Detection With Target-Induced Subspace Interference

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***Abstract*—In this letter, we consider the detection of a multichannel signal with an unknown amplitude in colored noise, when there is a covariance mismatch between the null and alternative hypotheses. Specifically, the covariance mismatch is caused by a target-induced subspace interference that is present only under the alternative hypothesis. According to the signal model, we propose a detector involving the following steps. The observation is first projected to the orthogonal complement of the signal to be detected, followed by a second projection to the interference subspace. Then, the energy of the doubly projected signal (residual) is computed. If the residual energy is small, the proposed detector reduces to the standard matched filter (MF), which ignores the subspace interference; otherwise, a modified test statistic is employed for additional interference cancellation. Simulation results are presented to demonstrate the effectiveness of the proposed detector.**

***Index Terms*—Adaptive detection, hypothesis test, subspace interference.**

I. Introduction

Detection of a deterministic multichannel signal known up to an unknown (complex) scaling factor in the presence of a colored noise is a fundamental problem in many applications, including wireless communications, seismic analysis, sonar and radar [1]. Given an $N\times 1$ complex output vector $\mathbf{x}$ from spatial and/or temporal sampling, the problem of interest involves a binary composite hypothesis testing [2]–[6]:

$$\begin{aligned} H_0: &\quad \mathbf{x} = \mathbf{w}, \\ H_1: &\quad \mathbf{x} = \alpha\mathbf{s} + \mathbf{w} \end{aligned} \tag{1}$$

where $\mathbf{s} \in \mathbb{C}^{N\times 1}$ is the *known* steering vector, α is an *unknown* complex-valued amplitude, $\mathbf{w}$ is a complex Gaussian noise with zero-mean and covariance matrix $\mathbf{R}$, i.e., $\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R})$.

If $\mathbf{R}$ is known, the generalized likelihood ratio test (GLRT) turns out to be the conventional matched filter (MF) [5]:

$$T_{\mathrm{MF}} = \frac{\left|\mathbf{s}^H\mathbf{R}^{-1}\mathbf{x}\right|^2}{\mathbf{s}^H\mathbf{R}^{-1}\mathbf{s}}. \tag{2}$$

If $\mathbf{R}$ has a subspace structure, i.e., $\mathbf{R} = \mathbf{H}\boldsymbol{\Sigma}\mathbf{H}^H + \sigma^2\mathbf{I}$ [2], [3], where the interference subspace is spanned by the columns of $\mathbf{H} \in \mathbb{C}^{N\times r}$, $r \leq N$, and σ^2 is the variance of the thermal noise, the GLRT turns out to be the decorrelated matched subspace detector [3, Case B, eq. (9)]. If $\mathbf{R}$ is unknown, training signals $\mathbf{x}_k$, $k = 1, \ldots, K$, with the same covariance matrix $\mathbf{R}$, are adaptively used to estimate the covariance matrix $\mathbf{R}$. The classical Kelly's GLRT [4] and adaptive matched filter (AMF) [5] are solutions in this category. If $\mathbf{R}$ has a subspace structure for adaptive detection, the maximum invariant framework can be applied [6].

We consider here a different scenario where the target incurs an *additional* subspace interference under the alternative hypothesis, which is absent from the null hypothesis. Mathematically, we have

$$\begin{aligned} H_0: &\quad \mathbf{x} = \mathbf{w}_0, \\ H_1: &\quad \mathbf{x} = \alpha\mathbf{s} + \mathbf{w}_1 \end{aligned} \tag{3}$$

