Distributed Target Detection Based on the Volume Cross-Correlation Function

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***Abstract*—This letter addresses the detection of a subspace distributed target signal obscured by disturbance. The disturbance consists of a clutter component with an unknown subspace structure and a white noise component with unknown noise power. A detection strategy is proposed based on the volume cross-correlation function, which provides a metric that measures the linear (in) dependency between two subspaces. Simulation results indicate that the proposed detector can achieve better performance than several peer methods, without resorting to secondary data and *a priori* knowledge about the clutter subspace including its rank.**

***Index Terms*—Distributed target detection, subspace target, volume cross-correlation (VCC) function.**

I. Introduction

With the improvement of radar range resolution, a target may be represented as a number of scattering centers across multiple range cells. Such a target is called a distributed target (also known as a range spread target or extended target) [1]. The detection of a distributed target has received extensive attention in recent years. Most of these works have been carried out on the basis of statistical hypothesis testing using different target and disturbance (accounting for clutter, interference, and noise) models. Oftentimes, parameters associated with the statistical models are required to be either known *a priori* or estimated. For instance, the target amplitudes and disturbance covariance matrix are required in widely used methods based on the generalized ratio test (GLRT), Rao, and Wald tests (see [1]–[17] and references therein).

Specifically, constant false alarm rate detectors based on the GLRT were devised in [2] and [3]. Distributed target detection was examined in [4] by modeling the disturbance as a spherically invariant random process (SIRP), and in [5] and [6] by using a complex Gaussian distribution. Furthermore, the Rao and Wald tests were employed to solve this problem in recent years [7]–[11]. In these detectors, a set of target-free data (henceforth referred to as secondary data) is often required to obtain an estimate of the disturbance covariance matrix. Moreover, detection schemes without secondary data and based on the GLRT were addressed in [1], [12], and [13], by restricting the disturbance covariance matrix to a specific set applying a constrained maximum-likelihood estimate. In the aforementioned works, the target echo is modeled as a rank-one signal.

For multirank distributed target detection, a GLRT and a two-step GLRT were derived in [14] under the condition when the distributed target is embedded in subspace interference plus homogeneous (or partially homogeneous) noise. It is assumed that the target and interference lie in two linearly independent subspaces, and a set of noise-only data is available. Two cases were considered in [14], with the interference subspace assumed to be either known or unknown (except for the dimension). Adaptive detection of subspace distributed target embedded in non-Gaussian clutter modeled as an SIRP was addressed in [15] and [16] by using the GLRT, and in [17] by using the Rao and Wald tests.

On one hand, we may have little *a priori* knowledge about the disturbance in some complex electromagnetic environments, where it is difficult to obtain an accurate estimate of the disturbance covariance matrix. On the other hand, since the uniformly most powerful test [18] does not exist for the problem, there is a need to explore different approaches, which can reduce the dependence on the detection environment or achieve higher detection performance. The volume cross-correlation (VCC) function, which provides a measure of the distance between two subspaces, was introduced for point-target detection in [19] and time difference of arrival estimation in [20]. For point target detection, it was assumed that multilook observations are available for each range cell. The target amplitude is assumed identically distributed in each observation. For the distributed target detection problem, observations are obtained from several range cells. There is little *a priori* knowledge on how many and which specific range cells the target occupies. This is due to the fact that the scattering geometry of the target is often unknown and may differ from target to target [21]. In other words, the target amplitude may have different characteristics across the range cells, or even there is no target echo present in some range cells. To our best knowledge, distributed target detection based on the VCC approach has not been examined.

In this letter, we address the problem of detecting a multirank subspace distributed target obscured by clutter plus white noise. The detection of whether the observation contains the target

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echo is first recast to the decision of whether the signal subspace intersects with the target subspace. Then, a detection strategy is proposed based on the VCC. Simulation results indicate that the proposed detector can achieve better performance than several peer methods, without resorting to secondary data and *a priori* knowledge about the disturbance covariance matrix. Furthermore, the proposed method shows stronger robustness with respect to the rank and certain geometric variations of the clutter subspace.

The remainder of this letter is organized as follows. Section II specifies the detection problem. The detection strategy based on VCC is presented in Section III. Simulation results are provided in Section IV followed by conclusions.

Notation: The Hermitian transpose is denoted by the superscript $(\cdot)^H$. The notation $\mathbb{C}^M$ and $\mathbb{C}^{M\times N}$ are the sets of M-dimensional vectors and $M \times N$ matrices of complex numbers, respectively. The range subspace of matrix $\mathbf{U}$ is denoted by span $(\mathbf{U})$.

