

Final Examination, Summer 2010

Name:

- There are 6 problems, for a total of 100 points.
- Before you start, make sure your exam is not missing any page.
- You may do the problems in any order you like.
- You can earn lots of partial credits if you show your work.
- You are allowed three pages of notes (both sides) and a calculator.
- Please verify your answers before handing in the exam. You should have sufficient time to do so.

For instructor's use only

Problem	Points	Score
1	15	
2	20	
3	10	
4	20	
5	20	
6	15	
Total	100	

1. (From “Ask Marilyn” column of *Parade Magazine*, August 9, 1992). Three of us couples are going to Lava Hot Springs next weekend. We’re staying two nights and we rented two studios, because each holds a maximum of only four people. One couple will get their own studio on Friday, a different couple on Saturday, and one couple will be out of luck. We’ll draw straws to see which are the two lucky couples. I told my wife we should just draw once, and the loser will be the couple out of luck both nights. I figure we’ll have a two-out-of-three ($66\frac{2}{3}\%$) chance of winning one of the two nights to ourselves. But she contends that we should draw straws twice - first on Friday and then, for the remaining two couples only, on Saturday – reasoning that a one-in-three ($33\frac{1}{3}\%$) chance for Friday and a one-in-two (50%) chance for Saturday will give us better odds. Which way should we go?

2. Please answer the following questions. Answers yes or no without justification will earn 0 points.

(a) Stevens library has 800,000 books. The librarians want to use a code word of three letters followed by two numbers. Is this a good idea?

(b) Calculate the number of divisors of 55,125. ($55,125 = 3^2 5^3 7^2$)

(c) In how many arrangements of the letter S T E V E N S are both E adjacent? The sequence EVEN in this order?

(d) A deck of 52 cards is dealt, 13 each, at random among players A, B, C, and D. What is the probability that A and B together get all the 4 aces?

3. Let X , the lifetime of a light bulb, be an exponential random variable with parameter λ . Is it possible that X satisfies the following relation?

$$P(X \leq 2) = 2P(2 < X < 3)$$

If so for what value of λ ?

4. For $\Omega = \{(x, y) \mid 0 < x + y < 1, 0 < x < 1, 0 < y < 1\}$, a certain region in the plane let:

$$f(x, y) = \begin{cases} 3(x + y), & \text{if } (x, y) \in \Omega \\ 0, & \text{otherwise} \end{cases}$$

be the joint probability density function of the variables X and Y . Find the marginal probability density functions of X and Y and $P(X + Y > 1/2)$.

5. A random sample of size 30 is taken from a distribution with probability density function

$$f(x) = \begin{cases} \frac{1}{9} \left(x + \frac{5}{2}\right), & \text{if } 1 < x < 3 \\ 0, & \text{otherwise} \end{cases}$$

Let \bar{X} be the sample mean.

- (a) Calculate $E[X]$ and $V[X]$ for the original variables.
- (b) Approximate $P(2 < \bar{X} < 2.15)$.
- (c) How large a sample size is needed if the width of a 95% confidence interval is to be no greater than 0.1?

6. Commonly named coast redwood, giant redwood and California redwood (*Sequoia sempervirens* - latin always green) is an evergreen, one of the longest and tallest living species of tree on the planet earth. It is one of the most valuable species for timber production. A sample of 15 trees is monitored for 20 years and their diameter at the base is obtained at the end of the period and produces the following numbers (in cm):

20.3	22.5	17.3	18.3	22.1
18.7	21.2	19.6	20	18.5
18.5	21.7	17.9	19.3	22.3

- (a) First, we assume that the diameters are normally distributed. Calculate a 95% confidence interval for the true average diameter of a 20 year old sequoia tree.
- (b) Suppose that the standard deviation is known and is equal to the value you calculated in part (a). Calculate another 95% confidence interval this time using the extra information. How does the confidence interval change?