where $\mathbf{w}_0 \sim \mathcal{CN}(\mathbf{0}, \mathbf{R})$ denotes a noise under H_0 which may collectively account for the thermal noise, clutter, and jamming signals, while $\mathbf{w}_1 \sim \mathcal{CN}(\mathbf{0}, \mathbf{R} + \mathbf{H}\boldsymbol{\Sigma}\mathbf{H}^H)$ denotes the noise under H_1 which, in addition to $\mathbf{w}_0$, includes a target-induced subspace interference. Alternatively, $\mathbf{w}_1$ is statistically equivalent to $\mathbf{w}_1 = \mathbf{w}_0 + \mathbf{H}\boldsymbol{\theta}$, where the target-induced interference subspace matrix $\mathbf{H} \in \mathbb{C}^{N\times L}$ is assumed to be known and $\boldsymbol{\theta} \in \mathbb{C}^{r\times 1}$ is complex Gaussian distributed with zero mean and *unknown* covariance matrix $\boldsymbol{\Sigma}$, i.e., $\boldsymbol{\theta} \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{\Sigma})$. We further assume that the range spaces of $\mathbf{s}$ and $\mathbf{H}$ are linearly independent. A related work on the covariance mismatch between the two hypotheses is [7], which considers a noise power mismatch, i.e., $\mathbf{w}_0 \sim \mathcal{CN}(\mathbf{0}, \sigma_0^2\mathbf{I})$ and $\mathbf{w}_1 \sim \mathcal{CN}(\mathbf{0}, \sigma_1^2\mathbf{I})$. In this letter, we consider a subspace model for the target-induced interference, which is different from the model used in [7].

We have several reasons to consider the subspace model of target-induced interference. One example is wireless communications in dense multipath (urban or indoor) environments, where in addition to the line-of-sight (LOS) signal component, there may exist a large number of multipath components arriving from different directions at different time delays. The LOS component is relatively strong and can be treated as a deterministic signal, whereas the target-induced multipath components consisting of many randomly attenuated and delayed copies of the LOS target signal are often considered to be stochastic. Such disturbance can be described using a properly selected subspace with unknown coordinates $\boldsymbol{\theta}$. Meanwhile, the covariance matrix $\mathbf{R}$ in this case may include other sources of disturbances (inter-cell interference, thermal noise, and jamming signals). Another example is the multiple-input multiple output (MIMO) radar which usually assumes the transmitters transmit orthogonal probing waveforms with zero cross-correlation. As recently shown in [8], such ideal waveform separation is impossible across all Doppler frequencies and time delays. Hence, target-induced residuals due to non-ideal waveform

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separations cannot be ignored. Since these target residuals only appear when a target is present (the alternative hypothesis), our subspace model of target-induced interference provides an effective way to represent such target residuals.

The purpose of this letter is to develop a detection scheme for the binary hypothesis testing problem in (3).

II. Proposed Detector

First, a pre-whitening process $\mathbf{y} = \mathbf{R}^{-1/2}\mathbf{x}$ converts (3) to

$$\begin{aligned} H_0: &\quad \mathbf{y} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}), \\ H_1: &\quad \mathbf{y} \sim \mathcal{CN}(\alpha\tilde{\mathbf{s}}, \tilde{\mathbf{H}}\boldsymbol{\Sigma}\tilde{\mathbf{H}}^H + \mathbf{I}) \end{aligned} \tag{4}$$

where $\tilde{\mathbf{s}} = \mathbf{R}^{-1/2}\mathbf{s}$ and $\tilde{\mathbf{H}} = \mathbf{R}^{-1/2}\mathbf{H}$. It is seen that the unknown parameters, i.e, the nuisance parameter $\boldsymbol{\Sigma}$ and the signal parameter α, are both within the alternative hypothesis. Subsequently, we apply the principle of GLRT and the detector takes the form of

$$T = \frac{\max_{\alpha,\boldsymbol{\Sigma}} f_1(\mathbf{y}\,|\alpha, \boldsymbol{\Sigma})}{f_0(\mathbf{y})} \tag{5}$$

where $f_1(\mathbf{y}\,|\alpha, \boldsymbol{\Sigma})$ and $f_0(\mathbf{y})$ are, respectively, the likelihood functions under both hypotheses:

$$\begin{aligned} f_1(\mathbf{y}\,|\alpha, \boldsymbol{\Sigma}) &= \frac{1}{\pi^N |\mathbf{C}|} \exp\left\{-(\mathbf{y} - \alpha\tilde{\mathbf{s}})^H \mathbf{C}^{-1}(\mathbf{y} - \alpha\tilde{\mathbf{s}})\right\}, \\ f_0(\mathbf{y}) &= \frac{1}{\pi^N} \exp\left\{-\mathbf{y}^H\mathbf{y}\right\} \end{aligned} \tag{6}$$

with a positive definite matrix $\mathbf{C} \triangleq \tilde{\mathbf{H}}\boldsymbol{\Sigma}\tilde{\mathbf{H}}^H + \mathbf{I}$.