II. Problem Formulation

We consider using a radar system equipped with N channels to detect a distributed target that may occupy up to K range cells. The channels may represent the elements of an array antenna [22], the pulses of a pulse Doppler radar [23], or a combination of both (such as in space-time adaptive processing [24]). The returns, denoted by $\mathbf{z}_k$, from the kth range cell, $k \in \mathcal{K} = \{1, 2, \ldots, K\}$, may consist of a disturbance signal $\mathbf{d}_k$ under the null hypothesis H_0 or a target echo $\mathbf{s}_k$ plus $\mathbf{d}_k$ under the alternative hypothesis H_1. Moreover, the disturbance $\mathbf{d}_k \in \mathbb{C}^N$ in general is a sum of mutually uncorrelated clutter $\mathbf{c}_k$ and noise $\mathbf{n}_k$.

For the kth range cell, the target $\mathbf{s}_k$ and clutter $\mathbf{c}_k$ are assumed to lie in two linearly independent subspaces spanned by the columns of full-rank matrices $\mathbf{P} \in \mathbb{C}^{N\times p}$ and $\mathbf{Q} \in \mathbb{C}^{N\times q}$, respectively; we assume $p + q \leq N$. Therefore, $\mathbf{s}_k$ and $\mathbf{c}_k$ can be represented by $\mathbf{s}_k = \mathbf{P}\boldsymbol{\alpha}_k$ and $\mathbf{c}_k = \mathbf{Q}\boldsymbol{\beta}_k$, where $\boldsymbol{\alpha}_k$ and $\boldsymbol{\beta}_k$ denote the coordinate vectors of the target and clutter, respectively. The noise $\mathbf{n}_k$ are modeled as independent and identically distributed (i.i.d.) zero-mean complex Gaussian vectors with power σ_0^2, i.e., $\mathbf{n}_k \sim \mathcal{CN}(\mathbf{0}, \sigma_0^2\mathbf{I}_N)$.

Based on the aforementioned assumptions, the detection problem involves the following hypothesis testing

$$\begin{cases} H_0 : \mathbf{z}_k = \mathbf{Q}\boldsymbol{\beta}_k + \mathbf{n}_k \\ H_1 : \mathbf{z}_k = \mathbf{P}\boldsymbol{\alpha}_k + \mathbf{Q}\boldsymbol{\beta}_k + \mathbf{n}_k \end{cases} \quad k \in \mathcal{K}. \tag{1}$$

In this work, we assume that $\mathbf{P}$ is known, while $\mathbf{Q}$ is unknown. In practical radar operation, target detection is performed on a set of angular/Doppler bins one by one. For detection at a specific angular/Doppler bin, $\mathbf{P}$ can be constructed based on the spatial and Doppler frequencies associated with that bin, and therefore is known *a priori* [24]. On the other hand, since the clutter response has spread in both range and Doppler, the associated matrix $\mathbf{Q}$ is in general unknown.

The problem of interest is to solve the hypothesis testing (1) by using observation data $\mathbf{Z} = [\mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_K]$, as well as the knowledge of the target subspace basis $\mathbf{P}$. The clutter subspace basis $\mathbf{Q}$, coefficient vectors $\boldsymbol{\alpha}_k$ and $\boldsymbol{\beta}_k$, and the noise power σ_0^2 are assumed unknown.

III. Proposed Approach

In this section, we briefly discuss the concept of VCC and then present our VCC-based solution to (1).

A. VCC

For a tall or square matrix $\mathbf{X} = [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_d] \in \mathbb{C}^{m\times d}$ ($d \leq m$), its k-dimensional volume ($k \leq d$) is given by [20], [25]

$$\text{vol}_k(\mathbf{X}) = \prod_{i=1}^{k} \gamma_i \tag{2}$$

where $\gamma_1 \geq \gamma_2 \geq \cdots \geq \gamma_d \geq 0$ are the singular values of $\mathbf{X}$. The d-dimensional volume $\text{vol}_d(\mathbf{X})$ can be considered as a description of the geometrical volume spanned by the columns of $\mathbf{X}$. In this letter, we extend the above definition for arbitrary matrices by defining $\text{vol}_d(\mathbf{X}) = 0$ if $d > m$.

