Math 623 Stochastic Processes

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Objectives

This is a second year graduate class. The students have to have a sufficient understanding of probability concepts to proceed with this class. MA611 Probability or an equivalent class would be enough for this purpose. The students are expected to understand basic techniques and methodologies used for discrete *and* continuous Stochastic processes. Normally, the material in this class would be covered in a year or more. My solution is to give solid definitions, examples and theorem statements while skipping the proofs for most theorems.

Textbook(s):

This semester we will follow:

- *First course in Stochastic Processes*, by Samuel Karlin, 2^{nd} edition, Academic Press, 1975. ISBN: 01239-8552-8

Future reading with a more in depth analysis of the same topics and further ones is the second volume of the textbook:

- *Second Course in Stochastic Processes*, by Samuel Karlin, Academic Press, 1981. ISBN: 01239-8650-8

The course will be supplemented with the following references:
Stochastic processes books:

- *Stochastic Processes*, by Sheldon Ross, 2nd edition, 1975, published by John Wiley and Sons. ISBN: 04711-2062-6
- *Probability with Martingales*, by David Williams, Cambridge University Press 1991
- later chapters of *Fundamentals of Probability with Stochastic Processes*, by Safed Ghahramani, 3rd edition, Academic Press, 2004. ISBN: 01314-5340-8

Probability books:

- *Probability: Theory and Examples*, by Richard Durrett, Thomson Learning 2004.
- *Probability and Measure*, by Patrick Billingsley, Wiley series in probability and mathematical statistics 1995, ISBN: 04710-3173-9.
- *A course in probability theory*, by Kai Lai Chung, Academic Press 2000, ISBN: 01217-4151-6.
- *Probability and Random Processes* by Geoffrey Grimmett and David Stirzaker, Oxford University Press 2001, ISBN: 01314-5340-8.

I will post lecture notes from my forthcoming book on the subject, these will contain the material in a very close form with what I will present in class. I only ask that you write on the printed notes any comments and unclarities you may have. I plan to collect these notes at the end of the semester.

Topics and plan of lectures:

- Probability review. (1 week)
- Basic notions of Stochastic processes, the Bernoulli, Poisson processes. (2 weeks)
- Markov Chains. Classification of states, limit theorems, random walk. (3 weeks)
- Renewal processes. (1 week)
- Continuous time Markov Chains (1 week)

- Martingales (2 weeks)
- Brownian Motion (2 weeks)
- Stationary processes, Branching processes[1]. (2 weeks)

Homework, Exams and Grading:

Proper assignment write-up

To understand the course material and get a good grade it is necessary (though not sufficient) to invest a substantial amount of time working on the assignments. Homework consisting of about 7-8 problems will be assigned in class and posted on the web every other week or so. They will be due on the specified due date at the specified time. *No late homework will be accepted under any circumstances.* The lowest homework grade will be dropped. I will grade two or three problems (selected by me) from each assignment which will count toward 60% of the homework grade, while casually reviewing the other problems for the remaining 40% of the homework grade.

You are encouraged to discuss homework; however, **all written homework must be written by you. Copying solutions from other students in the class, former students, tutors, or any other source is strictly forbidden**. Copying the solution of one or more problems from another source than your own brain is consider academic dishonesty/misconduct and will be dealt with according to the Stevens honor board policy.

Your solutions must be those that you fully understand and can produce again (and solve similar problems) without help. The ideal model to follow is first to work independently, then to discuss **issues** with your fellow students, and then to prepare the final write-up.

There are three stages in the preparation of the solution to a problem in this class:

1. Outline the steps.
2. Identify the mathematical techniques necessary to carry out those steps.
3. Carry out the mathematical techniques correctly.

[1] Time permitting

Comments about the first two steps. It is no surprise that in a mathematical course students spend most time on the final 3rd stage. However, the first two stages are equally important for a successful demonstration of understanding the course concepts. At the beginning of every course the problems are simple enough that the need for the first two stages seem unnecessary but by the end of the class the problems become complicated enough that this will not seem artificial (indeed it will be most helpful).

It is equally important that you do this for the test problems. During a test students have sometime difficulties carrying out all the mathematical analysis to completely solve the problem. If I can see that you know what steps you should be doing, then I can give you more credit than if you just cant carry out the steps and don't tell me anything. Thus, a clearly written plan of your solution method will help you earn a good test grade.

Comments for the third step. As a professional in a quantitative field, you will be expected to be mathematically sophisticated enough to know whether or not you are carrying out a mathematical technique correctly. I expect you to practice that sophistication in all material submitted in this course. For example, do not ever turn in a problem requiring an integration that you did not know how to do completely, so you just did it as far as you could and then wrote the answer you knew it should have, hoping the instructor or grader would not notice that the solution was not complete. Instead, find the help you need to fully carry out the solution correctly before you submit the paper, as you will do in your professional activities.

You must show all of your steps in carrying out the mathematical techniques. Explain what you are doing as if you are teaching it to someone. People who write journal articles often leave out most of the easy steps and just show the hardest steps. That is fine for journal articles, but it is not appropriate for a classroom situation where you need to be convincing the instructor that you understand the reasons for all the steps you are doing.

Exam and Projects policy

We will have one midterm and a final exam. Both exams will be in-class, closed-book, closed-notes. You will be allowed to bring a handwritten page containing whatever you think is relevant for the exam. You will have to show all the work to receive credit for the problems in the exam. The date for the midterm will be agreed on during the semester. The most weight for the final grade will be coming from the final examination.

There will be no individual make up exams. If you miss one of the exams, you may be allowed to take a *comprehensive* make up exam (location and time to be determined) at the end of the semester. To be allowed to take this make up exam you have to bring valid written documentation that explains the reason for the missed exam. The make up exam will replace at most one missing exam grade.

In addition we will have one project in this class split into two parts. For the first part of the project you will have to think about a real life situation involving randomness. This situation should involve something you like or are interested in. Randomness is present in all aspects of life so anything you think of it will probably involve randomness. For the second part you will have to pick a small portion of the problem you proposed and analyze it using tools learned in this class.

I may replace one of the exams with these projects.