A. Likelihood Function Under the Alternative Hypothesis

In the following, we compute $|\mathbf{C}|$ and $\mathbf{C}^{-1}$ for the likelihood function under H_1. Let $\tilde{\mathbf{H}}\boldsymbol{\Sigma}\tilde{\mathbf{H}}^H = \tilde{\mathbf{H}}(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}})^{-1/2}\boldsymbol{\Gamma}(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}})^{-1/2}\tilde{\mathbf{H}}^H$, where

$$\boldsymbol{\Gamma} \triangleq (\tilde{\mathbf{H}}^H\tilde{\mathbf{H}})^{1/2}\boldsymbol{\Sigma}(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}})^{1/2}. \tag{7}$$

Denote its eigenvalue decomposition (EVD) as $\boldsymbol{\Gamma} = \mathbf{U}\mathbf{E}\mathbf{U}^H$, with $\mathbf{U}$ being a unitary matrix and $\mathbf{E}$ a diagonal matrix. We have

$$\tilde{\mathbf{H}}\boldsymbol{\Sigma}\tilde{\mathbf{H}}^H = \bar{\mathbf{H}}\mathbf{E}\bar{\mathbf{H}}^H \tag{8}$$

where

$$\bar{\mathbf{H}} = \tilde{\mathbf{H}}(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}})^{-1/2}\mathbf{U} \tag{9}$$

is an $N \times r$ matrix with r orthonormal columns. Hence

$$\mathbf{C} = [\bar{\mathbf{H}} \quad \bar{\mathbf{H}}^{\perp}] \begin{bmatrix} \mathbf{E}+\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} [\bar{\mathbf{H}} \quad \bar{\mathbf{H}}^{\perp}]^H. \tag{10}$$

It follows that

$$\begin{aligned} \mathbf{C}^{-1} &= \bar{\mathbf{H}}(\mathbf{E}+\mathbf{I})^{-1}\bar{\mathbf{H}}^H + \bar{\mathbf{H}}^{\perp}\bar{\mathbf{H}}^{\perp H} \\ &= \bar{\mathbf{H}}(\mathbf{E}+\mathbf{I})^{-1}\bar{\mathbf{H}}^H + (\mathbf{I} - \mathbf{P}_{\bar{\mathbf{H}}}) \end{aligned} \tag{11}$$

where the projection matrix is independent of $\mathbf{U}$ since

$$\mathbf{P}_{\bar{\mathbf{H}}} = \bar{\mathbf{H}}\left(\bar{\mathbf{H}}^H\bar{\mathbf{H}}\right)^{-1}\bar{\mathbf{H}}^H = \tilde{\mathbf{H}}(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}})^{-1}\tilde{\mathbf{H}}^H = \mathbf{P}_{\tilde{\mathbf{H}}}. \tag{12}$$

The determinant of $\mathbf{C}$ is

$$|\mathbf{C}| = \prod_{i=1}^{r}(e_i + 1), \tag{13}$$

where e_i is the i-th diagonal element of $\mathbf{E}$. Let $\mathbf{z} = [\bar{\mathbf{H}} \quad \bar{\mathbf{H}}^{\perp}]^H(\mathbf{y} - \alpha\tilde{\mathbf{s}})$ and denote z_i as its i-th element. As a result, the negative log-likelihood function under H_1 is

$$\begin{aligned} &-\ln f_1(\mathbf{y}\,|\alpha, \mathbf{E}, \bar{\mathbf{H}}) \\ &\quad\propto \ln|\mathbf{C}| + (\mathbf{y} - \alpha\tilde{\mathbf{s}})^H\mathbf{C}^{-1}(\mathbf{y} - \alpha\tilde{\mathbf{s}}) \\ &\quad\propto \ln\prod_{i=1}^{r}(e_i+1) + \mathbf{z}^H \begin{bmatrix} (\mathbf{E}+\mathbf{I})^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \mathbf{z} \\ &\quad\propto \sum_{i=1}^{r}\ln(e_i+1) + \sum_{i=1}^{r}\frac{|z_i|^2}{e_i+1} + \sum_{i=r+1}^{N}|z_i|^2. \end{aligned} \tag{14}$$

Hence, the estimate of $\boldsymbol{\Sigma}$ can be determined from the estimates of $\bar{\mathbf{H}}$ (viz. $\mathbf{U}$ of (9)) and the diagonal matrix $\mathbf{E}$.

