Final Examination, Summer 2010
– *Makeup Exam–*

Name:

- There are 6 problems, for a total of 100 points.

- Before you start, make sure your exam is not missing any page.

- You may do the problems in any order you like.

- You can earn lots of partial credits if you show your work.

- You are allowed three pages of notes (both sides) and a calculator.

- Please verify your answers before handing in the exam. You should have sufficient time to do so.

For instructor's use only

Problem	Points	Score
1	10	
2	20	
3	20	
4	15	
5	20	
6	15	
Total	100	

1. The math department orders plastic utensils only from two different suppliers. A box of utensils from supplier A has 30% forks and 70% knifes, while a box from supplier B has 50% forks and 50% knifes. From the past the math department know it orders 20% of the time from supplier A. We have just received a box without label from one of the suppliers and we do not want to disturb the box very much. So we reach inside and pick one utensil completely at random from the box. It is a knife. Now, find the probability that the box comes from supplier A. Find the probability that the box is from supplier B.

2. I have a group of 9 graduate students. Five are men and four are women. There is an interesting conference coming up but I only have funds to support 6 students participating at the conference.

   (a) Without noticing gender, determine the number of ways of selecting 6 people to attend the conference.

   (b) How many ways are there of selecting 3 men and 3 women to attend a conference?

   (c) I need to select one student to be the head of a certain project and another to keep the books for the funds spent on the project. In how many ways the 2 positions may be filled?

   (d) If two of the students are selected at random, what is the probability that both are men?

3. A 12-in. bar that is clamped at both ends is to be subjected to an increasing amount of stress until it snaps. Let Y be the distance from the left end at which the break occurs. Suppose that Y has the pdf:

$$f(y) = \begin{cases} \frac{1}{24}y\left(1 - \frac{y}{12}\right), & 0 \leq y \leq 12 \\ 0, & \text{otherwise} \end{cases}$$

Compute the following:

(a) The CDF of Y and graph it.

(b) $P(Y \leq 4)$, $P(Y > 6)$ and $P(4 \leq Y \leq 6)$

(c) $E(Y)$, $E(Y^2)$ and $V(Y)$.

(d) The probability that the breakpoint occurs more than 2 in from the expected breaking point.

4. Suppose a randomly chosen individual's verbal score X and quantitative score Y on a nationally administered examination have joint pdf:

$$f(x, y) = \begin{cases} \frac{2}{5}(2x + 3y), & 0 \le x \le 1, 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) You are asked to predict a typical value for an individual's total score $X + Y$ taking this exam. What is your prediction?

(b) Is an individual taking this exam more likely to score better on the verbal or the quantitative part? (i.e., calculate $P(X \ge Y)$ and $P(X \le Y)$)

5. We know that the helium porosity (in percentage) of coal samples taken from any particular seam is normally distributed with known true standard deviation 0.75.

   (a) Compute a 95% confidence interval for the expected porosity if an experiment was conducted and 20 samples were produced with average porosity 4.85.

   (b) Compute a 98% confidence interval based on another sample from another location that yielded 16 observations with a sample average porosity of 4.56.

   (c) How large a sample size is required if the width of 95% confidence interval is to be no greater than 0.40?

(d) Suppose a new sample of 16 observations is produced taken from the Mars expedition and we have no idea if the known value of the standard deviation holds anymore. So we calculate the mean and variance of the 16 observations as 5.12 and 0.49 respectively. Give a 95% confidence interval for the Mars seam.

6. To obtain information on the corrosion-resistance properties of a certain material, 45 specimens are buried in soil for a 2-year period. The maximum penetration (in millimeters) for each specimen is them measured yielding a sample average penetration of 52.7mm. The standard deviation of the measurements is assumed to be known at 4.8mm. The specimens were manufactured with the specification that the true average penetration be at most 50mm. They will be mass produced unless it can be demonstrated conclusively that the specification has not been met. What would you conclude? (Base your conclusion on a confidence interval).