Final Examination, Summer 2011

Friday July 8, 2011

Name:

- There are 7 problems, for a total of 100 points.
- Before you start, make sure your exam is not missing any page.
- You may do the problems in any order you like.
- You can earn lots of partial credits if you show your work.
- You are allowed three pages of notes (both sides) and a calculator.
- Please verify your answers before handing in the exam. You should have sufficient time to do so.

For instructor's use only

Problem	Points	Score
1	15	
2	15	
3	20	
4	10	
5	10	
6	15	
7	15	
Total	100	

1. A group of 8 Romanians are traveling to Bulgarian silver sand beaches. Unfortunately, two of them have outdated passports. It is known that customs inspectors check the passports of 25% of the people in any group passing their desks. The group can go as a whole in a van (all eight) or they can split into two cars of four each. How should the members of the group arrange themselves to maximize the probability of getting by the inspectors without having the outdated passports be detected.

2. The percentage of impurities per batch in a certain type of industrial chemical can be modeled by a random variable X with the following probability density function:

$$f(x) = \begin{cases} 2(1-x) & \text{if } 0 \leq x \leq 1 \\ 0 & \text{else} \end{cases}$$

(a) Suppose that a batch with more than 30% impurities cannot be sold. What is the probability that a randomly selected batch cannot be sold because of this reason?

(b) Calculate expected percentage of impurities and the standard deviation of this percentage

(c) Suppose that the value of each batch is given by $V = 500(1 - X)$ dollars. Find the expected value and the variance of V.

(d) What percentage of impurities is exceeded in exactly 50% of the batches.

3. This problem refers to the percentage of impurities distribution introduced in the previous problem. Suppose that we take two such batches X and Y.

   (a) What is the joint density of the two variables?

   (b) What is the probability that the average impurity level of the two batches is less than 20%?

   Hint: To answer this question calculate the distribution of $X+Y$ *by first calculating the joint distribution of* $(X+Y, Y)$ *for example and then calculating the appropriate marginal density. Finally, think how you would express the required probability in terms of the density you just calculated.*

(c) Now suppose we take 50 such batches $X_1, \ldots, X_{50}$ and calculate the average impurity level in these batches call it $\bar{X}$. Give an approximate probability that the average impurity level $\bar{X}$ is less than 20%.

(d) Calculate an interval that will contain this sample average $\bar{X}$ with probability 0.95.

(e) How many such batches $X_1, X_2, \ldots, X_n$ should we take so that the total width of the interval calculated in part (d) is no larger than 0.01?

4. Fill the joint distribution table bellow so that the X and Y variables are independent.

		Y					
		0	5	10	15	25	p(x)
X	1	0.06	0.03	0.09	0.09	0.03	
	2						0.1
	3						0.6
	p(y)						

Then calculate:

$$P(X = 2 | Y \geq 10)$$

5. **Multiple choice problem.** You pay \$5 to play a game which consists of shuffling a deck of cards then drawing a card at random. If one of the 10, J, Q, K, or A appear you fail to win anything. However, you win \$1 for each draw in which one of the mentioned cards do not appear. The game ends the first time one of those cards are drawn. Your expected winnings in this game is:

   (a) E(X-1) where $X \sim$ Bin (52, 5/13)

   (b) E(5-X) where $X \sim$ Geom (5/13)

   (c) E(X-6) where $X \sim$ Geom (5/13)

   (d) E(X-5) where $X \sim$ NegBin (5, 5/13)

   (e) E(1-X) where $X \sim$ NegBin (1, 5/13)

   (f) E(4-X) where $X \sim$ Bin (5, 5/13)

6. Let X_1, X_2, X_n, be independent random variables from the same distribution. In each of the following cases find the constant that the sample mean $\bar{X}$ is likely to be close to if n is large.

   (a) The X's are exponential with density $f(x) = 2e^{-2x}$ for $x > 0$.

   (b) The X's have density $f(x) = \frac{24}{x^4}$ for $x > 2$.

7. In an attempt to determine what proportion of people living in Hoboken have mice in their apartments a survey of 1500 people was taken. Of these 1500 people, it was noted that 666 of them had mice visit their house frequently.

   (a) Construct a 99% confidence interval for the proportion of apartments that have a mice infestation.

   (b) State the conclusion you draw from the confidence interval above in plain English. Do not forget to state the meaning of confidence level.

   (c) Using the confidence interval, conduct a hypothesis test (at a 99% confidence level) to determine whether the proportion of people who have a mice infestation is different from 40%.