The concept of VCC was introduced in [20] to characterize the relation between two full-rank matrices. Specifically, the VCC of $\mathbf{X} \in \mathbb{C}^{m\times d}$ and $\mathbf{Y} \in \mathbb{C}^{m\times l}$ is defined as

$$\text{vcc}(\mathbf{X}, \mathbf{Y}) = \frac{\text{vol}_{d+l}([\mathbf{X}, \mathbf{Y}])}{\text{vol}_d(\mathbf{X})\,\text{vol}_l(\mathbf{Y})} \tag{3}$$

where $[\mathbf{X}, \mathbf{Y}]$ means concatenating the arrays $\mathbf{X}$ and $\mathbf{Y}$ along the column dimension.

It can be verified that $0 \leq \text{vol}_d(\mathbf{X}) \leq \|\mathbf{x}_1\| \cdot \|\mathbf{x}_2\| \cdot \cdots \cdot \|\mathbf{x}_d\|$. The equality holds that $\text{vol}_d(\mathbf{X}) = 0$ if and only if the vectors $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_d$ are linearly dependent, and $\text{vol}_d(\mathbf{X}) = \|\mathbf{x}_1\| \cdot \|\mathbf{x}_2\| \cdot \cdots \cdot \|\mathbf{x}_d\|$ if and only if the vectors $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_d$ are orthogonal to each other. In particular, if the columns of $\mathbf{X}$ are orthonormal basis vectors, then $\text{vol}(\mathbf{X}) = 1$. Furthermore, if $\text{span}(\mathbf{X}) \cap \text{span}(\mathbf{Y}) = \mathbf{0}$, we have $\text{vcc}(\mathbf{X}, \mathbf{Y}) > 0$; otherwise, $\text{vcc}(\mathbf{X}, \mathbf{Y}) = 0$.

From the above discussions, we can see that VCC provides a metric that measures the linear (in)dependency between two subspaces. Next, we explain how to utilize this property to develop a new detector for distributed target detection.

B. Distributed Target Detection

For illustration, we first consider the noise-free case in which the received data contains either the clutter or both the clutter and target. Let $\mathcal{H}^s$ denote the signal subspace, which is spanned by the received data. The detection can be considered as a decision between whether $\mathcal{H}^s$ intersects with the target subspace $\mathcal{H}^t$. In other words, the detection is formulated as

$$\begin{cases} H_0 : \mathcal{H}^s \cap \mathcal{H}^t = \mathbf{0} \\ H_1 : \mathcal{H}^s \cap \mathcal{H}^t \neq \mathbf{0}. \end{cases} \tag{4}$$

Let $\mathbf{B}^s$ and $\mathbf{B}^t$ denote the bases of the subspaces $\mathcal{H}^s$ and $\mathcal{H}^t$, respectively. Based on the analysis in Section III-A, the detection problem (4) can be recast as

$$\begin{cases} H_0 : \text{vcc}(\mathbf{B}^s, \mathbf{B}^t) > 0 \\ H_1 : \text{vcc}(\mathbf{B}^s, \mathbf{B}^t) = 0. \end{cases} \tag{5}$$

For the noisy case in (1), since the observation is contaminated by white noise, the basis vectors of the signal subspace are required. They can be obtained by resorting to several algorithms, e.g., MUSIC [26] and ESPRIT [27], in which the observation subspace is separated into a signal subspace and noise subspace by eigenvalue decomposition of the observation covariance matrix. The detection can then be made according to (5).

Main steps of the detection based on VCC is summarized as follows.

- *Step 1:* Obtain $\mathbf{M}^z$ based on the observation data matrix $\mathbf{Z}$ as

$$\mathbf{M}^z = \frac{1}{K}\mathbf{Z}\mathbf{Z}^H. \tag{6}$$

- *Step 2:* By computing the eigenvalue decomposition of $\mathbf{M}^z$, one can obtain the eigenvalues

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_r > \lambda_{r+1} = \cdots = \lambda_N \tag{7}$$

and the corresponding eigenvectors

$$\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_r, \mathbf{u}_{r+1}, \ldots, \mathbf{u}_N \tag{8}$$

where r is the dimension of the signal subspace. Denote $\mathbf{U}^s = [\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_r]$, which can be considered as an estimate of the orthonormal basis matrix spanning the signal subspace $\mathcal{H}^s$.

