Performance of Cooperative Relay With Binary Modulation in Nakagami-m Fading Channels

Qiang Zhao, Hongbin Li, *Member, IEEE*, and Pu Wang

***Abstract*—We consider a wireless cooperative relay system with one source, one relay, and one destination node. The average bit-error probability (BEP) of several coherent or differential relay schemes with binary modulation is derived for Nakagami-m fading channels and verified by computer simulation. Our analytical results provide a set of mathematical tools for the design and analysis of cooperative communication systems in general fading channels.**

***Index Terms*—Average bit-error probability (BEP), cooperative diversity, Nakagami fading, wireless relays.**

I. INTRODUCTION

Wireless cooperative communications have become of significant interest in recent years [1]–[17]. For wireless mobile networks, cooperative diversity (e.g., [1]–[3]) serves as an alternative to traditional space diversity, as the size of mobile nodes may not accommodate multiple transmit and receive antennas. Performance analysis, including the bit-error probability (BEP) analysis, of cooperative communications over Rayleigh fading channels has yielded many useful results to gain insight into nongenerative [amplify-and-forward (AF)] and generative (decode-and-forward) relaying schemes. Depending on whether the channel state information (CSI) is available or not, coherent or noncoherent modulation and detection may be applied at the participating cooperative nodes. An exact BEP, along with its upper and lower bounds, for coherent amplify-and-forward (CAF) in Rayleigh fading was obtained in [8]. The maximum-likelihood (ML) and piecewise-linear (PL) combiners and the corresponding BEP analysis for the coherent decode-and-forward (CDF) were developed in [9]. Although imperative for coherent cooperative schemes, channel estimation may not be a viable choice in fast-fading environments because of training overhead and convergence issues. On the contrary, noncoherent or differential cooperative schemes can achieve cooperative diversity and decent performance without channel estimation. The BEP of a cooperative noncoherent scheme with binary phase shift keying (BPSK) was investigated in [10]. Two differential cooperative schemes, which are referred to as differential amplify-and-forward (DAF) and differential decode-and-forward (DDF), were proposed and examined in terms of the BEP and the outage probability. For general multibranch multihop framework with CAF, closed-form approximations of the average symbol error probability at a high SNR were obtained in [11]. Distributed space-time block-encoded cooperation with multibranch and multihop transmissions was considered in [17] (and references therein).

Although a lot of works rely on a Rayleigh fading assumption, Nakagami-m fading, which covers a broader variety of fading scenarios, has received less attention. In [12], the BEP of a noncooperative two-hop system in Nakagami-m fading channels is derived. In this paper, we attempt to provide a unified BEP analysis for coherent and differential cooperative schemes using CAF, CDF, DAF, or DDF with binary modulation in Nakagami-m fading channels. Our focus here is, thus, different from [13], which covered only the differential scheme, or [14], both assuming Rayleigh fading.

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Q. Zhao is with LitePoint Corporation, Sunnyvale, CA 94085 USA (e-mail: qiang.zhao@litepoint.com).

H. Li and P. Wang are with the Department of Electrical and Computer Engineering, Stevens Institute of Technology, Hoboken, NJ 07030 USA (e-mail: hli@stevens.edu; pwang4@stevens.edu).

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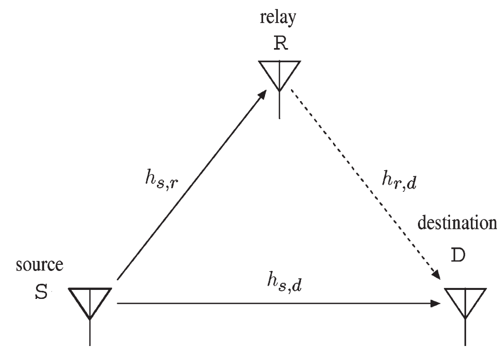

Fig. 1. Cooperative wireless relay system.

The rest of this paper is organized as follows. In Section II, we present a simple cooperative system and the assumptions that are used for the analysis. CAF, CDF, DAF, and DDF, as well as their corresponding detection/combining schemes, are briefly summarized in Section III. Section IV details the average BEP analysis for these four schemes. Following the numerical results in Section V, our conclusions are given in Section VI.

II. SYSTEM MODEL

Fig. 1 depicts a wireless relay network with one source S, one relay R, and one destination D node. Each node is equipped with one transmit and one receive antenna with a half-duplex transmission mode. A sequence of information symbols is to be transmitted from S to D with the aid of R. To eliminate mutual interference, we assume time-division multiplexing by which the transmission is divided into *two* distinct phases. During phase I, S transmits, whereas R and D listen, i.e.,

$$x_r(n) = h_{s,r}s(n) + w_r(n) \tag{1}$$

$$x_d(n) = h_{s,d}s(n) + w_d(n) \tag{2}$$

where $s(n) \in \{\pm 1\}$ denotes the BPSK symbols that are transmitted from S, $x_r(n)$ and $x_d(n)$ are the received signals, $h_{s,r}$ and $h_{s,d}$ are the corresponding fading coefficients, whereas $w_r(n)$ and $w_d(n)$ are the channel noises at R and D, respectively.

During phase II, S is silent, whereas R generates a *unit average power* signal $s_r(n)$ from $x_r(n)$ via amplifying or decoding and retransmits it to D, i.e.,

$$y_d(n) = h_{r,d}s_r(n) + u_d(n) \tag{3}$$

where $y_d(n)$, $h_{r,d}$, and $u_d(n)$ denote the received signal at D, the fading coefficient, and the noise, respectively. For notational simplicity, we use the same n to denote the index of symbols in phases I and II.

