A Deterministic Multiuser Code-Timing Estimator for Long-Code Band-Limited CDMA Systems

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***Abstract*—In this letter, we present a *deterministic* multiuser code-timing estimator for asynchronous direct-sequence (DS) code-division multiple-access (CDMA) systems with *aperiodic long* spreading codes and *band-limited* chip waveforms. A key feature of the proposed estimator is that it captures and capitalizes a deterministic structure of the overall interference, namely multiaccess interference (MAI) and intersymbol interference (ISI), in the frequency domain. This allows complete interference elimination in a deterministic manner, which is in general more effective and data-efficient than stochastic approaches. Numerical results show that the proposed estimator can achieve fast acquisition; it is also near-far resistant, providing accurate code acquisition for even *overloaded* systems (i.e., systems with more users than the processing gain) in multipath fading environments.**

***Index Terms*—Aperiodic/long spreading codes, band-limited chip waveforms, code-division multiple access (CDMA), parameter estimation, synchronization.**

I. INTRODUCTION

MULTIUSER code-timing estimation is a challenging task in code-division multiple access (CDMA) systems. Traditional techniques, typically based on matched-filter (MF) estimation [1, Ch.5] that ignores the inherent structure of the MAI, are found inadequate in multiuser environments. Recently, a variety of multiuser acquisition techniques have been proposed by exploiting the structure of the MAI (e.g., [2]–[5] and references therein). However, most of these schemes assume *short* (symbol-periodic) spreading codes. In contrast, most practical CDMA systems, including the IS-95 standard and the majority of 3G CDMA-based wireless networks, make use of *long* (symbol-aperiodic) spreading codes to randomize the interference.

Among limited studies on multiuser acquisition for long-code CDMA, Buzzi and Poor [6] proposed a centralized acquisition scheme for use at the base station (i.e, in the uplink). However, their method assumes *rectangular* chip waveform, which is not bandlimited. Meanwhile, most practical systems utilize bandlimited chip waveforms, such as the square-root raised-cosine pulse. As admitted by the authors of [6], it is nontrivial to extend their scheme to the bandlimited case. The purpose of this letter is to introduce an alternative centralized multiuser acquisition scheme that can deal with both long spreading codes and bandlimited chip waveforms.

Manuscript received January 9, 2004. This work was supported in part by the Army Research Office under Grant DAAD19-03-1-0184, and by the New Jersey Commission on Science and Technology. The associate editor coordinating the review of this letter and approving it for publication was Dr. J. Louveaux.

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Digital Object Identifier 10.1109/LCOMM.2004.833818

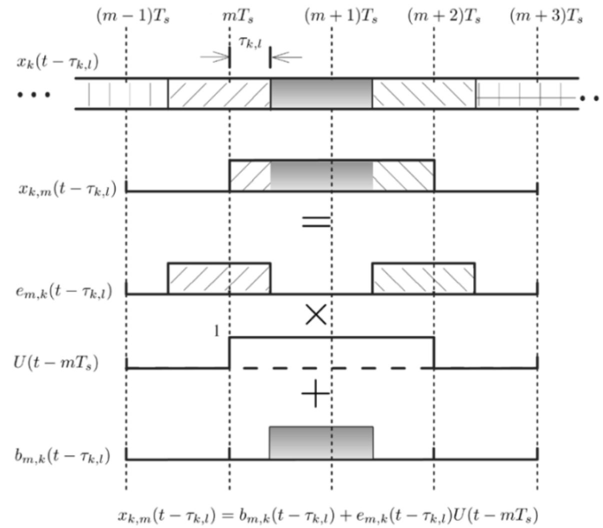

Fig. 1. Graphical illustration of the decomposition of (2).

Notation: Vectors (matrices) are denoted by boldface lower (upper) case letters; all vectors are column vectors; superscripts $(\cdot)^T$, and $(\cdot)^H$ denote transpose and conjugate transpose, respectively; $\star$ denotes convolution; $\mathrm{diag}\{\cdot\}$ denotes a diagonal matrix; $\mathbf{0}$ denotes an all-zero matrix or vector; finally, $\odot$ denotes the elementwise Hadamard product.

