

Midterm Exam 2, Summer 2011

Thursday June 16, 2011

Name:

- Please write neatly and clearly
- Each bonus is worth 10 points.
- All answers should include the necessary work in order to receive credit.
- The last page contains a normal distribution table.

For instructor's use only

Problem	Points	Score
1	45	
2	35	
3	20	
bonus 1	10	
bonus 2	10	
Total	100	

1. A mathematician spends a lot of time at a certain bar. Over time he notices that whenever a glass of whiskey is ordered, the amount in the glass is approximately normally distributed with a mean of 5 oz. and a standard deviation of .25 oz.. I sit down next to my friend the mathematician and order a whiskey.

(a) Calculate the probability that my glass of whiskey is less than 5.5 oz.

(b) What is the size of the smallest glass that can contain 99 percent of all pours?

(c) Assuming that each pour is independent, what is the probability that the next 3 glasses contain more than 5.5 oz.?

My friend the mathematician turns that analytic mind over to my drinking habits and proclaims that the number of glasses I drink an hour could be modeled using a Poisson random variable with mean of 3 drinks per hour.

(d) What is the probability that I don't have a drink for the next half hour?

(e) Assuming that I haven't had a drink for the past two hours, what is the probability that I don't have a drink for the next half hour?

Bonus 1. How many oz. of whiskey should I expect to drink in the next two hours if my friend the mathematician is to be believed?

2. The random variables X , Y , and Z have a joint pdf of

$$f(x, y, z) = \begin{cases} kxy^2z & \text{if } 0 \leq x, y \leq 1, 0 \leq z \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find k so that f is a proper density

(b) Are X , Y , and Z statistically independent?

(c) Find $P(X \leq 1/2, Y \geq 2/3, 1 \leq Z \leq 2)$?

(d) Find the covariance of X and Y .

3. Let's say the bar from problem 1 has exactly 400 drink orders in a night. Assume that each order is independent and the probability of any order being for whiskey is 0.2. Assume that each glass contains exactly 5 oz. of whiskey in it and that the whiskey comes in a 32 oz. bottle. The bar has ordered and received 15 bottles of whiskey in order to try and satisfy the demand. What is the probability that they run out of whiskey that night?

Bonus 2. The probability that a person orders absinthe is 0.01. Find the probability that no more than 3 orders for this drink are made that night.