B. Estimation of $\boldsymbol{\Sigma}$ (viz $\bar{\mathbf{H}}$ and $\mathbf{E}$) and α

For a given α, the cost function is

$$\min_{e_i, \bar{\mathbf{H}}} \sum_{i=1}^{r}\ln(e_i+1) + \sum_{i=1}^{r}\frac{|z_i|^2}{e_i+1} + \sum_{i=r+1}^{N}|z_i|^2, \quad \text{s.t. } e_i \geq 0. \tag{15}$$

Taking the derivative of the above cost function with respect to e_i and equating it to zero yield the ML estimate of e_i

$$\hat{e}_i = \max\{|z_i|^2 - 1, 0\}, \quad i = 1, 2, \ldots, r. \tag{16}$$

Substituting $\hat{e}_i + 1 = \max\{|z_i|^2, 1\}, i = 1, \ldots, r$ into the cost function (15), we have

$$\min_{\bar{\mathbf{H}}} \sum_{\substack{1\leq i\leq r: \\ |z_i|^2\leq 1}} |z_i|^2 + \sum_{\substack{1\leq i\leq r: \\ |z_i|^2 > 1}} \left(\ln|z_i|^2 + 1\right) + \sum_{i=r+1}^{N}|z_i|^2. \tag{17}$$

Recognizing that the last term

$$\begin{aligned} &\sum_{i=r+1}^{N}|z_i|^2 \\ &\quad= (\mathbf{y} - \alpha\tilde{\mathbf{s}})^H\bar{\mathbf{H}}^{\perp}\bar{\mathbf{H}}^{\perp H}(\mathbf{y} - \alpha\tilde{\mathbf{s}}) = (\mathbf{y} - \alpha\tilde{\mathbf{s}})^H\mathbf{P}_{\tilde{\mathbf{H}}}^{\perp}(\mathbf{y} - \alpha\tilde{\mathbf{s}}) \end{aligned}$$

where $\mathbf{P}_{\tilde{\mathbf{H}}}^{\perp} = \mathbf{I} - \mathbf{P}_{\tilde{\mathbf{H}}}$, is not a function of $\mathbf{U}$, the cost function reduces to

$$\min_{\bar{\mathbf{H}}} \sum_{1\leq i\leq r:|z_i|^2\leq 1} |z_i|^2 + \sum_{1\leq i\leq r:|z_i|^2>1} \left(\ln|z_i|^2 + 1\right) \tag{18}$$

where $\mathbf{z}_r = [z_1, z_2, \ldots, z_r]^T = \bar{\mathbf{H}}^H(\mathbf{y} - \alpha\tilde{\mathbf{s}})$ denotes the first r elements of $\mathbf{z}$. Similarly, the energy of $\mathbf{z}_r$, denoted as η:

$$\eta = \|\mathbf{z}_r\|_2^2 = \sum_{1\leq i\leq r}|z_i|^2 = (\mathbf{y} - \alpha\tilde{\mathbf{s}})^H\mathbf{P}_{\tilde{\mathbf{H}}}(\mathbf{y} - \alpha\tilde{\mathbf{s}}) \tag{19}$$

is not a function of $\mathbf{U}$. Subsequently, the optimization can proceed as follows according to the value of η.

