\right]^T \triangleq \left[\hat{\bar{d}}_k(2n),\quad \hat{\tilde{d}}_k(2n),\quad \hat{\bar{d}}_k(2n+1),\quad \hat{\tilde{d}}_k(2n+1)\right]^T.$$

Reversing the ST encoding process (15), we obtain the soft estimates of $\{b_k(2n), b_k(2n+1)\}$ as follows:

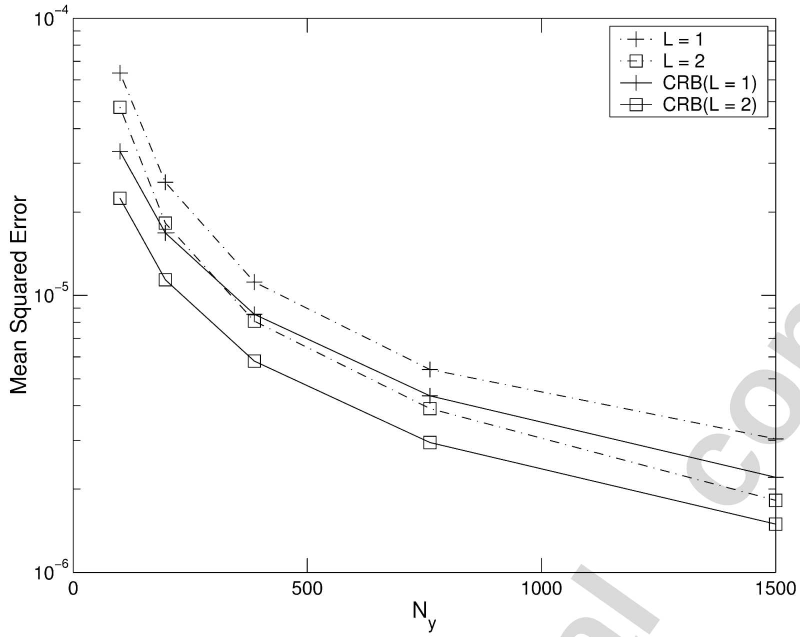
$$\hat{b}_k(2n) = \left[\hat{\bar{d}}_k(2n) + \hat{\tilde{d}}_k^*(2n+1)\right]/2, \qquad \hat{b}_k(2n+1) = \left[\hat{\tilde{d}}_k(2n) - \hat{\bar{d}}_k^*(2n+1)\right]/2.$$

Finally, the hard estimate $\hat{\hat{b}}_k(n)$ is obtained by comparing the soft estimate $\hat{b}_k(n)$ with every constellation point:

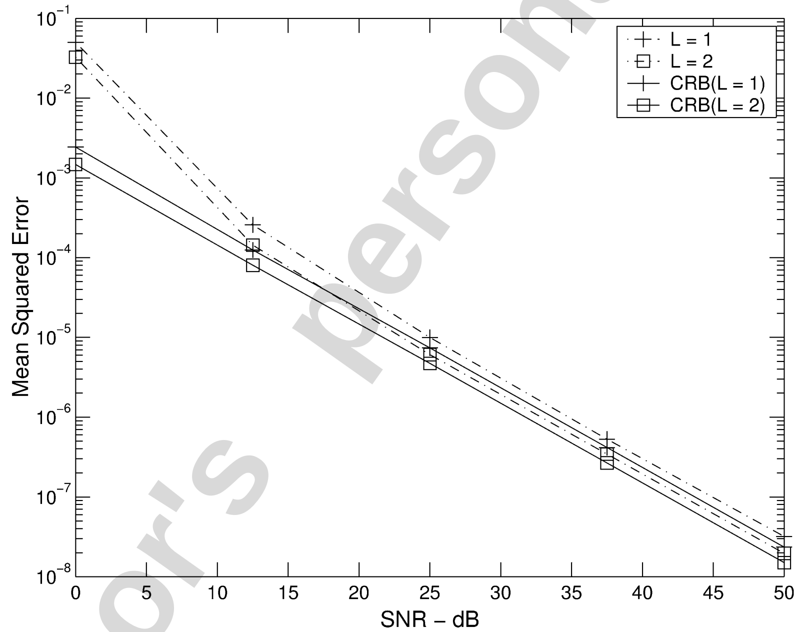
$$\hat{\hat{b}}_k(n) = \arg\min_{b\in\mathcal{B}} \left|\hat{b}_k(n) - b\right|.$$