In Nakagami-m channels, the *instantaneous SNR* between nodes i and j, which is denoted by $\gamma_{i,j} = |h_{i,j}|^2/N_0$, has a gamma distribution [18], i.e.,

$$p_{\gamma_{i,j}}(x) = \frac{1}{\Gamma(m_{i,j})}\left(\frac{m_{i,j}}{\bar{\gamma}_{i,j}}\right)^{m_{i,j}} x^{m_{i,j}-1}e^{-m_{i,j}x/\bar{\gamma}_{i,j}} \tag{4}$$

where $(i,j) \in \{(s,r),(s,d),(r,d)\}$, $m_{i,j}$ denotes the fading parameter, and $\bar{\gamma}_{i,j} = \Omega_{i,j}/N_0 = E\{|h_{i,j}|^2\}/N_0$ is the *average SNR* between nodes i and j. The moment generating function (MGF) of the instantaneous SNR $\gamma_{i,j}$ is given by [18]

$$\mathcal{M}_{\gamma_{i,j}}(s) = \left(\frac{m_{i,j}}{\bar{\gamma}_{i,j}s + m_{i,j}}\right)^{m_{i,j}}. \tag{5}$$

We further assume that for differential detection, channel coefficients remain (approximately) constant for two adjacent bits within each frame. The channel noises $w_r(n)$, $w_d(n)$, and $u_d(n)$ are independent identically distributed (i.i.d.) complex Gaussian random variables with zero mean and variance N_0 and independent of the channel coefficients.

III. Coherent and Differential Schemes

We briefly summarize the four coherent or differential cooperative schemes, namely, CAF, CDF, DAF, and DDF, which are analyzed in Section IV.

1) CAF: In phase II, R produces a scaled signal $s_r(n)$ to meet a *unit average power* constraint [9], i.e.,

$$s_r(n) = \frac{x_r(n)}{(N_0 + |h_{s,r}|^2)^{1/2}}. \tag{6}$$

After inserting (6) into (3), it is seen that $y_d(n)$ follows a complex Gaussian distribution with zero mean and variance $\check{\sigma}^2_{s,d} = |h_{r,d}|^2(N_0 + |h_{s,r}|^2)^{-1}N_0 + N_0$ conditioned on the channel coefficients. The decision variable of the maximal ratio combiner (MRC) at D is shown to be [9]

$$z(n) = \frac{h^*_{s,d}x_d(n)}{N_0} + \frac{\check{h}^*_{s,d}y_d(n)}{\check{\sigma}^2_{s,d}} \tag{7}$$

where $(\cdot)^*$ denotes complex conjugation, and $\check{h}_{s,d} \triangleq h_{s,r}h_{r,d}/(N_0 + |h_{s,r}|^2)^{1/2}$.

2) DAF: In the absence of the CSI $h_{s,r}$, an alternative AF scheme that facilitates differential implementation is suggested in [15], i.e.,

$$s_r(n) = \frac{x_r(n)}{(N_0 + \Omega_{s,r})^{1/2}}. \tag{8}$$

Using an ML approach to combine $x_d(n)$ and $y_d(n)$ leads to [13]

$$z(n) = x^*_d(n-1)x_d(n) + \frac{(1+\bar{\gamma}_{s,r})y^*_d(n-1)y_d(n)}{1+\bar{\gamma}_{s,r}+\gamma_{r,d}}. \tag{9}$$

However, the above decision variable requires the knowledge of the instantaneous SNR $\gamma_{r,d}$. It was suggested in [13] to replace $\gamma_{r,d}$ with $\bar{\gamma}_{r,d}$, which results in a truly differential combiner, i.e.,

$$z(n)=x^*_d(n-1)x_d(n)+\frac{(1+\bar{\gamma}_{s,r})y^*_d(n-1)y_d(n)}{1+\bar{\gamma}_{s,r}+\bar{\gamma}_{r,d}}. \tag{10}$$

Simulation results show that the performance of (10) is almost identical to that of detector (9). In the sequel, (9) is referred to as the DAF-I combiner, and (10) is referred to as the DAF-II combiner.

3) CDF: R first decodes the received signal

$$s_r(n) = \operatorname{sign}\left(\Re\left\{h^*_{s,r}x_r(n)\right\}\right) \tag{11}$$

which is then retransmitted to D. Here, $\Re\{\cdot\}$ is the real part of the argument. Since R may make a correct or wrong decision, the conditional probability density function (pdf) of $y_d(n)$ assumes a Gaussian mixture, i.e.,

$$p_{y_d(n)}(y) = (1-\epsilon)\Phi_c\left(y; h_{r,d}s(n), N_0\right) + \epsilon\Phi_c\left(y; -h_{r,d}s(n), N_0\right) \tag{12}$$

where $\Phi_c(y;\mu,\sigma^2)$ denotes the pdf of a complex Gaussian random variable with mean μ and variance σ^2, and ϵ is the average bit-error rate of BPSK in Nakagami-m fading channels [18]. Maximizing the joint probability density of $x_d(n)$ and $y_d(n)$, the ML detector is shown to be [9]

$$f(t_1) + t_0 \underset{-1}{\overset{1}{\gtrless}} 0 \tag{13}$$

where

$$f(t_1) = \ln\frac{(1-\epsilon)e^{t_1}+\epsilon}{\epsilon e^{t_1}+1-\epsilon} \tag{14}$$

$$t_1 = \left(h^*_{r,d}y_d(n) + h_{r,d}y^*_d(n)\right)/N_0 \tag{15}$$

$$t_0 = \left(h^*_{s,d}x_d(n) + h_{s,d}x^*_d(n)\right)/N_0. \tag{16}$$

It is observed that the nonlinear function $f(t_1)$ can be approximated by a PL function [9], i.e.,

$$f(t_1) \approx f_{\mathrm{PL}}(t_1) \triangleq \begin{cases} -T_1, & t_1 \le -T_1 \\ t_1, & -T_1 \le t_1 \le T_1 \\ T_1, & t_1 \ge T_1. \end{cases} \tag{17}$$

Replacing $f(t_1)$ in (13) by $f_{\mathrm{PL}}(t_1)$ leads to the PL detector.