- $\eta \leq 1$: This means that $|z_i|^2 \leq 1$, $i = 1, \ldots, r$. The cost function (18) reduces to

$$\min_{\bar{\mathbf{H}}} \sum_{1 \leq i \leq r} |z_i|^2 = \min_{\bar{\mathbf{H}}} \eta. \quad (20)$$

Since η is not a function of $\mathbf{U}$, the estimate of $\bar{\mathbf{H}}$ is

$$\hat{\bar{\mathbf{H}}}_1 = \tilde{\mathbf{H}}(\tilde{\mathbf{H}}^H \tilde{\mathbf{H}})^{-1/2} \hat{\mathbf{U}} \quad (21)$$

where $\hat{\mathbf{U}}$ is an arbitrary $r \times r$ unitary matrix. Then, under this condition, the cost function (17) reduces to

$$\min_{\alpha} \sum_{i=1}^{r} |z_i|^2 + \sum_{i=r+1}^{N} |z_i|^2 = \min_{\alpha} \|\mathbf{y} - \alpha \tilde{\mathbf{s}}\|^2 \quad (22)$$

and the amplitude α can be estimated as

$$\hat{\alpha}_1 = \frac{\tilde{\mathbf{s}}^H \mathbf{y}}{\tilde{\mathbf{s}}^H \tilde{\mathbf{s}}}. \quad (23)$$

- $\eta > 1$: Under this condition, the estimate of $\bar{\mathbf{H}}$ has one column given by (without loss of generality, we assume the first column)

$$\hat{\bar{\mathbf{H}}}_2 [:, 1] = \frac{\mathbf{P}_{\tilde{\mathbf{H}}} (\mathbf{y} - \alpha \tilde{\mathbf{s}})}{\|\mathbf{P}_{\tilde{\mathbf{H}}} (\mathbf{y} - \alpha \tilde{\mathbf{s}})\|} \quad (24)$$

and the remaining $(r-1)$ columns are orthonormal to $\hat{\bar{\mathbf{H}}}_2 [:, 1]$. It follows $\mathbf{z}_r = [\sqrt{\eta}, 0, \ldots, 0]^T$ and (18) reduces to

$$C_0 = \ln \eta + 1 = \ln (\mathbf{y} - \alpha \tilde{\mathbf{s}})^H \mathbf{P}_{\tilde{\mathbf{H}}} (\mathbf{y} - \alpha \tilde{\mathbf{s}}) + 1. \quad (25)$$

In Appendix, we prove that the minimized cost function of (18) is (25), which is obtained by $\hat{\bar{\mathbf{H}}}_2$ of (24). With (18) reducing to (25), the cost function (17) becomes

$$\ln (\mathbf{y} - \alpha \tilde{\mathbf{s}})^H \mathbf{P}_{\tilde{\mathbf{H}}} (\mathbf{y} - \alpha \tilde{\mathbf{s}}) + 1 + (\mathbf{y} - \alpha \tilde{\mathbf{s}})^H \mathbf{P}_{\tilde{\mathbf{H}}}^{\perp} (\mathbf{y} - \alpha \tilde{\mathbf{s}}) \quad (26)$$

as a function of α. The minimizer of (26) (found via searching) gives the estimate of α, denoted as $\hat{\alpha}_2$, when $\eta > 1$.

Remark: The above estimates of α and $\bar{\mathbf{H}}$ are obtained on a condition on η of (19) which is a function of α and hence cannot be checked. To address this issue, we can use an estimate of η along with some estimate of α. For simplicity, we use $\hat{\alpha}_1$ of (23) in (19):

$$\hat{\eta} = \mathbf{y}^H \mathbf{P}_{\tilde{\mathbf{s}}}^{\perp} \mathbf{P}_{\tilde{\mathbf{H}}} \mathbf{P}_{\tilde{\mathbf{s}}}^{\perp} \mathbf{y} = \left\| \mathbf{P}_{\tilde{\mathbf{H}}} \mathbf{P}_{\tilde{\mathbf{s}}}^{\perp} \mathbf{y} \right\|^2 \quad (27)$$

with $\mathbf{P}_{\tilde{\mathbf{s}}}^{\perp} = \mathbf{I} - \tilde{\mathbf{s}}\tilde{\mathbf{s}}^H / (\tilde{\mathbf{s}}^H \tilde{\mathbf{s}})$. Now (27) provides a way to check which pair of estimates, $(\hat{\alpha}_1, \hat{\bar{\mathbf{H}}}_1)$ versus $(\hat{\alpha}_2, \hat{\bar{\mathbf{H}}}_2)$, should be used. Clearly, this is an *ad hoc* procedure, although numerical results show it works well.

