

Midterm Exam 1, Summer 2010

Thursday, June 2, 2011

Name:

- There are 5 problems, worth a total of 100 points.
- Before you start, make sure your exam is not missing any pages.
- You may do the problems in any order you like.
- Be very specific with your random variables and your definitions. Showcase your work.
- You are allowed one page of notes (both sides) and a calculator.

For instructor's use only

Problem	Points	Score
1	15	
2	20	
3	30	
4	15	
5	20	
Total	100	

1. Every morning my alarm clock sounds exactly at 6:45, I then take a shower and leave for the subway. This takes anywhere between 15 and 20 minutes. Next, I take the train to Hoboken which takes between 30 and 40 minutes. Along the way to school I buy coffee from Starbucks, Kings or Dunkin' Donuts and this walk (including coffee buying) lasts between 10 and 15 minutes.

(a) I am interested in the time at which I arrive at school and the type of coffee I end up with. Write a sample space to describe the possible outcomes.

(b) While walking to school I smoke between 2 and 7 cigarettes. Write a sample space which describes the number of cigarettes smoked and the type of coffee bought. How many elements are in the sample space?

(c) If smoking any number of cigarettes is equally likely, what is the probability that I smoke more than 5 cigarettes and end up with a Starbucks coffee?

2. About 8.3% of the adult population in the US has diabetes. It is known that diabetes may be inherited. Suppose we know that if one of the parents (regardless of gender) has diabetes, the chance that the child develops diabetes up to the age of 30 is 20%. If both parents have diabetes the chance increases to 40%. If neither parent has diabetes, the chance of the child developing diabetes is 4%.

(a) What is the probability that neither one of my parents has diabetes? What is the probability that only my dad has diabetes? What is the probability that both have diabetes?

(b) Suppose I am an only child and *I just discovered that I have the disease*. I have no idea whether my dad or mom have diabetes. Obviously I have to tell them to test themselves but right now I can calculate the probability that neither one of my parents have diabetes. What is this probability? Also calculate the probability that at least one has the disease.

3. The hard drive in my computer keeps breaking. I have investigated the quality control process at the manufacturer's company and I discovered the testing procedure. Drives are packaged in bulk boxes containing exactly 25 drives. The quality control procedure is performed three times in succession by three different inspectors. For each procedure a drive is selected at random from the box and tested and then placed back into the box. If a defective drive is found in one of the tests, the entire box is sent back immediately and the batch is rejected. The box is shipped only if the inspectors found nothing wrong with the drives analyzed.

(a) Find the probability that a box containing 5 defective items passes the inspection.

(b) What is the probability that a box with one bad hard drive is rejected?

Now suppose that the company changes its policy by having only one inspector selecting a sample of 3 drives randomly from the box and tested instead of the three previous procedures. If any defective item is found, the entire box is rejected.

- (c) Find the probability that a box containing 5 defective items passes the inspection in this case.

- (d) What is the probability that a box with one bad hard drive is rejected?

- (e) Which quality control procedure would you prefer as the consumer? Justify your answer.

4. It is known that the hard drives produced by a certain manufacturer are defective in a proportion of 1 in 100 drives. Suppose I test every drive coming out from the assembly line in order and suppose each drive is independent of all others.

(a) What is the probability that the fifth item I test is the first defective one I find?

(b) What is the probability that the 10th drive tested is also the third defective I find?

(c) Suppose I take a sample of 10 drives. What is the probability that this sample contains exactly 3 defective ones?

5. Considering the situation outlined in the previous problem, define the following random variables:
X = the number of hard drives sampled until the first defective one is discovered
Y = the number of hard drives sampled until the third defective one is discovered
Z = the number of defective drives in a sample of 10 taken from the line

(a) Find the expected value and variance of X, Y, and Z.

(b) Suppose that for every defective item found in a sample of ten, the company pays an extra \$10 per item. How much does the company expect to pay for the defective units in a sample of ten? A sample of 100?