4) DDF: At R, the received signal is differentially decoded, i.e.,

$$\tilde{d}(n) = \operatorname{sign}\left(\Re\left\{x^*_r(n-1)x_r(n)\right\}\right), \quad n = 1,2,\ldots,N. \tag{18}$$

The decoded bits are then re-encoded via a differential encoder as $s_r(n) = s_r(n-1)\tilde{d}(n)$ with $s_r(0) = 1$. Due to hard decoding at R, the conditional pdf of $y_d(n)$ also takes the form of a Gaussian mixture, i.e.,

$$p_{y_d(n)}(y) = (1-\epsilon)\Phi_c\left(y; y_d(n-1)d(n), 2N_0\right) + \epsilon\Phi_c\left(y; -y_d(n-1)d(n), 2N_0\right). \tag{19}$$

In analogy to the CDF, the ML and PL detectors for DDF take the same form of (13) but with t_1 and t_0 replaced by the following [13]:

$$t_1 = \left(y^*_d(n-1)y_d(n) + y_d(n-1)y^*_d(n)\right)/N_0 \tag{20}$$

$$t_0 = \left(x^*_d(n-1)x_d(n) + x_d(n-1)x^*_d(n)\right)/N_0. \tag{21}$$

IV. BEP Analysis

In this section, we examine the BEP of the four cooperative schemes.

1) CAF: The instantaneous SNR of the MRC that is specified in (7) is the sum of the instantaneous SNRs of the *direct link* and the *relay link*, viz., $\gamma_{\mathrm{CAF}} = \gamma_{s,d} + \gamma_{s,r,d}$, where $\gamma_{s,r,d} = (\gamma_{s,r}\gamma_{r,d})/(\gamma_{s,r} + \gamma_{r,d} + 1)$ is the instantaneous SNR of the relay link. For the coherent detection of BPSK signals, the conditional BEP is given by [19]

$$\begin{aligned} P_e(\gamma_{\mathrm{CAF}}) &= Q\left(\sqrt{2(\gamma_{s,d}+\gamma_{s,r,d})}\right) \\ &= \frac{1}{\pi}\int_0^{\pi/2} \exp\left(-\frac{\gamma_{s,d}+\gamma_{s,r,d}}{\sin^2\theta}\right) d\theta \end{aligned} \tag{22}$$

where $Q(x)$ is the standard Q-function, and, in the second equality, we used the alternative form of the Q-function [16]. Since the relay link and the direct link are statistically independent, using the

MGF-based method, we can average out the channel statistics and obtain the average BEP as follows:

$$\bar{P}_e = \frac{1}{\pi}\int_0^{\pi/2} \mathcal{M}_{\gamma_{s,d}}(\sin^{-2}\theta)\mathcal{M}_{\gamma_{s,r,d}}(\sin^{-2}\theta)d\theta \tag{23}$$

where $\mathcal{M}_\gamma(\cdot)$ denotes the MGF of a random variable γ. The MGF of $\gamma_{s,d}$ is given by (5). For the case of i.i.d. $h_{s,r}$ and $h_{r,d}$ (i.e., $m_{s,r} = m_{r,d}$ and $\bar{\gamma}_{s,r} = \bar{\gamma}_{r,d}$), the MGF of $\gamma_{s,r,d}$ can be approximated as follows [12]:

$$\mathcal{M}_{\gamma_{s,r,d}}(s) \approx {}_2F_1\left(m_{s,r}, 2m_{s,r}; m_{s,r}+\frac{1}{2}; -\frac{\bar{\gamma}_{s,r}}{4m_{s,r}}s\right) \tag{24}$$

where ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ denotes the Gauss hypergeometric function defined in [20, eq. (9.100)], and the approximation is due to $\gamma_{s,r,d} \approx \gamma_{s,r}\gamma_{r,d}/(\gamma_{s,r}+\gamma_{r,d})$, which holds at a high SNR. In general, the evaluation of (23) needs the help of numerical techniques using, for example, the Gaussian quadratic numerical integration.

2) DAF: The analysis of DAF-II is prohibitively complex. We consider here the BEP of DAF-I, which serves as a performance gauge for DAF-II. The decision variable (9) is a quadratic form in complex Gaussian random variables given $h_{i,j}$. Using the results in [19, eqs. (12.1)–(12.13)] for differential BPSK with two i.i.d. channels, the conditional BEP is given by

$$P_e = \frac{1}{8}(4+\gamma_{s,r,d}+\gamma_{s,d})e^{-(\gamma_{s,r,d}+\gamma_{s,d})} \tag{25}$$

where $\gamma_{s,r,d}$ is given by $\gamma_{s,r,d} = (\gamma_{s,r}\gamma_{r,d})/(\bar{\gamma}_{s,r}+1+\gamma_{r,d})$. Note that $\gamma_{s,r,d}$ of DAF-I is different from its counterpart in CAF. The pdf of $\gamma_{s,r,d}$ is given by the following, which is derived in Appendix A:

$$\begin{aligned} p_{\gamma_{s,r,d}}(x) &= \frac{x^{m_{s,r}-1}}{\Gamma(m_{s,r})\Gamma(m_{r,d})}\left(\frac{m_{s,r}}{\bar{\gamma}_{s,r}}\right)^{m_{s,r}}\left(\frac{m_{r,d}}{\bar{\gamma}_{r,d}}\right)^{m_{r,d}} \\ &\quad\times e^{-m_{s,r}x/\bar{\gamma}_{s,r}}\int_0^\infty \frac{(1+\bar{\gamma}_{s,r}+t)^{m_{s,r}}}{t^{1+m_{s,r}-m_{r,d}}} \\ &\quad\times e^{-c_1 t - c_2/t}dt \end{aligned} \tag{26}$$