C. Proposed Detector

Replacing the above estimates in likelihood function (14) under H_1 and with the likelihood function (6) under H_0, the proposed detector of (5) becomes

$$T = \begin{cases} \mathbf{y}^H \mathbf{y} - \mathbf{y}^H \mathbf{P}_{\tilde{\mathbf{s}}}^{\perp} \mathbf{y} \triangleq T_0, & \text{if } \hat{\eta} \leq 1 \\ \mathbf{y}^H \mathbf{y} - \ln \beta_1 - \beta_2 - 1 \triangleq T_1, & \text{if } \hat{\eta} > 1 \end{cases} \quad (28)$$

where $\hat{\eta}$ is given by (27): $\beta_1 = (\mathbf{y} - \hat{\alpha}_2 \tilde{\mathbf{s}})^H \mathbf{P}_{\tilde{\mathbf{H}}} (\mathbf{y} - \hat{\alpha}_2 \tilde{\mathbf{s}})$ and $\beta_2 = (\mathbf{y} - \hat{\alpha}_2 \tilde{\mathbf{s}})^H \mathbf{P}_{\tilde{\mathbf{H}}}^{\perp} (\mathbf{y} - \hat{\alpha}_2 \tilde{\mathbf{s}})$. It is easy to show that T_0 and T_1 can be expressed in terms of $\mathbf{x}$ and $\mathbf{R}$:

$$T_0 = \frac{\left| \mathbf{s}^H \mathbf{R}^{-1} \mathbf{x} \right|^2}{\mathbf{s}^H \mathbf{R}^{-1} \mathbf{s}}, \; T_1 = \mathbf{x}^H \mathbf{R}^{-1} \mathbf{x} - \ln \beta_1 - \beta_2 - 1 \quad (29)$$

and $\hat{\eta} = \left\| \mathbf{P}_{\mathbf{R}^{-1/2}\mathbf{H}} \mathbf{P}_{\mathbf{R}^{-1/2}\mathbf{s}}^{\perp} \mathbf{R}^{-1/2} \mathbf{x} \right\|^2$, where T_0 coincides with the conventional MF. It is noted that the quantity $\hat{\eta}$ represents the energy of the observed signal after double projection into 1) the orthogonal complement of the whitened steering vector (via $\mathbf{P}_{\tilde{\mathbf{s}}}^{\perp}$ or $\mathbf{P}_{\mathbf{R}^{-1/2}\mathbf{s}}^{\perp}$) and 2) the whitened interference subspace $\langle \mathbf{R}^{-1/2} \mathbf{H} \rangle$ (via $\mathbf{P}_{\mathbf{R}^{-1/2}\mathbf{H}}$). It is seen that the proposed detector uses a two-step procedure: 1) compute the quantity $\hat{\eta}$ from the observation $\mathbf{x}$ and compare it with the integer 1 (note that 1 is the energy of the whitened noise under H_0); 2) if $\hat{\eta} \leq 1$, a conventional matched filter T_0 is used; otherwise, the modified detector T_1 is used.

III. Performance Evaluation

In this section, simulation results are provided to demonstrate the performance of the proposed detector (28). We compare it with 1) the clairvoyant MF (denoted as **MF1**) which takes into account the covariance mismatch and also has knowledge of the subspace covariance matrix $\mathbf{\Sigma}$; and 2) the conventional MF of (2) (denoted as **MF2**) which does not take into account the covariance mismatch between H_0 and H_1. It is expected that the MF1, albeit practically inapplicable, gives a performance benchmark or upperbound on the proposed detector and the MF2. In all simulation examples, we consider the case where $N = 16$ and the steering vector $\mathbf{s}$ is given by the Fourier basis vector $\mathbf{u}(f) = [1, e^{-j2\pi f}, \ldots, e^{-j2\pi(N-1)f}]^T / \sqrt{N}$ with $f = 1.8/N$, i.e., $\mathbf{s} = \mathbf{u}(1.8/N)$. The signal-to-noise ratio (SNR) is defined as

