A Robust Approach to Channel Estimation and Detection for Multi-Carrier CDMA

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***Abstract*— We present a robust approach to channel estimation and multiuser detection (MUD) for multi-carrier CDMA by explicitly taking into account prior channel estimation errors. Specifically, robust channel estimates and MUD receivers are obtained by optimizing the worst-case performance over a properly selected bounded uncertainty set. Although the prior channel estimation error is not bounded, it is beneficial to refine the estimate over a properly chosen bounded uncertainty set. Numerical results show that linear MUD detectors resulting from our robust approach yield improved performance over those that ignore the prior estimation errors.**

***Index Terms*— Robust estimation, multi-carrier (MC) code-division multiple-access (CDMA), multiuser detection (MUD).**

I. Introduction

MUD for MC-CDMA has received a lot interests recently. Numerous MUD schemes have been proposed (see [1] and references there). While these schemes are derived from different principles, they usually rely on prior channel estimates. However, channel estimates always contain some estimation errors, and most MUD schemes are sensitive to such errors.

We develop herein robust channel estimation and MUD schemes by explicitly considering prior channel estimation errors. Our schemes build on recent developments in robust adaptive beamforming, which deals with bounded uncertainty in prior knowledge of the steering vector of the antenna array [2]– [4]. While uncertainty on the steering vector in array processing is usually bounded, channel estimation errors have an infinite support and is generally unbounded. Even so, we found it beneficial to optimize the worst-case performance over a properly chosen bounded uncertainty set. Our strategy is to improve detection robustness against small to moderate channel estimation errors within the bounded set. We discuss how to choose a bounded parameter set for our problem by using the Chebyshev inequality.

A different robust detector is proposed in [5]. The idea is to minimize the interference subject to a set of probability bounded constraints that take into account channel estimation errors. The authors showed that the underlying stochastic programming problem can be simplified to a convex nonlinear programming problem or approximated by a second-order cone programming problem. In contrast, our approach is different and involves only scalar optimization. Finally, we note that only channel uncertainty is considered in the sequel. Robust detectors that deal with channel and covariance estimation errors is considered in [6].

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II. Data Model

Consider a baseband MC-CDMA (uplink) system with P sub-carriers and K active users. The mth information symbol for the kth user, $d_k(m)$, is modulated by a $P \times 1$ spreading code $\mathbf{c}_k$. After a P-point inverse discrete Fourier transform (IDFT) and parallel-to-serial (P/S) conversion, a cyclic prefix (CP) is inserted to avoid inter-symbol interference. Then, the signal is sent out from the transmitter and passes through a channel with impulse response $\mathbf{h}_k \in \mathbb{C}^{L\times 1}$, which lumps together the transmitter/receiver filters and the physical channel. At the receiver, the received signal is first serial-to-parallel (S/P) converted, followed by CP removal and discrete Fourier transform (DFT). We assume that the delay spread is within the CP duration, so that the output of the DFT processor during the mth symbol interval can be expressed as

$$\mathbf{y}(m) = \mathbf{S}\mathbf{d}(m) + \mathbf{e}(m),$$

where $\mathbf{d}(m) = [d_1(m),\ \ldots,\ d_K(m)]^T$, $\mathbf{e}(m)$ contains the additive channel noise and possibly unmodeled interference, and $\mathbf{S} = [\mathbf{s}_1,\ \cdots,\ \mathbf{s}_K]$ with $\mathbf{s}_k = \mathbf{C}_k\boldsymbol{\mathcal{F}}\mathbf{h}_k$, for $k = 1,\ \ldots,\ K$, $\mathbf{C}_k$=diag$\{\mathbf{c}_k\}$, and $\boldsymbol{\mathcal{F}} \in \mathbb{C}^{P\times L}$ denoting the DFT matrix with the klth element given by $P^{-1/2}\exp\{-j2\pi(k-1)(l-1)/P\}$. The problem of interest is to develop robust linear MUD schemes by taking into account imperfect channel estimates $\hat{\mathbf{h}}_k$ obtained by some standard channel estimator.