where $c_1 = m_{r,d}/\bar{\gamma}_{r,d}$, and $c_2 = m_{s,r}x(1+\bar{\gamma}_{s,r})/\bar{\gamma}_{s,r}$. When $m_{s,r}$ is a positive integer, (26) can be simplified using the binomial theorem and with the help of [20, eq. (3.478.4)], i.e.,

$$\begin{aligned} p_{\gamma_{s,r,d}}(x) &= \frac{2x^{m_{s,r}-1}}{\Gamma(m_{s,r})\Gamma(m_{r,d})}\left(\frac{m_{s,r}}{\bar{\gamma}_{s,r}}\right)^{m_{s,r}}\left(\frac{m_{r,d}}{\bar{\gamma}_{r,d}}\right)^{m_{r,d}} \\ &\quad\times e^{-m_{s,r}x/\bar{\gamma}_{s,r}}\Bigg\{\sum_{k=0}^{m_{s,r}}\binom{m_{s,r}}{k}(\bar{\gamma}_{s,r}+1)^k \\ &\quad\times\left(\frac{m_{s,r}(1+\bar{\gamma}_{s,r})\bar{\gamma}_{r,d}x}{m_{r,d}\bar{\gamma}_{s,r}}\right)^{\frac{m_{r,d}-k}{2}} \\ &\quad\times K_{m_{r,d}-k}\left(2\sqrt{\frac{m_{s,r}m_{r,d}(1+\bar{\gamma}_{s,r})x}{\bar{\gamma}_{s,r}\bar{\gamma}_{r,d}}}\right)\Bigg\} \end{aligned} \tag{27}$$

where $K_\nu(\cdot)$ denotes the νth-order modified Bessel function of the second kind [20, eq. (8.432)].

Now, we compute the average BEP by averaging the conditional BEP with respect to the joint pdf of $\gamma_{s,r,d}$ and $\gamma_{s,d}$. We have (see Appendix B)

$$\bar{P}_e = \frac{1}{2}\bar{P}_{e1}\bar{P}_{e2} + \frac{1}{8}\bar{P}_{e3}\bar{P}_{e2} + \frac{1}{8}\bar{P}_{e1}\bar{P}_{e4} \tag{28}$$

where

$$\begin{aligned} \bar{P}_{e1} &= \sum_{k=0}^{m_{s,r}}\binom{m_{s,r}}{k}(1+\bar{\gamma}_{s,r})^{(m_{r,d}+k-1)/2} \\ &\quad\times\frac{\Gamma(m_{s,r}+m_{r,d}-k)}{\Gamma(m_{r,d})}e^{\frac{m_{s,r}m_{r,d}(1+\bar{\gamma}_{s,r})}{2\bar{\gamma}_{r,d}(m_{s,r}+\bar{\gamma}_{s,r})}} \\ &\quad\times m_{s,r}^{-u}m_{r,d}^{(m_{r,d}+k-1)/2} \\ &\quad\times\bar{\gamma}_{r,d}^{\frac{1-m_{r,d}-k}{2}}(m_{s,r}+\bar{\gamma}_{s,r})^u W_{u,v}(\beta) \end{aligned} \tag{29}$$

$$\bar{P}_{e2} = \left(\frac{m_{s,d}}{m_{s,d}+\bar{\gamma}_{s,d}}\right)^{m_{s,d}} \tag{30}$$

$$\begin{aligned} \bar{P}_{e3} &= \sum_{k=0}^{m_{s,r}}\binom{m_{s,r}}{k}(1+\bar{\gamma}_{s,r})^{(m_{r,d}+k-1)/2} \\ &\quad\times\frac{\Gamma(m_{s,r}+m_{r,d}+1-k)}{\Gamma(m_{r,d})}e^{\frac{m_{s,r}m_{r,d}(1+\bar{\gamma}_{s,r})}{2\bar{\gamma}_{r,d}(m_{s,r}+\bar{\gamma}_{s,r})}} \\ &\quad\times m_{s,r}^{-u+1}m_{r,d}^{\frac{m_{r,d}+k-1}{2}}\bar{\gamma}_{s,r}\bar{\gamma}_{r,d}^{\frac{1-m_{r,d}-k}{2}} \\ &\quad\times(m_{s,r}+\bar{\gamma}_{s,r})^{u-1}W_{u-1,v}(\beta) \end{aligned} \tag{31}$$

$$\bar{P}_{e4} = \frac{\Gamma(m_{s,d}+1)}{\Gamma(m_{s,d})}\frac{m_{s,d}^{m_{s,d}}\bar{\gamma}_{s,d}}{(m_{s,d}+\bar{\gamma}_{s,d})^{m_{s,d}+1}} \tag{32}$$

where $W_{\mu,\nu}(\cdot)$ denotes the Whittaker function [20, eq. (9.220)], $u = (2m_{s,r}+m_{r,d}-k-1)/2$, $v = (m_{r,d}-k)/2$, and $\beta = (m_{s,r}m_{r,d}(1+\bar{\gamma}_{s,r}))/(\bar{\gamma}_{r,d}(m_{s,r}+\bar{\gamma}_{s,r}))$. It can be shown that, when $m_{s,r} = m_{s,d} = m_{r,d} = 1$, the closed-form BEP that is given in (28) reduces to [13, eq. (16)] for the Rayleigh fading case.

