Assignment 7

11. Let $N(t)$ be the number of shooting stars observed up to time t. Let one minute be the unit of time. Then $\{N(t) : t \geq 0\}$ is a Poisson process with $\lambda = 1/12$. We have that

$$P(N(30) = 3) = \frac{e^{-30/12}(30/12)^3}{3!} = 0.21.$$

13. Let $N(t)$ be the number of wrong calls up to t. If one day is taken as the time unit, it is reasonable to assume that $\{N(t) : t \geq 0\}$ is a Poisson process with $\lambda = 1/7$. By the independent increment property and stationarity, the desired probability is

$$P(N(1) = 0) = e^{-(1/7)\cdot 1} = 0.87.$$

4. Let X be the time between the first and second heart attacks. We are given that $P(X \leq 5) = 1/2$. Since exponential is memoryless, the probability that a person who had one heart attack five years ago will not have another one during the next five years is still $P(X > 5)$ which is $1 - P(X \leq 5) = 1/2$.

8. The number of documents typed by the secretary on a given eight-hour working day is Poisson with parameter $\lambda = 8$. So the answer is

$$\sum_{i=12}^{\infty} \frac{e^{-8}8^i}{i!} = 1 - \sum_{i=0}^{11} \frac{e^{-8}8^i}{i!} = 1 - 0.888 = 0.112.$$

9. The answer is

$$E\big[350 - 40N(12)\big] = 350 - 40\Big(\frac{1}{18} \cdot 12\Big) = 323.33.$$

1. Yes, it is a probability density function of a beta random variable with parameters $\alpha = 2$ and $\beta = 3$. Note that $\dfrac{1}{B(2,3)} = \dfrac{4!}{1!\,2!} = 12$. We have

$$E(X) = \frac{2}{5}, \qquad \text{Var}X = \frac{6}{6(5^2)} = \frac{1}{25}.$$

3. Let $\alpha = 5$ and $\beta = 6$. Then f is the probability density function of a beta random variable with parameters 5 and 6 for

$$c = \frac{1}{B(5,6)} = \frac{10!}{4!\,5!} = 1260.$$

For this value of c,

$$E(X) = \frac{5}{11}, \qquad \text{Var}X = \frac{30}{12(11^2)} = \frac{5}{242}.$$

11. Let X be the grade of a randomly selected student.

$$P(X \geq 90) = P\Big(Z \geq \frac{90-72}{7}\Big) = 1 - \Phi(2.57) = 0.0051.$$

Similarly,

$$\begin{aligned}
P(80 \leq X < 90) &= P(1.14 \leq Z < 2.57) = 0.122,\\
P(70 \leq X < 80) &= P(-0.29 \leq Z < 1.14) = 0.487,\\
P(60 \leq X < 70) &= P(-1.71 \leq Z < -0.29) = 0.3423,\\
P(X < 60) &= P(Z < -1.71) = 0.0436.
\end{aligned}$$

Therefore, approximately 0.51% will get A, 12.2% will get B, 48.7% will get C, 34.23% D, and 4.36% F.