III. Preliminaries

A. Linear Multiuser Detection

Suppose user signature $\mathbf{S}$ is known (by using some channel estimator), the zero-forcing (ZF) detector is given by $\mathbf{W}_{\text{ZF}} = \mathbf{S}(\mathbf{S}^H\mathbf{S})^{-1}$, where we have ignored any possible unmodeled interference in $\mathbf{e}(m)$. If any unmodeled interference is present, we can use the ZF minimum variance (ZF-MV) criterion:

$$\begin{aligned}\mathbf{W}_{\text{ZF-MV}} &= \arg\min_{\mathbf{W}} \operatorname{tr}\{\mathbf{W}^H\mathbf{R}_y\mathbf{W}\}, \quad \text{s.t. } \mathbf{W}^H\mathbf{S} = \mathbf{I}_K \\ &= \mathbf{R}_y^{-1}\mathbf{S}(\mathbf{S}^H\mathbf{R}_y^{-1}\mathbf{S})^{-1},\end{aligned} \tag{1}$$

where $\mathbf{R}_y \triangleq E\{\mathbf{y}(m)\mathbf{y}^H(m)\}$. The ZF-MV will reduce to the MV detector if only a single user, say user 1 is detected:

$$\mathbf{w}_{\text{MV}} = \mathbf{R}_y^{-1}\mathbf{s}_1(\mathbf{s}_1^H\mathbf{R}_y^{-1}\mathbf{s}_1)^{-1}. \tag{2}$$

For the ZF-MV detector, the minimum output power is

$$V \triangleq \operatorname{tr}\{(\mathbf{S}^H\mathbf{R}_y^{-1}\mathbf{S})^{-1}\}, \tag{3}$$

which will be used for robust receiver design in Section IV-B.

B. Channel Estimation

Suppose M training symbols are available. Let $\mathbf{y} \triangleq [\mathbf{y}^T(1), \ldots, \mathbf{y}^T(M)]^T$, which can be expressed as

$$\mathbf{y} = \begin{bmatrix} d_1(1)\mathbf{C}_1\boldsymbol{\mathcal{F}} & \cdots & d_K(1)\mathbf{C}_K\boldsymbol{\mathcal{F}} \\ \vdots & \ddots & \vdots \\ d_1(M)\mathbf{C}_1\boldsymbol{\mathcal{F}} & \cdots & d_K(M)\mathbf{C}_K\boldsymbol{\mathcal{F}} \end{bmatrix} \begin{bmatrix} \mathbf{h}_1 \\ \vdots \\ \mathbf{h}_K \end{bmatrix} + \begin{bmatrix} \mathbf{e}(1) \\ \vdots \\ \mathbf{e}(M) \end{bmatrix} \triangleq \mathbf{A}\mathbf{h} + \mathbf{e}.$$

An initial channel estimate can be obtained by least-squares: $\hat{\mathbf{h}} = (\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H\mathbf{y}$, whose estimation error is $\Delta\mathbf{h} = \hat{\mathbf{h}} - \mathbf{h} = (\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H\mathbf{e}$. We assume $\mathbf{e}$ is zero-mean white Gaussian with $E\{\mathbf{e}\mathbf{e}^H\} = \sigma^2\mathbf{I}$. While this assumption may be violated if $\mathbf{e}$ contains unmodeled interference, it allows a simple way to bound the uncertainty set (see Section IV-A). Hence, $\Delta\mathbf{h}$ is zero-mean Gaussian with covariance matrix

$$\operatorname{cov}\{\Delta\mathbf{h}\} = \sigma^2(\mathbf{A}^H\mathbf{A})^{-1}. \quad (4)$$

Let $\boldsymbol{\mathcal{A}} = (\mathbf{A}^H\mathbf{A})$ and $\boldsymbol{\mathcal{A}}_{ij}$ the ijth $L \times L$ submatrix of $\boldsymbol{\mathcal{A}}$: $\boldsymbol{\mathcal{A}}_{ij} = \boldsymbol{\mathcal{F}}^H\mathbf{C}_i\mathbf{C}_j\boldsymbol{\mathcal{F}}\sum_{m=1}^{M} d_i^*(m)d_j(m)$, $i, j = 1, \ldots, K$. For independent and identically distributed (i.i.d.) training symbols, the *sample cross-correlation* $r_{ij} \triangleq \frac{1}{M}\sum_{m=1}^{M} d_i^*(m)d_j(m) = O(M^{-1/2})$, $i \neq j$, which means that r_{ij} approaches zero (which is the statistical cross-correlation) at rate $M^{-1/2}$ as M increases. Hence, for large M, $\boldsymbol{\mathcal{A}} = M(\mathbf{I}_{KL} + O(M^{-1/2}))$ and, in turn, $\boldsymbol{\mathcal{A}}^{-1} = \frac{1}{M}\mathbf{I}_{KL} + O(M^{-3/2})$ [7, p. 58] (i.e., inverse of a perturbed matrix). Therefore, for large M, we have $\operatorname{cov}\{\Delta\mathbf{h}\} \approx \frac{\sigma^2}{M}\mathbf{I}_{KL}$, and $\operatorname{cov}\{\Delta\mathbf{h}_k\} \approx \frac{\sigma^2}{M}\mathbf{I}_L$. Let $\beta \triangleq \|\Delta\mathbf{h}_k\|^2$. Then, β is χ^2 distributed with $2L$ degrees of freedom [8], whose mean μ_β and variance σ_β^2 are given by