6. Simulation results

In this section, we present simulation results to illustrate the performance of the proposed subspace-based blind channel estimation algorithm and the linear multiuser detection schemes for ST-coded CDMA systems. We consider systems equipped with $L = 1$ or $L = 2R\times$'s. The spreading codes are randomly generated with processing gain $P = 32$. The channels $\{f^{(l)}(p)\}$ and $\{\tilde{f}^{(l)}(q)\}$ are modeled as FIR filters with duration $W = 30$ chips; the individual taps are generated as independent complex Gaussian random variables with zero-mean and variance $1/W$. The information symbols are drawn from a unit-energy QPSK constellation. The SNR is defined as $\mathrm{SNR} = 10\log_{10} 1/\sigma_v^2$ in dB.



(a)



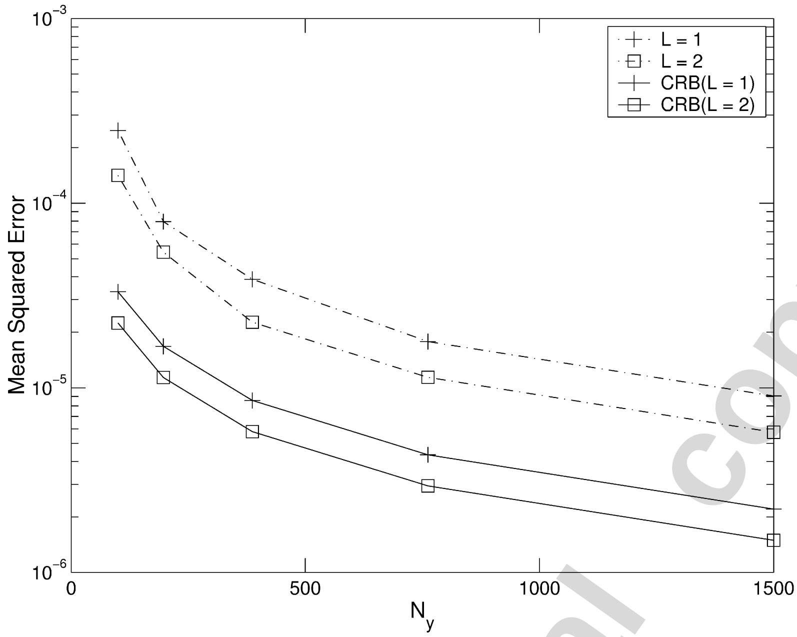
(b)

Fig. 2. (a) MSE and CRB versus N_y using all users' spreading codes when SNR $= 25$ dB, $K = 5$, $P = 32$, and $W = 30$; (b) MSE and CRB versus SNR using all users' spreading codes when $N_y = 500$, $K = 5$, $P = 32$, and $W = 30$.