II. DATA MODEL

Consider a baseband asynchronous (uplink) K-user DS-CDMA system with *long* spreading codes. Let $p(t)$ be the *band-limited* chip waveform. The transmitted signal for user k is $x'_k(t) = \sum_{m=0}^{M-1} d_k(m)s'_{k,m}(t-mT_s)$, where M is the number of symbols used for acquisition, T_s the symbol period, $d_k(m)$ the mth symbol of user k, and $s'_{k,m}(t)$ the spreading waveform: $s'_{k,m}(t) = \sum_{n=0}^{N-1} c_{k,m}(n)p(t-nT_c)$, with $T_c = T_s/N$ denoting the chip interval, N the *processing gain*, and $\{c_{k,m}(n)\}_{n=0}^{N-1}$ the spreading codes for the mth symbol of user k. Signal $x'_k(t)$ passes through a frequency-selective channel with L_k paths. The signal received at the base station, after chip-matched filtering, is

$$y(t) = \sum_{k=1}^{K}\sum_{l=1}^{L_k} \alpha_{k,l}x_k(t-\tau_{k,l}) + w(t) \tag{1}$$

where $\alpha_{k,l}$ and $\tau_{k,l}$ are, respectively, the attenuation and *code-timing* for the lth path of user k, $w(t)$ the channel noise, and $x_k(t)$ the output of the chip-matched filter given input $x'_k(t)$: $x_k(t) \triangleq x'_k(t) \star p(T_c-t) = \sum_{m=0}^{M-1} d_k(m)s_{k,m}(t-mT_s)$, where $s_{k,m}(t) \triangleq s'_{k,m}(t) \star p(T_c-t)$.

The problem is to estimate the multiuser multipath code-timings $\tau_{k,l}$, $\forall\, k$, $\forall\, l$, from $y(t)$. Once we have the code-timing

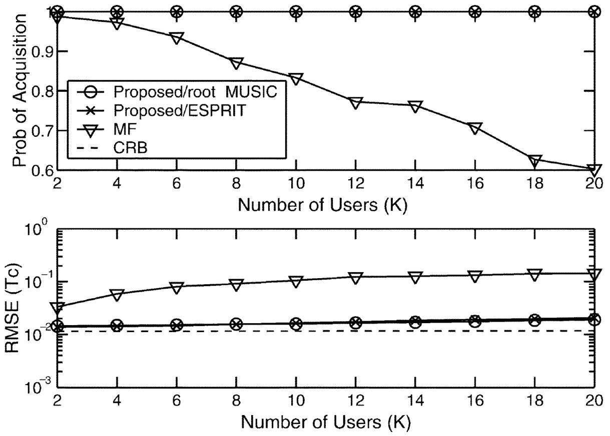

Fig. 2. Performance versus user capacity K when $N = 16$, $M = 100$, $\mathrm{SNR} = 15$ dB, and $\mathrm{NFR} = 10$ dB in multipath channels.

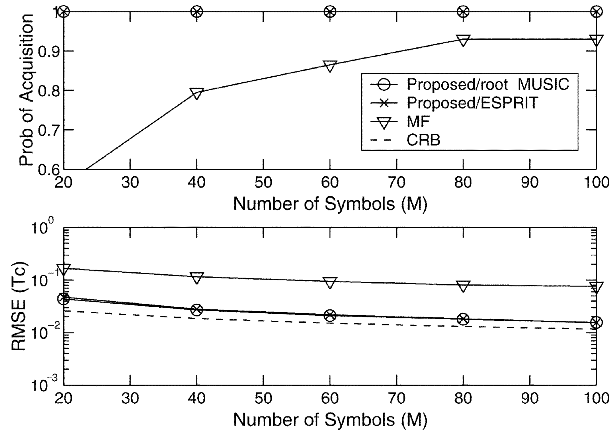

Fig. 3. Performance versus observation time M when $N = 16$, $K = 5$, $\mathrm{SNR} = 15$ dB, and $\mathrm{NFR} = 10$ dB in multipath channels.

estimates, $\alpha_{k,l}$ can be estimated by least-squares. Similar to earlier works (e.g., [2]–[6]), we assume $\max_{k,l} \tau_{k,l} < T_s$; extension to larger delays is possible but skipped for space limitation. Also similar to the centralized scheme of [6], we assume that the data symbols and spreading codes for all users are known. This may correspond an asynchronous CDMA packet network with all users jointly transmitting known preambles before real communication starts [6].

III. Proposed Scheme

First, we split $y(t)$ into *overlapping* blocks, $y_m(t)$, $m = 0, \ldots, M-2$, each with duration $2T_s$ and every two adjacent blocks are offset by T_s seconds. (Note that $M-1$ such overlapping blocks can be formed from the transmission of M symbols.) Specifically, let $U(t)$ be a rectangular window function with duration $2T_s$: $U(t) = 1$, $0 \le t < 2T_s$, and zero, elsewhere. Then, we have $y_m(t) \triangleq y(t)U(t - mT_s)$. Likewise, let $x_{k,m}(t-\tau_{k,l}) \triangleq x_k(t-\tau_{k,l})U(t-mT_s)$. Clearly, $y_m(t)$ is a superposition of $x_{k,m}(t-\tau_{k,l})$, $\forall\, k, \forall\, l$, plus the channel noise.