$$\mu_\beta = E\{\beta\} \approx \sigma^2 L/M; \quad \sigma_\beta^2 = \operatorname{var}\{\beta\} \approx \sigma^4 L/M^2, \quad (5)$$

which are used to bound the uncertainty set. Note that approximation is not needed for orthogonal training. Although for non-orthogonal training and small M (which are used in Section V), our approximation may be inaccurate, it still offers a guideline how to set the ambiguity set. Simulation results show that the proposed approach is insensitive to the size of the uncertainty set.

IV. Robust Channel Estimation and MUD

A. Bounding Channel Estimation Error

Although $\beta = \|\Delta\mathbf{h}_k\|^2$ is unbounded, by the Chebyshev inequality, the channel estimation error is bounded in probability: $P_\beta(|\beta - \mu_\beta| > \delta_\beta) \leq \sigma_\beta^2/\delta_\beta^2$, where δ_β is any positive number. For sufficiently large δ_β, we can ignore the *unbounded channel estimation error*, which is a small-probability event, and consider a bounded set $P_\beta(\beta \leq \mu_\beta + \delta_\beta) \geq 1 - \sigma_\beta^2/\delta_\beta^2$. Let $\epsilon_k \triangleq \mu_\beta + \delta_\beta$ denote a chosen boundary of β. Then

$$P_\beta(\|\Delta\mathbf{h}_k\|^2 \leq \epsilon_k) \geq 1 - \sigma_\beta^2/\delta_\beta^2, \quad (6)$$

where $P_\beta(\|\Delta\mathbf{h}_k\|^2 \leq \epsilon_k)$ is henceforth referred to as the *Chebyshev bounding probability*. Recall our strategy is to improve detection robustness against small to moderate channel estimation errors within a bounded set. For a chosen Chebyshev bounding probability P_β (say, $P_\beta = 0.9$), we can determine the corresponding boundary ϵ_k by setting the two sides of (6) equal. In particular, we have $\epsilon_k = \mu_\beta + \sqrt{\sigma_\beta^2/(1-P_\beta)}$. In light of (5), we can rewrite ϵ_k as $\epsilon_k = \frac{1}{M}\left(L + \sqrt{L/(1-P_\beta)}\right)\sigma^2$.

B. Robust Channel Estimation and MUD

Since the ZF-MV detector is known to be sensitive to signal mismatch due to errors in $\hat{\mathbf{h}}$, we consider *robust channel estimation* by *maximizing* the following multi-channel output power (cf. (3)): $V(\mathbf{h}) = \operatorname{tr}\{[\mathbf{S}^H(\mathbf{h})\mathbf{R}_y^{-1}\mathbf{S}(\mathbf{h})]^{-1}\}$. Note that $V(\mathbf{h})$ is a highly nonlinear function in $\mathbf{h}$ (due to the outer matrix inversion). Instead, we utilize an upper bound of $V(\mathbf{h})$ that is tight for high SNR (see [9]). Specifically, rather than maximizing $\operatorname{tr}\{\left[\mathbf{S}^H(\mathbf{h})\mathbf{R}_y^{-1}\mathbf{S}(\mathbf{h})\right]^{-1}\}$, we can minimize $\operatorname{tr}\left\{\mathbf{S}^H(\mathbf{h})\mathbf{R}_y^{-1}\mathbf{S}(\mathbf{h})\right\}$.

Our robust channel estimate is obtained by the following constrained optimization:

$$\tilde{\mathbf{h}} = \arg\min_{\mathbf{h}} \operatorname{tr}\left[\mathbf{S}^H(\mathbf{h})\mathbf{R}_y^{-1}\mathbf{S}(\mathbf{h})\right], \text{ s.t. } \|\hat{\mathbf{h}} - \mathbf{h}\|^2 \leq \epsilon, \quad (7)$$