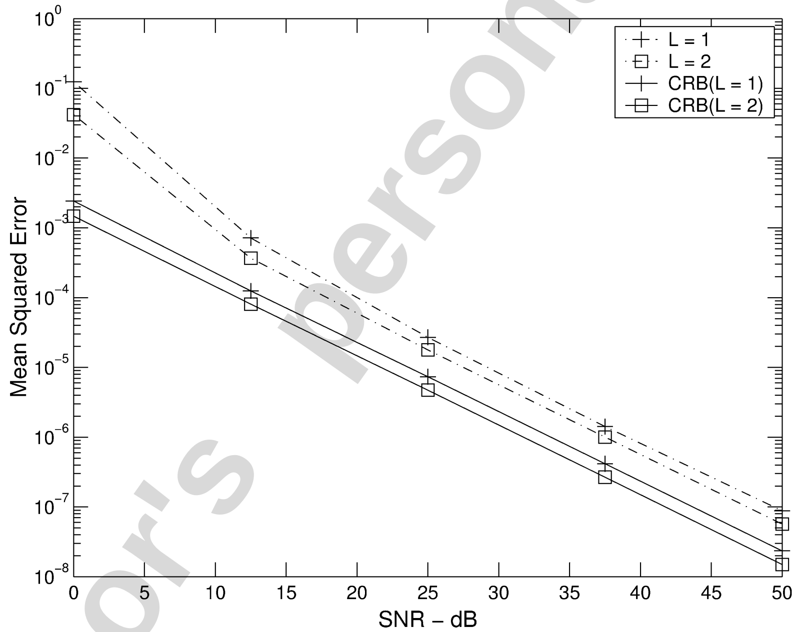
6.1. Channel estimation

The performance measure is the averaged MSE (mean-squared error) of the channel estimates, defined as: $\mathrm{MSE}(\hat{\bar{\mathbf{f}}}) = \frac{1}{LW-1}\sum_{i=1}^{LW-1}\mathrm{MSE}(\hat{\bar{f}}_i)$, where $\hat{\bar{f}}_i$ denotes the estimate of the ith element of $\bar{\mathbf{f}}$; $\mathrm{MSE}(\hat{\tilde{\mathbf{f}}})$ is similarly defined. The channel estimates $\hat{\bar{\mathbf{f}}}$ and $\hat{\tilde{\mathbf{f}}}$ are normalized with respect to the first element of $\bar{\mathbf{f}}$ and $\tilde{\mathbf{f}}$, respectively, in order to remove the inherent scalar ambiguity in the estimates. The smoothing factor $N = 3$ is used for channel estimation. The MSE results presented below are averaged over 100 independent Monte Carlo trials.

Figure 2a depicts the MSE of the channel estimates versus the number of data samples N_y when $K = 5$ and SNR $= 25$ dB, whereas Fig. 2b shows the MSE versus the SNR when $K = 5$ and $N_y = 500$. The MSEs in these



(a)



(b)

Fig. 3. (a) MSE (and CRB) versus N_y using one user's spreading codes when SNR $= 25$ dB, $K = 5$, $P = 32$, and $W = 30$; (b) MSE (and CRB) versus SNR using one user's spreading codes when $N_y = 500$, $K = 5$, $P = 32$, and $W = 30$.

figures correspond to channel estimates that use the spreading codes of all users [i.e., (33)–(34)]. The MSEs of the channel estimates that use the spreading codes of only the desired user [i.e., (31)–(32)] are depicted in Figs. 3a and 3b. We also plot the unconditional CRB derived in Section 4 in these figures to illustrate the lower performance bound. We note that channel estimates are *consistent* in the sense that as the SNR or N_y increases, the MSE decreases monotonically. The channel estimates for the $2R\times$ system are seen to be always more accurate than those for the $1R\times$ system, even though the number of unknown channel coefficients doubles in the former system. We also note that the channel estimation accuracy differs noticeably between using all users' spreading codes and using only one user's spreading codes. It should be stressed that the CRB assumes the knowledge of the all spreading codes; it is thus unlikely for any estimators that use the spreading codes of only the desired user, e.g., the estimator as in (31)–(32), to achieve the CRB.

