Midterm Exam 1, Summer 2010

Name:

- There are 5 problems, each worth 20 points for a total of 100.
- Before you start, make sure your exam is not missing any page.
- You may do the problems in any order you like.
- Be very specific with your random variables and your definitions. Showcase your work.
- You are allowed one page of notes (both sides) and a calculator.

For instructor's use only

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1. My Grandmother has 7 children all grown up now. She invites all of them for the fourth of July. Will they go? Well, each child will go with probability 80%, and they will all decide independently of all others. Let Z be a random variable such that $Z = 1$ if my father (the eldest child) spends the holiday with Grandma, and $Z = 0$ otherwise. Also let X be the number of children that spend fourth of July with Grandma.

   (a) What is distribution of Z? (Name the distribution and give the numerical value of its parameters.)

   (b) What is distribution of X? (Name the distribution and give the numerical value of its parameters.)

   (c) What is the probability Grandma will spend fourth of July alone?

   (d) What is the expected number of her children that will spend the respective holiday with Grandma?

2. In Hoboken parking lots charge \$22 per day and \$6/hour for parking. A car that is illegally parked on the street will be fined \$65 if caught. The probability that a car is caught is 0.6 during any hour and the parking officials only work from 9am to 9pm.

   (a) If I have to stay at Stevens (in Hoboken) for about an hour, should I park illegally on the street or in a parking garage?

   (b) What is the probability that I get caught if I park on the street an entire day (I get to Stevens at 11:00am and leave at 11:00pm)? What is the expected amount of the fine if I leave the car parked for the period?

   (c) Suppose today I park illegally but I have total amnesia and do not remember how long I left the car on the street. I am sure however it is not more than three hours (equally likely either hour). I come back and I find a ticket on the car. Given this fact, calculate the probabilities that the car was parked illegally 1, 2 or 3 hours.

3. Every year, each person in Hoboken gets a Poisson number of colds with mean equal to 2 colds per year. Suppose that different people are independent as far as colds are concerned.

   (a) What is the probability that Prof. Florescu will get exactly one cold next year?

   (b) What is the probability that he will get at least one cold next year?

   (c) What is the probability that nobody in the Florescu family gets any cold (he is married and has a boy)?

   (d) What is the expected number of colds within the Florescu family in a one-year period?

4. Tomorrow, I have to interview 4 candidates A, B, C, D for a research assistantship. To save time they will all be interviewed at the same time (10:30am) and be given a different test each in a separate room. So I assign rooms 1,2,3,4 respectively to each of the candidates. Our secretary without my knowledge assigns one of the four rooms at random to each candidate and leads the candidate to the respective room.

   (a) What is the probability that all candidates get to the respective room I assigned?

   (b) What is the probability that none of the candidates reaches the room I assigned?

   (c) Given that candidates A and B get to the assigned rooms 1 and 2 what is the probability that candidates C, D will be assigned rooms 3 respectively 4?

5. A continuous random variable has probability density function:

$$f(x) = \begin{cases} k(x+2) & \text{if } 2 \leq x \leq 4 \\ 0 & \text{else} \end{cases}$$

(a) Find k which makes the function above a true probability density function.

(b) Find the CDF of the random variable

(c) Find the probability that the random variable is less than 3.

(d) Find the expected value and variance of the random variable