where $\epsilon = \sum_{k=1}^{K}\epsilon_k$ denotes the total amount of uncertainty for all users. The cost function can be simplified as follows: $\operatorname{tr}\left[\mathbf{S}^H(\mathbf{h})\mathbf{R}_y^{-1}\mathbf{S}(\mathbf{h})\right] = \sum_{k=1}^{K}\mathbf{h}_k^H\boldsymbol{\mathcal{F}}^H\mathbf{C}_k\mathbf{R}_y^{-1}\mathbf{C}_k\boldsymbol{\mathcal{F}}\mathbf{h}_k$. Let $\boldsymbol{\Phi}_k \triangleq \boldsymbol{\mathcal{F}}^H\mathbf{C}_k\mathbf{R}_y^{-1}\mathbf{C}_k\boldsymbol{\mathcal{F}}$, Note that $\boldsymbol{\Phi}_k \geq 0$, and they are independent of each other. Then, the problem in (7) reduces to K separate optimization problems:

$$\tilde{\mathbf{h}}_k = \arg\min_{\mathbf{h}_k} \mathbf{h}_k^H\boldsymbol{\Phi}_k\mathbf{h}_k, \text{ s.t. } \|\hat{\mathbf{h}}_k - \mathbf{h}_k\|^2 \leq \epsilon_k. \quad (8)$$

Since the solution of (8) will evidently occur on the boundary of the uncertainty set, we can solve (8) by the Lagrange multiplier [3] as below:

$$\tilde{\mathbf{h}}_k = \hat{\mathbf{h}}_k - (\mathbf{I} + \lambda\boldsymbol{\Phi}_k^{-1})^{-1}\hat{\mathbf{h}}_k,$$

where λ can be determined by setting $\|(\mathbf{I}+\lambda\boldsymbol{\Phi}_k^{-1})^{-1}\hat{\mathbf{h}}_k\|^2 = \epsilon_k$ (see [3] for details). Once $\tilde{\mathbf{h}}_k$ is obtained, the signature vectors are updated as: $\tilde{\mathbf{s}}_k = \mathbf{C}_k\boldsymbol{\mathcal{F}}\tilde{\mathbf{h}}_k$, and $\tilde{\mathbf{S}} = [\tilde{\mathbf{s}}_1, \cdots, \tilde{\mathbf{s}}_K]$. The robust ZF-MV detector is given by [cf. (1)]

$$\tilde{\mathbf{W}}_{\text{robust ZF-MV}} = \mathbf{R}_y^{-1}\tilde{\mathbf{S}}(\tilde{\mathbf{S}}^H\mathbf{R}_y^{-1}\tilde{\mathbf{S}})^{-1}. \quad (9)$$

When only a single spreading code is available, the robust ZF-MV detector reduces to a robust MV detector [cf. (2)]

$$\tilde{\mathbf{w}}_{\text{robust MV}} = \mathbf{R}_y^{-1}\tilde{\mathbf{s}}_1(\tilde{\mathbf{s}}_1^H\mathbf{R}_y^{-1}\tilde{\mathbf{s}}_1)^{-1}. \quad (10)$$

Compared with the conventional MUD schemes, our robust approach requires the following extra computations:

1) Compute $\boldsymbol{\Phi}_k = \boldsymbol{\mathcal{F}}^H\mathbf{C}_k\mathbf{R}_y^{-1}\mathbf{C}_k\boldsymbol{\mathcal{F}}$. $\Rightarrow$ $O(P^2L)$ flops
2) Compute $\boldsymbol{\Phi}_k^{-1}$. $\Rightarrow$ $O(L^3)$ flops
3) Solve (8) for λ by a Newton search scheme.

Both the conventional and robust approaches require computing $\mathbf{R}_y$ and its inverse, which involve $O(P^3)$ flops and dominates the overall complexity (since in practice $P \gg L$). The complexity involved in the Newton search is negligible since the cost function is scalar and monotonically decreasing (see [3]). As such, the robust approach is only slightly more involved than the conventional approach.

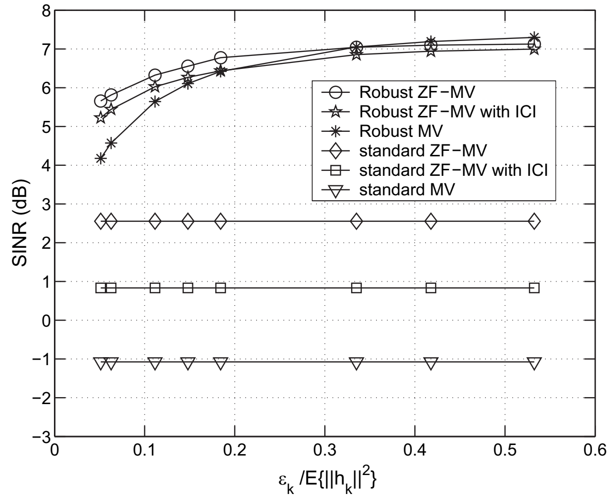

Fig. 1. Receiver output SINR versus the normalized uncertainty $_k$ $\{\|\mathbf{h}_k\|^2\}$ when SNR=10dB and $= 15$.