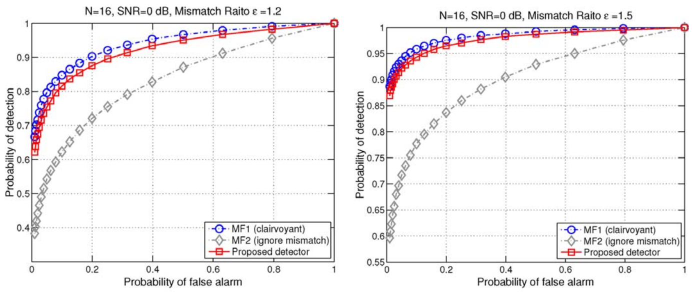
$$\text{SNR} = |\alpha|^2 \mathbf{s}^H \mathbf{R}^{-1} \mathbf{s} \quad (30)$$

where the noise covariance matrix $\mathbf{R}$ is chosen as $[\mathbf{R}]_{\ell\kappa} = \rho^{|\ell - \kappa|}$, with $\rho = 0.9$. The target-induced subspace interference with $r = 3$ is generated by using $\mathbf{H} = [\mathbf{u}(f_1), \mathbf{u}(f_2), \mathbf{u}(f_3)]$ with $\{f_i\}_{i=1,2,3} = [1/N, 2/N, 3/N]$ and the covariance matrix $\mathbf{\Sigma}$ is chosen as $[\mathbf{\Sigma}]_{\ell\kappa} = \gamma \rho^{|\ell - \kappa|}$ with $\rho = 0.6$, where γ is properly chosen to meet the preset covariance mismatch ratio

$$\epsilon = \frac{\text{tr}\{\mathbf{H}\mathbf{\Sigma}\mathbf{H}^H\} + \text{tr}\{\mathbf{R}\}}{\text{tr}\{\mathbf{R}\}} = \frac{\text{tr}\{\mathbf{\Sigma}\}}{\text{tr}\{\mathbf{R}\}} + 1 \geq 1. \quad (31)$$

The performance is evaluated in terms of the receiver operating characteristic (ROC) by using Monte-Carlo trials.

Fig. 1 shows the ROC performance of the proposed detector when $\text{SNR} = 0$ dB in two cases of covariance mismatch (a) $\epsilon = 1.2$; and (b) $\epsilon = 1.5$. The results confirm that the proposed detector has better performance than the MF2 detector which is unaware of the target-induced interference. Comparing Fig. 1(a) with (b) also reveals that the performance gain over the MF2 detector is higher when the covariance mismatch is larger. In both cases, the MF1 detector provides a reference on the optimal detection performance. It is seen that the proposed detector is closer to the optimal bound when the mismatch is larger.



(a) (b)

Fig. 1. Receiver operating characteristic (ROC) curves when $N = 16$ and SNR $= 0$ dB in cases of (a) $\epsilon = 1.2$; and (b) $\epsilon = 1.5$.

IV. Conclusion and Future Work

We have considered a binary hypothesis testing with a covariance mismatch between the two hypotheses, caused by target-induced subspace interference. The proposed detector involves a two-step procedure: First, it computes the energy of the doubly projected received signal. Conditioned on the energy, a conventional matched filter or a modified detector is then chosen to compute the test statistic. Simulation results show the effectiveness of the proposed detector. It is noted that our estimator involves an *ad-hoc* but simple procedure in determining the energy of the residual, and hence is only an approximate ML estimator. A future topic of interest is to find and compare with the exact ML estimator.

Appendix

In this appendix, we use *mathematical induction* to prove that the minimized cost function (18) is given by C_0 of (25), which is obtained by the estimate $\hat{\hat{\mathbf{H}}}_2$ of (24).