To quantify the overall interference deterministically, we need find an analytical expression for $x_{k,m}(t-\tau_{k,l})$. To do so, let $b_{m,k}(t-\tau_{k,l}) \triangleq d_k(m)s_{k,m}(t-mT_s-\tau_{k,l})$, $e_{m,k}(t-\tau_{k,l}) \triangleq d_k(m-1)s_{k,m-1}(t-(m-1)T_s-\tau_{k,l}) + d_k(m+1)s_{k,m+1}(t-(m+1)T_s-\tau_{k,l})$, and note the latter denotes the intersymbol interference (ISI) observed in $y_m(t)$. As illustrated in Fig. 1, $x_{k,m}(t-\tau_{k,l})$ can be analytically expressed as

$$x_{k,m}(t-\tau_{k,l}) = b_{m,k}(t-\tau_{k,l}) + e_{m,k}(t-\tau_{k,l})U(t-mT_s). \tag{2}$$

Let $w_m(t) \triangleq w(t)U(t-mT_s)$. It follows from (2) that

$$y_m(t) = \sum_{k=1}^{K}\sum_{l=1}^{L_k} \alpha_{k,l}[b_{m,k}(t-\tau_{k,l}) + e_{m,k}(t-\tau_{k,l})U(t-mT_s)] + w_m(t). \tag{3}$$

For digital processing, $y_m(t)$ is sampled with a sampling interval $T_i = T_c/Q$: $y_m(n) = y_m(t)|_{t=mT_s+nT_i}$, $n = 0, \ldots, 2NQ-1$, where Q is an integer; typically, $Q = 1$ or $Q = 2$. Let $\bar{\mathbf{y}}_m \triangleq [\mathbf{0}_{1\times NQ}, y_m(0), \ldots, y_m(2NQ-1), \mathbf{0}_{1\times NQ}]^T$, viz., it is formed by samples of $y_m(t)$ with NQ zeros padded at the beginning and end. Likewise, let $\bar{\mathbf{b}}_{m,k}(\tau_{k,l})$, $\bar{\mathbf{e}}_{m,k}(\tau_{k,l})$, $\bar{\mathbf{u}}$, and $\bar{\mathbf{w}}_m$ be $4NQ \times 1$ vectors formed from samples of $b_{m,k}(t-\tau_{k,l})$, $e_{m,k}(t-\tau_{k,l})$, $U(t-mT_s)$ and $w_m(t)$, respectively, all treated as signals of duration $4T_s$ with possibly zeros padded at the beginning and end (cf. Fig. 1). Then, the discrete-time form of (3) is

$$\bar{\mathbf{y}}_m = \sum_{k=1}^{K}\sum_{l=1}^{L_k} \alpha_{k,l}\left[\bar{\mathbf{b}}_{m,k}(\tau_{k,l}) + \bar{\mathbf{e}}_{m,k}(\tau_{k,l}) \odot \bar{\mathbf{u}}\right] + \bar{\mathbf{w}}_m. \tag{4}$$

Next, we convert the signals to the frequency domain. Let $\mathbf{y}_m \triangleq \mathcal{F}\{\bar{\mathbf{y}}_m\}$, $\mathbf{u} \triangleq \mathcal{F}\{\bar{\mathbf{u}}\}$, and $\mathbf{w}_m \triangleq \mathcal{F}\{\bar{\mathbf{w}}_m\}$, where $\mathcal{F}\{\cdot\}$ denotes the $4NQ$-point DFT operator. By the time-shifting property of Fourier transform, we have[1]

$$\begin{aligned}\mathbf{y}_m &= \sum_{k=1}^{K}\sum_{l=1}^{L_k} \alpha_{k,l}\left[\mathrm{diag}(\mathbf{b}_{m,k}) + \mathbf{U}\mathrm{diag}(\mathbf{e}_{m,k})\right]\boldsymbol{\phi}(\tau_{k,l}) + \mathbf{w}_m \\ &\triangleq \sum_{k=1}^{K} \mathbf{G}_{m,k}\boldsymbol{\beta}_k(\boldsymbol{\alpha}_k, \boldsymbol{\tau}_k) + \mathbf{w}_m \end{aligned} \tag{5}$$