3) CDF: Without loss of generality, assume that $s(n) = 1$ is transmitted at R. The conditional BEP for the PL detector is given by

$$\begin{aligned} P_e &= \Pr\{t_0 - T_1 < 0 | t_1 < -T_1, s(n)=1\} \\ &\quad\times\Pr\{t_1 < -T_1 | s(n)=1\} \\ &\quad+\Pr\{t_0 + T_1 < 0 | t_1 > T_1, s(n)=1\} \\ &\quad\times\Pr\{t_1 > T_1 | s(n)=1\} \\ &\quad+\Pr\{t_0 + t_1 < 0, -T_1 \le t_1 \le T_1 | s(n)=1\}. \end{aligned} \tag{33}$$

Conditioned on $s(n) = 1$ and $h_{i,j}$, t_0 is a real Gaussian random variable with mean $2\gamma_{s,d}$ and variance $2\gamma_{s,d}$, and t_1 is a Gaussian mixture with distribution $p_{t_1}(t_1) = (1-\epsilon)\Phi(t_1; 2\gamma_{r,d}, 2\gamma_{r,d}) + \epsilon\Phi(t_1; -2\gamma_{r,d}, 2\gamma_{r,d})$, where $\Phi(x;\mu,\sigma^2)$ denotes the pdf of a real Gaussian random variable with mean μ and variance σ^2. Therefore, the conditional BEP is given by

$$P_e = P_{e1}P_{e2} + P_{e3}P_{e4} + P_{e5} \tag{34}$$

where

$$P_{e1} = Q\left(\frac{2\gamma_{s,d} - T_1}{\sqrt{2\gamma_{s,d}}}\right) \quad (35)$$

$$P_{e2} = (1-\epsilon)Q\left(\frac{2\gamma_{r,d} + T_1}{\sqrt{2\gamma_{r,d}}}\right) + \epsilon Q\left(\frac{T_1 - 2\gamma_{r,d}}{\sqrt{2\gamma_{r,d}}}\right) \quad (36)$$

$$P_{e3} = Q\left(\frac{T_1 + 2\gamma_{s,d}}{\sqrt{2\gamma_{s,d}}}\right) \quad (37)$$

$$P_{e4} = (1-\epsilon)Q\left(\frac{T_1 - 2\gamma_{r,d}}{\sqrt{2\gamma_{r,d}}}\right) + \epsilon Q\left(\frac{T_1 + 2\gamma_{r,d}}{\sqrt{2\gamma_{r,d}}}\right) \quad (38)$$

$$\begin{aligned} P_{e5} &= \int_{-T_1}^{T_1} p_{t_1}(t_1)dt_1 \int_{-\infty}^{-t_1} p_{t_0}(t_0)dt_0 \\ &= \int_{-T_1}^{T_1} p_{t_1}(t_1) Q\left(\frac{t_1 + 2\gamma_{s,d}}{\sqrt{2\gamma_{s,d}}}\right) dt_1. \end{aligned} \quad (39)$$

The average BEP is obtained by averaging out the channel statistics, i.e.,

$$\bar{P}_e = \int_0^\infty \int_0^\infty P_e(\gamma_1, \gamma_2) p_{\gamma_{s,d}}(\gamma_1) p_{\gamma_{r,d}}(\gamma_2) d\gamma_1 d\gamma_2. \quad (40)$$

4) DDF: We note that it is generally more difficult to analyze decode-based schemes than amplify-based ones due to the hard decision made at R. This leads to a mixture distribution of the received signal [cf. (19)]. The analysis is further complicated by a decision statistic that involves quadratic forms in Gaussian and Gaussian mixture variates [see (20) and (21)]. Closed-form expressions of the distributions of such quadratic forms, in general, are known only via series expansion [21]. Similar to [13], we adopt a series expansion approach and examine the BEP of DDF with the PL detector, i.e., (13) with the approximation of $f(t_1)$ in (17) and parameters t_0 and t_1 specified in (20) and (21). The analysis of the nonlinear ML detector is significantly more involved and not considered. The conditional BEP of DDF with PL detection, which is conditioned on the channel instantaneous SNR, has been obtained in [13, Appendix B] as follows:

$$\begin{aligned} P_e(\gamma_{s,d}, \gamma_{r,d}) = &(P_{e1}P_{e2} + P_{e3}P_{e4} + P_{e7})(1-\epsilon) \\ &+ (P_{e1}P_{e3} + P_{e2}P_{e4} + P_{e8})\epsilon \end{aligned} \quad (41)$$

where the error probabilities P_{ei}, $i = 1, \ldots, 4, 7, 8$, are given by [13, eqs. (39)–(44)]. Note that although [13] considers Rayleigh channels, the conditional BEP is independent of the channel fading and can be applied to the current analysis.

The average BEP for DDF is obtained by averaging (41) using the distribution of $\gamma_{r,d}$ and $\gamma_{s,d}$ pertaining to Nakagami-m fading, i.e.,

$$\bar{P}_e = \int_0^\infty \int_0^\infty P_e(\gamma_1, \gamma_2) p_{\gamma_{s,d}}(\gamma_1) p_{\gamma_{r,d}}(\gamma_2) d\gamma_1 d\gamma_2. \quad (42)$$

The 2-D integration in (42) is separable. Using [20, eq. (3.351.3)], we have the following closed-form expression of the average BEP for DDF:

$$\begin{aligned} \bar{P}_e = &(\bar{P}_{e1}\bar{P}_{e2} + \bar{P}_{e3}\bar{P}_{e4} + \bar{P}_{e7})(1-\epsilon) \\ &+ (\bar{P}_{e1}\bar{P}_{e3} + \bar{P}_{e2}\bar{P}_{e4} + \bar{P}_{e8})\epsilon \end{aligned} \quad (43)$$

where $\bar{P}_{ei}$, $i = 1, \ldots, 4, 7, 8$ are given by

$$\begin{aligned} \bar{P}_{e1} = &1 - \frac{1}{2\Gamma(m_{s,d})}\left(\frac{m_{s,d}}{2\bar{\gamma}_{s,d} + m_{s,d}}\right)^{m_{s,d}} \sum_{k=0}^{\infty}\sum_{n=0}^{k} \\ &\times \frac{2^{k-n}(k + m_{s,d} - 1)!\bar{\gamma}_{s,d}^k \Gamma(k+1-n, T_1)}{k!(k-n)!(2\bar{\gamma}_{s,d} + m_{s,d})^k} \end{aligned} \quad (44)$$