- *Step 3:* Find an orthonormal set of basis vectors $\mathbf{U}^t$ for $\mathbf{P}$ by, e.g., the Gram–Schmidt orthogonalization.
- *Step 4:* Calculate the VCC of $\mathbf{U}^s$ and $\mathbf{U}^t$ as

$$\text{vcc}\left(\mathbf{U}^s, \mathbf{U}^t\right) = \text{vol}_{r+p}\left(\left[\mathbf{U}^s, \mathbf{U}^t\right]\right). \tag{9}$$

The decision rule is

$$\frac{1}{\text{vcc}\left(\mathbf{U}^s, \mathbf{U}^t\right)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta_V \tag{10}$$

where η_V denotes a test threshold.

It should be noticed that span$\left(\mathbf{U}^s\right)$ is an estimate of the true signal subspace, which is asymptotically statistically efficient [28]. In the finite observation case, the eigenvalues $\lambda_{r+1}, \lambda_{r+2}, \ldots, \lambda_N$ in (7) are often not equal in practice. Then, the dimension of the signal subspace r must be estimated. This can be accomplished by resorting to several source number estimation methods, such as the Akaike information criterion (AIC) [29] and Bayesian information criterion (BIC) [30]. To provide a reliable estimate for r and basis matrix of signal subspace $\mathcal{H}^s$, it is supposed that $K \geq N$. Moreover, when the error between the estimated and true signal subspace is not negligible, the detection in (5) is modified as testing whether vcc$\left(\mathbf{U}^s, \mathbf{U}^t\right)$ is close to zero, or equivalently, whether its reciprocal is sufficiently large, as shown in (10).

IV. Numerical Results

In this section, some numerical results are presented to verify the performance of the proposed detector. In the simulations, the radar employs a uniform linear array with $N = 20$ elements and half-wavelength spacing $d_0 = \lambda_0/2$. We assume that the target basis matrix $\mathbf{P}$ has a Vandermonde structure with the ith column given by $[1, e^{j2\pi d_0 \sin\theta_i/\lambda_0}, \ldots, e^{j(N-1)2\pi d_0 \sin\theta_i/\lambda_0}]^T$; in addition, the clutter subspace basis matrix $\mathbf{Q}$ has a similar structure with a different set of angle parameters θ_i. The angles of the target basis matrix are uniformly distributed over the interval $[0, 10^\circ]$. The clutter coefficient vectors $\boldsymbol{\beta}_k$, $k \in \mathcal{K}$, are modeled as i.i.d. zero-mean complex Gaussian vectors with power σ_c^2. The target coefficient vectors $\boldsymbol{\alpha}_k$, $k = 1, 2, \ldots, 10$, are modeled as i.i.d. zero-mean complex Gaussian vector with power σ_s^2. Furthermore, it is assumed that $\boldsymbol{\alpha}_{11} = \boldsymbol{\alpha}_{12} = \cdots = \boldsymbol{\alpha}_K = 0$. The clutter-to-noise ratio σ_c^2/σ_0^2 is set to 20 dB, and the signal-to-clutter-plus-noise ratio (SCNR) is defined as

$$\text{SCNR} = \frac{1}{NK}\sum_{k=1}^{K}\left(\mathbf{P}\boldsymbol{\alpha}_k\right)^H \mathbf{M}^{-1}\left(\mathbf{P}\boldsymbol{\alpha}_k\right) \tag{11}$$

where $\mathbf{M}$ is the covariance matrix of the disturbance (i.e., clutter plus noise). Moreover, the probability of false alarm (P_{FA}) is $P_{\text{FA}} = 10^{-4}$ and the predetermined threshold is set via Monte Carlo simulations of $100/P_{\text{FA}}$ independent trials.

A. Benchmark Methods

For comparison purposes, we consider four other detectors. The first detector is the optimal receiver (denoted as OPT) in the multirank target and deterministic scatterer model (i.e., unknown deterministic $\boldsymbol{\alpha}_k$) case [31]

$$\sum_{k=1}^{K}\left(\mathbf{z}_k^H\mathbf{M}^{-1}\mathbf{P}\right)\left(\mathbf{P}^H\mathbf{M}^{-1}\mathbf{P}\right)^{-1}\left(\mathbf{z}_k^H\mathbf{M}^{-1}\mathbf{P}\right)^H \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta_O. \tag{12}$$

The second is the adaptive matched filter (AMF) [31], which is obtained by replacing $\mathbf{M}$ in (12) with the sample covariance matrix $\hat{\mathbf{M}}$ formed from disturbance-only secondary data $\mathbf{z}_k$ ($k = K+1, K+2, \ldots, K+S_1$)

$$\hat{\mathbf{M}} = \frac{1}{S_1}\sum_{k=K+1}^{K+S_1}\mathbf{z}_k\mathbf{z}_k^H. \tag{13}$$