where $\mathbf{U}$ is a $4NQ \times 4NQ$ circular matrix with the first row given by $\mathbf{u}$, $\boldsymbol{\phi}(\tau_{k,l}) \triangleq \left[1, \phi^{\tau_{k,l}}, \ldots, \phi^{(4NQ-1)\tau_{k,l}}\right]^T$, with $\phi = e^{-j(2\pi/4NQ)}$, and $\mathbf{b}_{m,k}$ and $\mathbf{e}_{m,k}$ are $4NQ \times 1$ vectors formed by the $4NQ$-point DFT of samples of $b_{m,k}(t)$ and $e_{m,k}(t)$ with zero delays. In the second equality of (5), $\boldsymbol{\alpha}_k$ and $\boldsymbol{\tau}_k$ are $L_k \times 1$ vectors of the unknown attenuations and code-timings, $\boldsymbol{\beta}_k(\boldsymbol{\tau}_k, \boldsymbol{\alpha}_k) \triangleq \sum_{l=1}^{L_k} \boldsymbol{\phi}(\tau_{k,l})\alpha_{k,l}$, and $\mathbf{G}_{m,k} \triangleq \mathrm{diag}(\mathbf{b}_{m,k}) + \mathbf{U}\mathrm{diag}(\mathbf{e}_{m,k})$. Note that $\mathbf{G}_{m,k}$ is known and independent of the unknown $\boldsymbol{\alpha}_k$ and $\boldsymbol{\tau}_k$.

Remark 1: Equation (5) is instrumental to our proposed scheme. It captures the *deterministic* structure of the overall interference, including the ISI due to $e_{m,k}(t-\tau_{k,l})$, whereas many earlier acquisition schemes for short-code systems (e.g., [2]–[5] and references therein) often treat all or part of the interference as a stochastic process. Equation (5) enables deterministic and complete elimination of both ISI and MAI, which is in general more effective than stochastic approaches. It also leads to faster code acquisition due to no need for second-order statistics estimation that is vital to stochastic approaches and which typically converges slowly.

Based on (5), we have a two-step estimator that first obtains an *unstructured* linear estimate, $\hat{\boldsymbol{\beta}}_k$ of $\boldsymbol{\beta}_k$, followed by imposing the structure of $\boldsymbol{\beta}_k$ on $\hat{\boldsymbol{\beta}}_k$ to produce estimates of the parameters of interest. Specifically, let $\mathbf{y} \triangleq \left[\mathbf{y}_0^T, \ldots, \mathbf{y}_{M-2}^T\right]^T$,

[1]Equation (5) holds only approximately because of the aliasing caused by the windowing $U(t)$. The aliasing is in general negligible compared to the interference and noise induced estimation error [7].

$\boldsymbol{\beta} \triangleq [\boldsymbol{\beta}_1^T, \ldots, \boldsymbol{\beta}_K^T]^T$, and $\mathbf{w} \triangleq [\mathbf{w}_0^T, \ldots, \mathbf{w}_{M-2}^T]^T$. Then, we have $\mathbf{y} = \mathbf{G}\boldsymbol{\beta} + \mathbf{w}$, where $\mathbf{G}$ is a $4MNQ \times 4KNQ$ matrix with the mkth block given by $\mathbf{G}_{m,k}$, $m = 0, \ldots, M-2$, $k = 1, \ldots, K$. Assuming $\mathbf{G}$ is tall and has full rank, we have

$$\hat{\boldsymbol{\beta}} = (\mathbf{G}^H\mathbf{G})^{-1}\mathbf{G}^H\mathbf{y} \tag{6}$$

from which we can obtain $\hat{\boldsymbol{\beta}}_k$. Next, observe that $\hat{\boldsymbol{\beta}}_k$ is composed of L_k complex sinusoids

$$\hat{\beta}_k(n) = \sum_{l=1}^{L_k} \alpha_{k,l} e^{-j(2\pi\tau_{k,l} n/4NQ)} + v_k(n), \quad n = 0, 1, \ldots, 4NQ-1 \tag{7}$$

where $\hat{\beta}_k(n)$ is the nth element of $\hat{\boldsymbol{\beta}}_k$, and $v_k(n)$ denotes the estimation error incurred in (6). Hence, the problem reduces to a sinusoidal parameter estimation problem, and a wealth of good methods can be used to estimate the code-timing $\tau_{k,l}$ and attenuation $\alpha_{k,l}$. See, e.g., [8, Ch.4], for details.