$$\bar{P}_{e2} = \frac{(m_{r,d} - 1)!}{2\Gamma(m_{r,d})}\left(\frac{m_{r,d}}{\bar{\gamma}_{r,d} + m_{r,d}}\right)^{m_{r,d}} e^{-T_1} \quad (45)$$

$$\begin{aligned} \bar{P}_{e3} = &\frac{1}{2\Gamma(m_{r,d})}\left(\frac{m_{r,d}}{2\bar{\gamma}_{r,d} + m_{r,d}}\right)^{m_{r,d}} \sum_{k=0}^{\infty}\sum_{n=0}^{k} \\ &\times \frac{2^{k-n}(k + m_{r,d} - 1)!\bar{\gamma}_{r,d}^k \Gamma(k+1-n, T_1)}{k!(k-n)!(2\bar{\gamma}_{r,d} + m_{r,d})^k} \end{aligned} \quad (46)$$

$$\bar{P}_{e4} = \frac{(m_{s,d} - 1)!}{2\Gamma(m_{s,d})}\left(\frac{m_{s,d}}{\bar{\gamma}_{s,d} + m_{s,d}}\right)^{m_{s,d}} e^{-T_1} \quad (47)$$

$$\begin{aligned} \bar{P}_{e7} = &\frac{(m_{r,d} - 1)!}{2\Gamma(m_{r,d})}\left(\frac{m_{r,d}}{\bar{\gamma}_{r,d} + m_{r,d}}\right)^{m_{r,d}} (1 - e^{-T_1}) \\ &- \frac{(m_{r,d} - 1)!}{8\Gamma(m_{s,d})\Gamma(m_{r,d})}\left(\frac{m_{s,d}}{2\bar{\gamma}_{s,d} + m_{s,d}}\right)^{m_{s,d}} \\ &\times \left(\frac{m_{r,d}}{\bar{\gamma}_{r,d} + m_{r,d}}\right)^{m_{r,d}} \sum_{k=0}^{\infty}\sum_{n=0}^{k}\sum_{m=0}^{k-n} \\ &\frac{2^{k-n-m}(k + m_{s,d} - 1)!\bar{\gamma}_{s,d}^k \gamma(m+1, 2T_1)}{k!m!(2\bar{\gamma}_{s,d} + m_{s,d})^k} \\ &+ \frac{(m_{s,d} - 1)!}{8\Gamma(m_{s,d})\Gamma(m_{r,d})}\left(\frac{m_{s,d}}{\bar{\gamma}_{s,d} + m_{s,d}}\right)^{m_{s,d}} \\ &\times \left(\frac{m_{r,d}}{2\bar{\gamma}_{r,d} + m_{r,d}}\right)^{m_{r,d}} \sum_{k=0}^{\infty}\sum_{n=0}^{k} \\ &\frac{(k + m_{r,d} - 1)!\bar{\gamma}_{r,d}^k \gamma(k-n+1, 2T_1)}{k!(k-n)!(2\bar{\gamma}_{r,d} + m_{r,d})^k} \end{aligned} \quad (48)$$

$$\begin{aligned} \bar{P}_{e8} = &\frac{1}{2\Gamma(m_{r,d})}\left(\frac{m_{r,d}}{2\bar{\gamma}_{r,d} + m_{r,d}}\right)^{m_{r,d}} \sum_{k=0}^{\infty}\sum_{n=0}^{k} \\ &\frac{2^{k-n}(k + m_{r,d} - 1)!\bar{\gamma}_{r,d}^k \gamma(k+1-n, T_1)}{k!(k-n)!(2\bar{\gamma}_{r,d} + m_{r,d})^k} \\ &- \frac{1}{8\Gamma(m_{s,d})\Gamma(m_{r,d})}\left(\frac{m_{s,d}}{2\bar{\gamma}_{s,d} + m_{s,d}}\right)^{m_{s,d}} \\ &\times \left(\frac{m_{r,d}}{2\bar{\gamma}_{r,d} + m_{r,d}}\right)^{m_{r,d}} \sum_{k=0}^{\infty}\sum_{i=0}^{\infty}\sum_{n=0}^{k}\sum_{j=0}^{i}\sum_{m=0}^{i-j} \\ &\frac{(k + m_{r,d} - 1)!(i + m_{s,d} - 1)!\bar{\gamma}_{r,d}^k (2\bar{\gamma}_{s,d})^i}{(2\bar{\gamma}_{r,d} + m_{r,d})^k (2\bar{\gamma}_{s,d} + m_{s,d})^i} \\ &\times \frac{\gamma(k+m+1-n, 2T_1)}{k!(k-n)!i!m!2^{j+m}} + \frac{(m_{s,d} - 1)!}{8\Gamma(m_{s,d})} \\ &\times \frac{(m_{r,d} - 1)!}{\Gamma(m_{r,d})}\left(\frac{m_{s,d}}{\bar{\gamma}_{s,d} + m_{s,d}}\right)^{m_{s,d}} \\ &\times \left(\frac{m_{r,d}}{\bar{\gamma}_{r,d} + m_{r,d}}\right)^{m_{r,d}} (1 - e^{-2T_1}). \end{aligned} \quad (49)$$

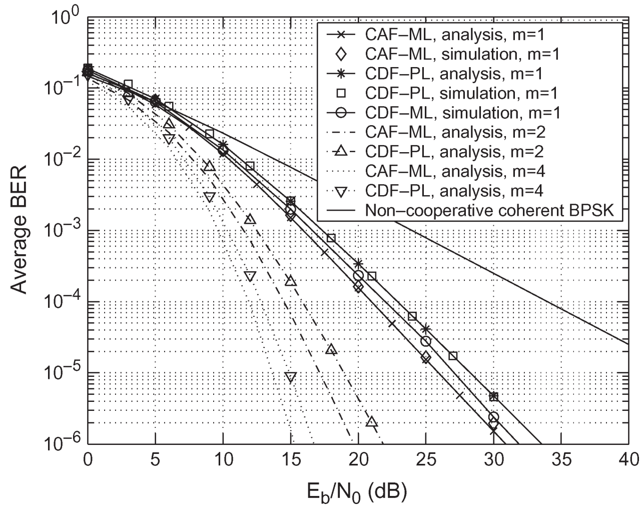

Fig. 2. Average BEP of cooperative and noncooperative coherent modulation in symmetric Nakagami-m fading channels.