Detection of distributed targets embedded in subspace interference plus noise is addressed in [14]. It assumes that q is known, and a set of noise-only data $\mathbf{z}_k$ ($k = K+S_1+1, K+S_1+2, \ldots, K+S_1+S_2$) is available. The method, denoted as SPM, employs a subspace projection matrix and is given by

$$\sum_{i=1}^{N-q}\ln\frac{1+\sigma_i^2}{1+\eta_i^2} \underset{H_0}{\overset{H_1}{\gtrless}} \eta_S \tag{14}$$

where σ_i^2 and η_i^2 ($i = 1, 2, \ldots$) are the eigenvalues arranged in an increasing order of $\mathbf{R}^{-\frac{1}{2}}\mathbf{Z}\mathbf{Z}^H\mathbf{R}^{-\frac{1}{2}}$ and $(\mathbf{I}_N - \mathbf{H}_{P_R})\mathbf{R}^{-\frac{1}{2}}\mathbf{Z}\mathbf{Z}^H\mathbf{R}^{-\frac{1}{2}}(\mathbf{I}_N - \mathbf{H}_{P_R})$; $\mathbf{R} = \sum_{k=K+S_1+1}^{K+S_1+S_2}\mathbf{z}_k\mathbf{z}_k^H$, $\mathbf{P}_R = \mathbf{R}^{-\frac{1}{2}}\mathbf{P}$, and $\mathbf{H}_{P_R}$ denotes the projection matrix onto the range subspace of $\mathbf{P}_R$.

The fourth method is a GSM detector based on a Gaussian scatterer model (i.e., $\boldsymbol{\alpha}_k$ are complex Gaussian random vectors), which is given by [15]

$$N\sum_{k=1}^{K}\ln\left(1+\epsilon_k\right) - \sum_{k=1}^{K}\ln\left[\left(N-p\right)\epsilon_k - \left(p-1\right)\right] \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta_G \tag{15}$$

where $\epsilon_k = \mathbf{z}_k^H\mathbf{V}\mathbf{z}_k/\mathbf{z}_k^H\left(\hat{\mathbf{M}}^{-1} - \mathbf{V}\right)\mathbf{z}_k$, $\mathbf{V} = \hat{\mathbf{M}}^{-1}\mathbf{S}\left(\mathbf{S}^H\hat{\mathbf{M}}^{-1}\mathbf{S}\right)^{-1}\mathbf{S}^H\hat{\mathbf{M}}^{-1}$, and $\mathbf{S} \in \mathbb{C}^{N\times p}$ is the left singular vector of $\mathbf{P}$.

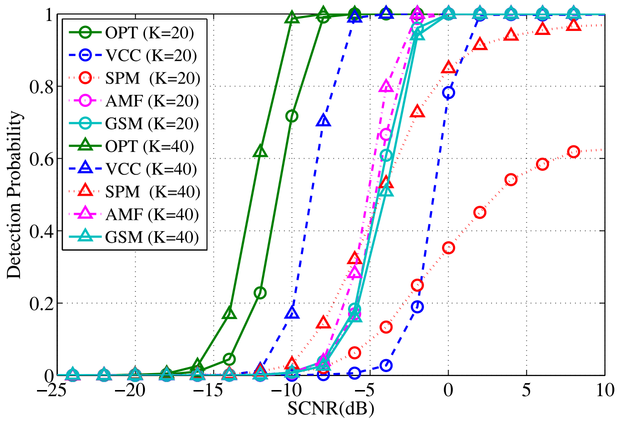

Fig. 1. Detection performance comparison with different numbers of range cells.

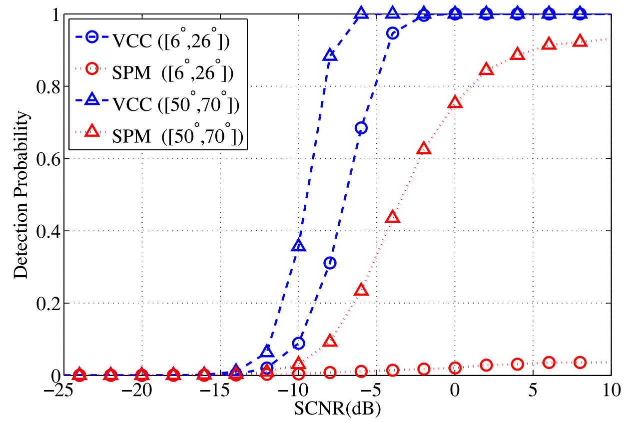

Fig. 2. Detection performance comparison of VCC and SPM with different clutter geometry cases.