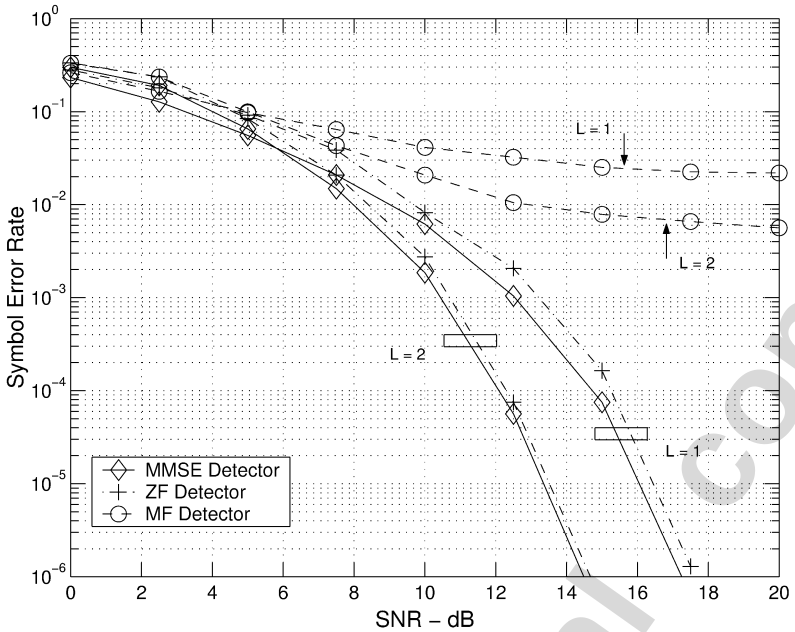


Fig. 4. Symbol error rate versus SNR using estimated channels when $K = 5$, $N = 3$, $P = 32$, and $W = 30$.

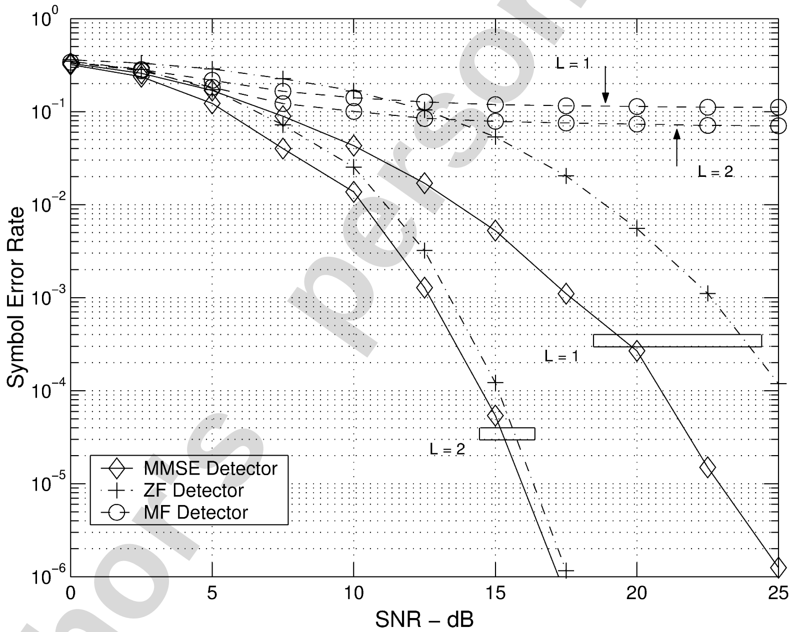


Fig. 5. Symbol error rate versus SNR using estimated channels when $K = 10$, $N = 3$, $P = 32$, and $W = 30$.

6.2. Multiuser detection

We next examine the performance of the linear detectors discussed in Section 5 by using the blind channel estimates in these detectors. The channel coefficients are estimated with $N_y = 1000$ samples of data, using a smoothing factor of $N = 3$ and *the spreading codes of only the desired user*. The following SER (symbol error rate) results are averaged over 500 *independent* channel realizations to emulate a Rayleigh fading environment.