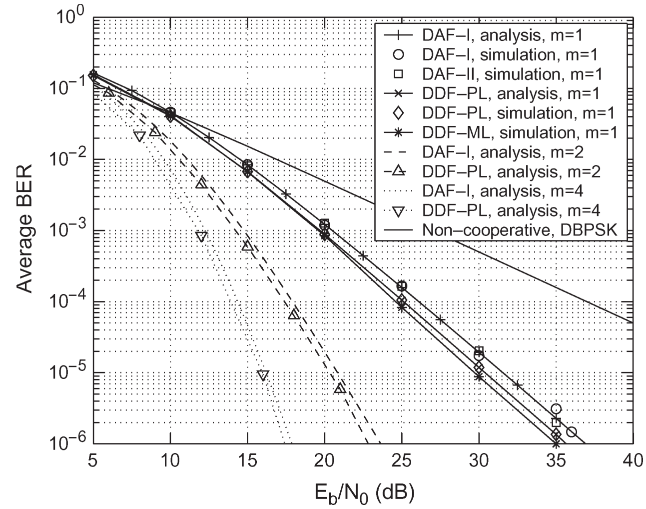

Fig. 3. Average BEP of cooperative and noncooperative differential modulation in symmetric Nakagami-m fading channels.

V. Numerical Results

We consider the cooperative wireless system shown in Fig. 1 with Nakagami-m fading channels. We first consider a symmetric scenario where the average SNRs of all hops are identical: $\bar{\gamma}_{s,d} = \bar{\gamma}_{s,r} = \bar{\gamma}_{r,d}$. We compare the cooperative schemes with their conventional noncooperative counterparts that involve direct transmission from S to D. For a fair comparison, we set $\bar{\gamma}_{s,d} = \bar{\gamma}_{s,r} = \bar{\gamma}_{r,d} = 0.5E_b/N_0$ so that the sum of the transmitted energy from both S and R for the cooperative systems is identical to that of the noncooperative systems.

Fig. 2 depicts the average BEPs of CAF and CDF obtained by analysis and, respectively, by simulation when $m_{i,j} = 1, 2,$ and 4. In addition, the conventional noncooperative BPSK with coherent reception in Rayleigh fading is also shown in Fig. 2 for comparison. We observe that our analytical average BEPs for CAF/CDF match those obtained by simulations. By examining the slope of the BEP–E_b/N_0 curve, the diversity gain is achieved by CAF/CDF. For the symmetric case considered, CAF slightly outperforms CDF. At an average BEP of 10^{-5}, CDF-PL incurs an SNR penalty of 2.4, 1.7, and 1.3 dB compared with CAF-ML for the three fading scenarios. Last, for CDF, the ML detector slightly outperforms the PL detector, although they are fairly close.

For a similar symmetric scenario, Fig. 3 depicts the average BEPs of DAF-I, DAF-II, and DDF obtained via analysis and by simulation. Also included in Fig. 3 is the noncooperative differential BPSK (DBPSK) in Rayleigh fading channels. For DAF-I, simulation results are seen to be in agreement with the analytical results; the average BEP of DAF-II also matches that of DAF-I. Fig. 3 indicates that DDF-PL is slightly worse than DDF-ML. At an average BEP of 10^{-4}, DDF-ML has an advantage of about 0.5 dB compared with DDF-PL when $m_{i,j} = 1$. In contrast to the coherent detection case, DDF slightly outperforms DAF. We see an advantage of about 1 dB of DDF-PL compared with DAF-I/DAF-II.

Last, we consider an asymmetric case with $\bar{\gamma}_{s,d} = 0.25E_b/N_0$ and $\bar{\gamma}_{s,r} = \bar{\gamma}_{r,d} = 0.5E_b/N_0$, which simulates a scenario where R sits between S and D. In addition, we assume nonidentical fading parameters with $m_{s,d} = 1$ and $m_{s,r} = m_{r,d} = 2$ (again, the relay link has a better channel than the direct link). Fig. 4 depicts the average BEP of CAF, CDF, DAF, and DDF, which is obtained by analysis and simulation. We see that the analytical results match the simulation results. It is noted that the differential cooperative schemes have an SNR loss of 2–4.5 dB compared with their coherent counterparts for the case considered.

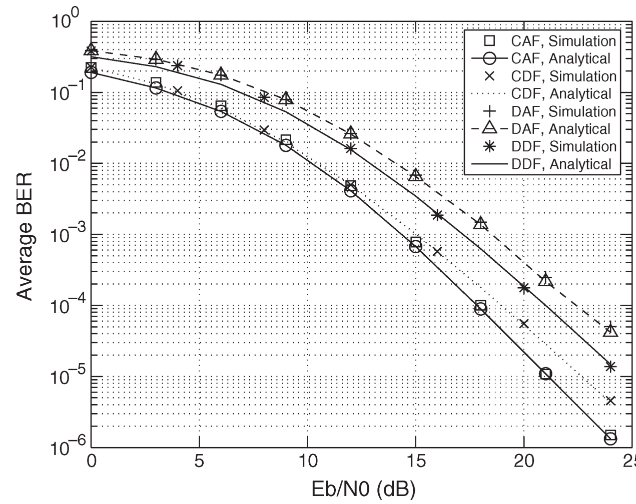

Fig. 4. Average BEP of cooperative coherent and differential modulation in an asymmetric scenario.