B. Results

We examine the impact of the number of range cells, and the geometry and dimension of the clutter subspace on the detection performance. The results are shown in Figs. 1–3, where we fix $p = 3$, and $S_1 = S_2 = 40$. The dimension of the signal subspace r in VCC is estimated by using the AIC. Additionally, we have tried AIC and BIC, both yielding similar results.

In Fig. 1, the angles of clutter basis matrix are uniformly distributed over the interval $[40°, 60°]$, and detection performances are shown with $K = 20$ and $K = 40$. It can be seen that the performances of VCC and SPM improve significantly as K increases from N to $2N$, while OPT and AMF improve only slightly. It is shown in (6) and (14) that the estimates of the signal and disturbance covariance matrix in VCC and SPM are obtained by utilizing the data from cells under test, while that of AMF is obtained from the secondary data. Then, the number of range cells K affects the estimation accuracy of VCC and SPM but not AMF. Thus, the number of range cells has a stronger influence on the detection performance of VCC and SPM than that of OPT and AMF.

Next, we examine the impact of the geometry and dimension of the clutter subspace on the detection performance. Since these factors have little impact on OPT, AMF, and GSM, we will focus on VCC and SPM for comparison. In Fig. 2, the number of range cells is set to $K = 40$. Detection performances are shown when the angle intervals of the clutter basis matrix are $[6°, 26°]$ and $[50°, 70°]$. The purpose is to illustrate the performances of VCC and SPM in two cases: 1) when the angle range of the clutter overlaps with that of the target, and 2) when the angle ranges of the clutter and the target are widely separated. It is seen that when the angle range of the clutter overlaps with that of the target, the performance of VCC degrades slightly, while SPM suffers a more significant degradation, with respect to the case where the angle ranges of the clutter and target are widely separated. It is noted from (14) that an orthogonal projection matrix $(\mathbf{I}_N - \mathbf{H}_{P_R})$ is used in SPM. When the angle ranges of clutter and target (namely the subspaces of the clutter and target) are overlapping, the clutter energy projected onto the orthogonal complement of the target subspace will be significantly reduced, which requires a higher detection threshold. For VCC, as long as the clutter angles are not the same with those of the target, (10) approaches infinity under H_1 while a finite value under H_0. In other words, VCC is less sensitive to the geometric relation of the target and clutter than SPM.

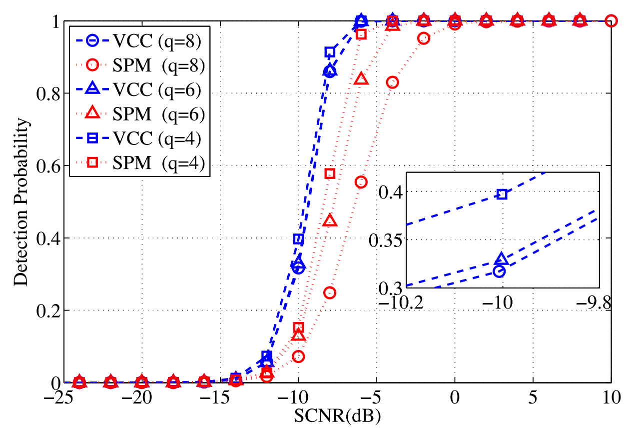

Fig. 3. Detection performance of VCC and SPM with different dimensions of clutter subspace.

In Fig. 3, a further performance comparison of VCC and SPM is provided, in which $K = 40$, $p = 3$, and the angles of clutter basis matrix are uniformly distributed over the interval $[40°, 60°]$. It is noted that the dimension of the clutter subspace q has little influence on the performance of VCC. The performance of SPM improves significantly with the decrease of q, as it can be seen from (14) that the $(N - q)$ minimum eigenvalues of the two matrix matter in the detection.

V. Conclusion

In this letter, we have examined the problem of detecting a multirank distributed target obscured by disturbance (clutter plus white noise) with unknown characteristics. The detection of whether the observation data contains the target echo has been recast as the decision of whether the signal subspace intersects with the target subspace. A detection strategy based on the VCC has been proposed by utilizing the geometric relation between the target and clutter subspaces. The performance of this algorithm is validated by simulation results.

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