First, we consider the *base case* when there is only one nonzero element in $\mathbf{z}_r$, say $\mathbf{z}_r = [z_1, 0, \ldots, 0]^T$ with $\eta = \|\mathbf{z}_r\|_2^2 = |z_1|^2 > 1$. In this case, the cost function (18) reduces to $C_1 = \ln|z_1|^2 + 1 = \ln\eta + 1$ which is the same as the minimum cost function C_0 of (25), achieved by $\hat{\hat{\mathbf{H}}}_2$ in (24).

Next, we assume that the cost function (18) is minimized when there are l nonzero elements in $\mathbf{z}_r$, i.e.,

$$C_l = \sum_{\substack{1\le i\le l:\\ |z_i|^2\le 1}} |z_i|^2 + \sum_{\substack{1\le i\le l:\\ |z_i|^2> 1}} \left(\ln|z_i|^2 + 1\right) \ge \ln\eta_l + 1 \quad (32)$$

with $\eta_l = \|\mathbf{z}_r\|_2^2 = \sum_{i=1}^{l}|z_i|^2 > 1$. We need to prove that, when there are $(l+1) \le r$ nonzero elements in $\mathbf{z}_r$, $C_{l+1} \ge C_0 = \ln\eta_{l+1} + 1$, with $\eta_{l+1} = \sum_{i=1}^{l+1}|z_i|^2 > 1$. Depending on the sum of the first l elements of $\mathbf{z}_r$ and the $(l+1)$-st entry, we have the following four cases.

Case A – $\sum_{i=1}^{l}|z_i|^2 \le 1$ *and* $|z_{l+1}|^2 \le 1$*:* the cost function (18) reduces to $C_{l+1} = \sum_{i=1}^{l+1}|z_i|^2 = \eta_{l+1}$ and we always have $C_{l+1} = \eta_{l+1} > C_0 = \ln\eta_{l+1} + 1$, due to $\eta_{l+1} > 1$ and the inequality $a > \ln a + 1$ if $a > 1$ as applied to $a = \eta_{l+1}$.

Case B – $\sum_{i=1}^{l}|z_i|^2 \le 1$ *and* $|z_{l+1}|^2 > 1$*:* the cost function (18) reduces to $C_{l+1} = \sum_{i=1}^{l}|z_i|^2 + \ln|z_{l+1}|^2 + 1$ with $\eta_{l+1} > 1$, which is still larger than the minimum cost function $C_0 = \ln\eta_{l+1} + 1$ in (25) because, in this case

$$C_{l+1} - C_0 = \sum_{i=1}^{l}|z_i|^2 - \ln\left(1 + |z_{l+1}|^{-2}\sum_{i=1}^{l}|z_i|^2\right)$$
$$\overset{|z_{l+1}|^2>1}{>} \sum_{i=1}^{l}|z_i|^2 - \ln\left(1 + \sum_{i=1}^{l}|z_i|^2\right) \ge 0 \quad (33)$$

since $a \ge \ln(a+1)$ if $0 \le a \le 1$.

Case C – $\sum_{i=1}^{l}|z_i|^2 > 1$ *and* $|z_{l+1}|^2 \le 1$*:* we have the cost function (18)

$$C_{l+1} = C_l + |z_{l+1}|^2 \ge \ln\eta_l + 1 + |z_{l+1}|^2$$
$$\ge \ln\sum_{i=1}^{l+1}|z_i|^2 + 1 = \ln\eta_{l+1} + 1 = C_0 \quad (34)$$

where the first inequality is due to (32) and the second inequality follows from the inequality $\ln a + b \ge \ln(a+b)$, $a > 1$ and $0 \le b \le 1$, applied to components $a = \eta_l$ and $b = |z_{l+1}|^2$.

Case D – $\sum_{i=1}^{l}|z_i|^2 > 1$ *and* $|z_{l+1}|^2 > 1$*:* we have

$$C_{l+1} = C_l + \ln|z_{l+1}|^2 + 1 \ge \ln\sum_{i=1}^{l}|z_i|^2 + \ln|z_{l+1}|^2 + 2$$
$$> \ln\sum_{i=1}^{l+1}|z_i|^2 + 1 = C_0 \quad (35)$$

since $\ln a + \ln b + 1 > \ln(a+b)$ if $a > 1$ and $b > 1$.

As a result, this completes the *inductive step* and closes the proof.

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