Figure 4 depicts the SER as a function of the SNR of the linear MF, ZF, and MMSE detectors when $K = 5$ users are transmitting simultaneously, whereas Fig. 5 shows the SER results when $K = 10$. We note that the multiuser ZF and MMSE detectors outperform significantly the single-user MF detector. The poor performance of the MF detector is due to the MUI which is treated as noise and not exploited for detection. We also note that the MMSE detector outperforms the ZF detector, especially when $L = 1$ and $K = 10$. Since the ZF detector requires the spreading codes

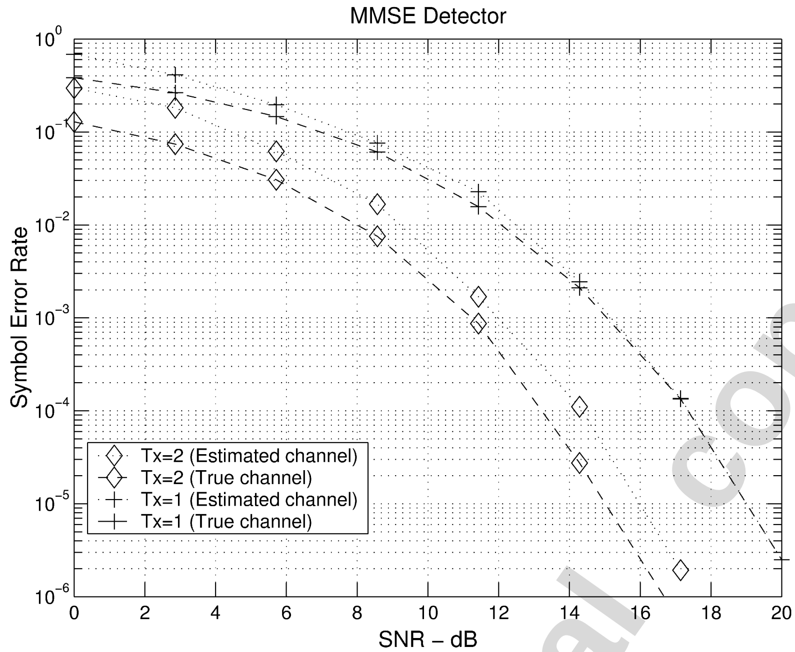


Fig. 6. Symbol error rate of the MMSE detector with/without ST coding versus SNR using true and estimated channels when $K = 5$, $N = 3$, $P = 32$, and $W = 30$.

of all users while the MMSE detector requires the spreading codes of only the desired user (see Sections 5.2 and 5.3), the latter is clearly more attractive in downlink applications.

6.3. Diversity advantage

Finally, we consider the diversity advantage offered by ST coding. We compare the ST-coded CDMA system with the conventional CDMA system using $1T\times$ without ST coding. Figure 6 depicts the SER of these two systems using MMSE detection when $K = 5$ and $L = 1$ (i.e., $1R\times$ for both systems) in Rayleigh fading channels. The MMSE detection is implemented by using both the *true* and *estimated* CSI. The CSI for the ST-coded system is estimated by our proposed channel estimator whereas the CSI for the conventional CDMA system is estimated by the approach in [25], both using a total of $N_y = 500$ data samples. Figure 6 indicates a diversity gain of more than 3 dB achieved by the ST-coded system, which clearly motivates the use of ST coding in CDMA systems. We also note that the performance between using the true and estimate CSI is quite small.

7. Conclusions

We have investigated the problem of blind channel identification and linear multiuser detection for ST-coded CDMA systems operating in frequency-selective fading environments. The classical MIMO identification results have been utilized to determine the identifiability conditions for ST-coded CDMA systems. Under these identifiability conditions, a subspace-based blind channel estimator has been proposed, which yields consistent channel estimates up to a scaling factor. We have also derived an unconditional CRB for the blind channel identification problem. The CRB is not conditioned on the unknown information symbols, which makes it a more suitable performance bench mark for blind estimators than CRBs which are conditioned on the information symbols. The proposed channel estimator has been used with three linear detectors for ST-coded CDMA systems, namely the single-user MF detector and the multiuser ZF and MMSE detectors, and the performance of the resulting receivers has been compared with one another in multipath Rayleigh fading channels. The performance gain achieved by the ST-coded CDMA systems over conventional CDMA systems without ST coding has also been demonstrated.

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