VI. Conclusion

In this paper, we have presented a unified BEP analysis for both coherent and differential cooperative modulation schemes using either nonregenerative or regenerative relays. Compared with some earlier studies, our results have been obtained in a broader fading scenario by considering Nakagami-m fading. We have considered only binary modulation in this paper, although some of our results may be extended to higher constellations, e.g., M-ary phase-shift keying (PSK). For the nonregenerative schemes, i.e., CAF and DAF, since the decision variable before thresholding is the same for both binary and M-ary PSK, the symbol error rate (SER) for M-ary PSK can easily be obtained by using our results and modifying the integration region of the decision variable. For the regenerative CDF and DDF, however, the signal that is received at D due to transmission from R has a Gaussian mixture pdf with M instead of two components, and hence, implementations of the ML or PL detector, as well as their SER analysis, are considerably more involved. One possible way to simplify the evaluation is to consider only the nearest-neighbor errors instead of all the possible errors.

APPENDIX A
PDF OF $\gamma_{s,r,d}$ FOR DAF

Let $X = \gamma_{s,r}\gamma_{r,d}$ and $Y = \bar{\gamma}_{s,r} + \gamma_{r,d} + 1$. The pdf of $\gamma_{s,r,d}$ is determined as follows (e.g., [22]):

$$p_{\gamma_{s,r,d}}(\gamma) = \int_0^\infty |\bar{\gamma}_{s,r} + 1 + t| \times p_{X,Y}\left(\gamma(\bar{\gamma}_{s,r} + 1 + t), \bar{\gamma}_{s,r} + 1 + t\right) dt \quad (50)$$

where the joint pdf $p_{X,Y}(x, Y) = p_{X|Y}(x|y)p_Y(y)$. The marginal pdf of Y is given by

$$p_Y(y) = \frac{1}{\Gamma(m_{r,d})}\left(\frac{m_{r,d}}{\bar{\gamma}_{r,d}}\right)^{m_{r,d}}(y - \bar{\gamma}_{s,r} - 1)^{m_{r,d}-1} \times e^{-m_{r,d}(y-\bar{\gamma}_{s,r}-1)/\bar{\gamma}_{r,d}} \quad (51)$$

whereas the conditional pdf of $p_{X|Y}(x|y)$ is given by

$$p_{X|Y}(x|y) = \frac{1}{|y - \bar{\gamma}_{s,r} - 1|\Gamma(m_{s,r})}\left(\frac{m_{s,r}}{\bar{\gamma}_{s,r}}\right)^{m_{s,r}} \times e^{-\frac{m_{s,r}x}{\bar{\gamma}_{s,r}(y-\bar{\gamma}_{s,r}-1)}}\left(\frac{x}{y - \bar{\gamma}_{s,r} - 1}\right)^{m_{s,r}-1}. \quad (52)$$

Substituting (51) and (52) into (50), and after some manipulations, we reach (26).

APPENDIX B
AVERAGE BEP OF DAF

Averaging the conditional BEP across the joint pdf of $\gamma_{s,r,d}$ and $\gamma_{s,d}$, we have

$$\begin{aligned}\bar{P}_e = &\frac{1}{2}\int_0^\infty e^{-x}p_{\gamma_{s,r,d}}(x)dx\int_0^\infty e^{-y}p_{\gamma_{s,d}}(y)dy \\ &+ \frac{1}{8}\int_0^\infty xe^{-x}p_{\gamma_{s,r,d}}(x)dx\int_0^\infty e^{-y}p_{\gamma_{s,d}}(y)dy \\ &+ \frac{1}{8}\int_0^\infty e^{-x}p_{\gamma_{s,r,d}}(x)dx\int_0^\infty ye^{-y}p_{\gamma_{s,d}}(y)dy. \end{aligned}\quad (53)$$

Changing variable $\sqrt{x} = u$ and substituting (27) into the first integration in (53), we have

$$\begin{aligned}\bar{P}_{e1} = &\sum_{k=0}^{m_{s,r}}\frac{4(\bar{\gamma}_{s,r}+1)^k}{\Gamma(m_{s,r})\Gamma(m_{r,d})}\left(\frac{m_{s,r}}{\bar{\gamma}_{s,r}}\right)^{m_{s,r}}\left(\frac{m_{r,d}}{\bar{\gamma}_{r,d}}\right)^{m_{r,d}} \\ &\times\left(\frac{m_{s,r}(1+\bar{\gamma}_{s,r})\bar{\gamma}_{r,d}}{m_{r,d}\bar{\gamma}_{s,r}}\right)^{\frac{m_{r,d}-k}{2}}\binom{m_{s,r}}{k} \\ &\times\int_0^\infty u^{2m_{s,r}+m_{r,d}-k-1}e^{-(m_{s,r}+\bar{\gamma}_{s,r})u^2/\bar{\gamma}_{s,r}} \\ &\times K_{m_{r,d}-k}\left(2u\sqrt{\frac{m_{s,r}m_{r,d}(1+\bar{\gamma}_{s,r})}{\bar{\gamma}_{s,r}\bar{\gamma}_{r,d}}}\right)du. \end{aligned}\quad (54)$$

With the help of [20, eq. (6.631.3)], (54) can be simplified to (29). Similarly, (31) is obtained for the third integration in (53). Substituting (4) into the second and fourth integrations in (53), followed by applying [20, eq. (3.478.1)], yields (30) and (32), respectively.

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