Remark 2: We need $M > K$ so that $\mathbf{G}$ is tall. This is necessary but not sufficient for $\mathbf{G}$ to be full-rank. Note that $\mathbf{G}$ is formed by the Fourier spectra of the spreading waveforms; with aperiodic spreading codes that are linearly independent for different users, $\mathbf{G}$ in general has full rank. It should also be noted that $\mathbf{G}$ is known to the base station, and its rank can be predetermined. In the event that it is rank deficient, we can discard certain rows of $\mathbf{G}$ (likewise the corresponding elements of $\mathbf{y}$) to ensure invertability. In our simulations, independent spreading codes are used and we never encountered the rank-deficient problem.

Remark 3: Given that $\mathbf{G}$ is full-rank, the identifiability for the sinusoidal parameters in (7) has been well studied (see [8, Ch.4] and references therein). Specifically, we need $\tau_{k,l}$ to be distinct for different l to avoid ambiguity. When subspace-based estimators, such as root-MUSIC or ESPRIT [8, Ch.4], are used for sinusoidal parameter estimation, we further need $L_k < 2NQ$, i.e., the number of resolvable paths for any user should be less than half the number of samples in $\hat{\boldsymbol{\beta}}_k$, to ensure proper estimation of the noise subspace. This is generally satisfied in practice.

Remark 4: In case of oversampling $(Q > 1)$, the DFT of the chip pulse has small tails, which shall be discarded to avoid noise amplification that may be caused by the matrix inverse in (6) [7]. Specifically, for $Q = 2$, we discard 50% of the frequency samples in (5) at the tail frequencies, as suggested in [7], and retain only the middle frequency samples for frequency deconvolution.

Finally, we mention that the block structure of $\mathbf{G}$ can be exploited for efficient computation. We skip the details for space limitation. Since $v_k(n)$ in (7) is free of any residual interference, the proposed estimator is near–far resistant.

IV. Numerical Results and Discussions

We consider a K-user asynchronous CDMA system using BPSK modulation and randomly generated long spreading codes with processing gain $N = 16$. The chip waveform is a square-root raised-cosine pulse with roll-off factor 0.95 oversampled with $Q = 2$. We focus on a near-far environment whereby the total (from all paths) average power for the desired user is scaled to unity, whereas the $K-1$ interfering users transmit at a power level P dB higher than the desired user. The near-far ratio (NFR) is defined as P in decibels. The code-timings are uniformly distributed in $[0, T_s)$, which are generated once and fixed in simulation to facilitate comparison with the Cramér–Rao bound (CRB), a lower bound for all unbiased estimators (e.g., [8]). The data bits and channel noise are changed independently from trial to trial. Two performance measures are considered: one is the probability of acquisition (PA), defined as the probability of the event that the code-timing estimate is within $T_c/2$ of the true value; the other is the root-mean-squared error (RMSE) of the code-timing estimate, given correct acquisition. For the multipath case, we evaluate the performance measures for each path, and present the average results averaged over all path estimates. The following simulation results are based on 400 trials.

We compare the proposed estimator with the MF scheme (e.g., [1, Ch.5]), which is implemented in the frequency-domain by first computing the DFT of $y(t)$, performing frequency deconvolution [7] to remove the data modulation and spreading for the desired user, computing the DFT of the resulting signal and, finally, locating the peaks of the DFT, which are the delay estimates. Note that we do not compare with the centralized scheme of [6] since it cannot deal with bandlimited chip pulses. For the proposed estimator, we use both the root-MUSIC [8, pp. 158–160] and ESPRIT [8, pp. 163–164] algorithms to solve the sinusoidal parameter estimation problem (7). The resultant estimators are labeled as *Proposed/root-MUSIC* and *Proposed/ESPRIT*, respectively, in the figures. Fig. 2 depicts the *user capacity* of both schemes when $M = 100$, $L_k = 2, \forall\, k$, $\mathrm{SNR} = 15$ dB, and $\mathrm{NFR} = 10$ dB. Also shown in the lower half of Fig. 2 is CRB. It is seen that the proposed estimators are similar to one another, and insensitive to the number of users in the system. Remarkably, they can support even *overloaded* systems with $K > N$, while experiencing little degradation in terms of both PA and RMSE. Fig. 3 illustrates the *observation time* that is needed for both methods, when $K = 5$, M is varying and the other parameters are identical to the previous example. We can see that the proposed schemes achieves much faster acquisition than the MF estimator. In particular, it is seen that the former can produce good acquisition even with $M = 20$ bits.

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