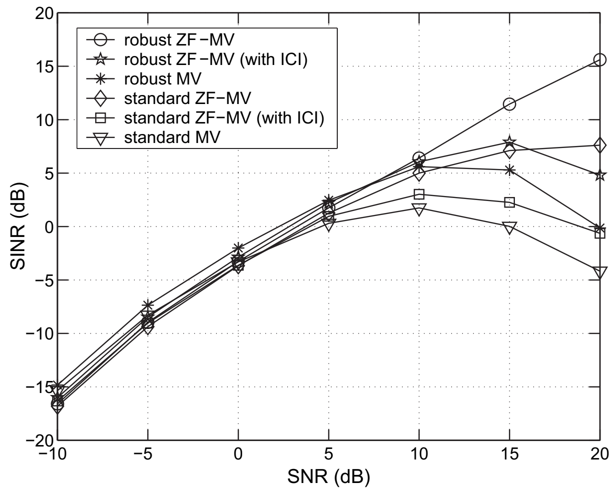

Fig. 2. SINR versus input SNR when $= 50$ and $_k$ $\{\|\mathbf{h}_k\|^2\} = 0\ 1.$

V. Numerical Results

We consider a K-user MC-CDMA (uplink) with BPSK modulation, K=10, P=32, L=7 for $\mathbf{h}_k$ and random spreading codes $\mathbf{c}_k$. The channel vector $\mathbf{h}_k$ is generated as $L \times 1$ independent complex Gaussian random variables with zero mean and identical variance $\frac{1}{L}$, which are varied independently from trial to trial. We have two sets of MUD detectors to compare with. The first set consists of our proposed robust ZF-MV detector (9) and the robust MV detector (10). The other includes the standard ZF-MV detector (1) and the MV detector (2), which ignore the estimation error of the prior channel estimate. For both the conventional ZF-MV and robust ZF-MV detectors, we further consider two different scenarios: one has knowledge of the spreading codes and training symbols of all 10 users; the other has knowledge of 6 out of 10 users, i.e., we assume the presence of inter-cell interference (ICI) with 4 out of 10 users coming from a different cell.

Fig. 1 depicts the receiver output signal-to-interference-and-noise ratio (SINR) for user 1 versus the normalized uncertainty $\epsilon_k/E\{\|\mathbf{h}_k\|^2\}$ when M=15 and SNR $=$10dB. Since the conventional MUD detectors ignore the prior estimation errors,

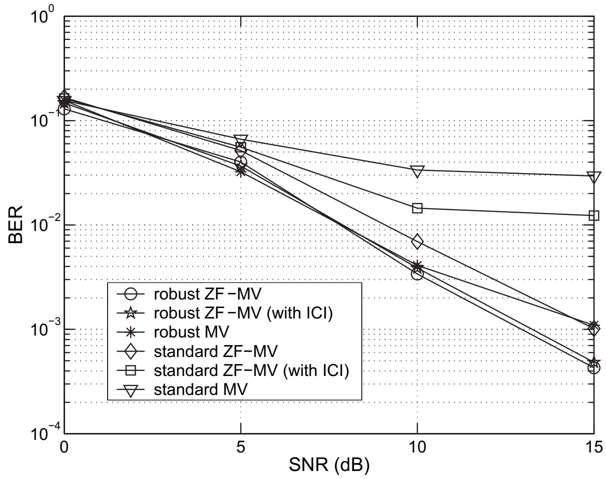

Fig. 3. BER versus input SNR when $= 50$ and $_k$ $\{\|\mathbf{h}_k\|^2\} = 0\ 1.$

they are independent of ϵ_k. It is seen that the robust detectors are insensitive to the choice of ϵ_k. We notice significant improvements for all the robust detectors relative to their non-robust counterparts. We next examine the output SINR and BER respectively of all the six detectors versus the input SNR, when $M = 50$ training symbols are used to obtain initial channel estimates and we set $\epsilon_k/E\{\|\mathbf{h}_k\|^2\} = 0.1$. Fig. 2 shows that, as the input SNR increases, the performance gap between the robust detectors and the non-robust detectors increases. For the average BER performance shown in Fig. 3, the conventional ZF-MV with ICI and MV detectors have an irreducible error floor due to poor initial channel estimates.

VI. Conclusions

We have proposed a robust approach to channel estimation and MUD for MC-CDMA. Numerical results illustrate that the proposed robust detector yields improved performance over the standard detector that ignores the prior estimation errors.

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